Inter-Hierarchy Comparison in HCLP[1]

Roman Barták

Department of Theoretical Computer Science
Charles University
Malostranské náměstí 2/25
Praha 1, Czech Republic

e-mail: bartak@kti.mff.cuni.cz
URL: http://kti.ms.mff.cuni.cz/~bartak/
phone: +420-2 2191 4242
fax: +420-2 2191 4323

Abstract

Constraint hierarchies have been proposed to describe over-constrained systems of constraints by specifying constraints with hierarchical preferences, i.e., hard and soft constraints. While the hard (required) constraints must hold, the soft (preferential) constraints should be satisfied as much as possible depending on the criterion used. Currently, constraint hierarchies are mostly applied to the areas of graphical user interfaces and geometric layout, but the area of usage is much more wider.

The integration of constraint hierarchies with Constraint Logic Programming (CLP) is called Hierarchical Constraint Logic Programming (HCLP). In the original definition of HCLP, only alternate solutions to one constraint hierarchy are compared and the best solutions are returned. The later extension of HCLP also supports comparison of solutions to more constraint hierarchies arising from different choices of rules in HCLP program. This extension, called inter-hierarchy comparison, extends usefulness of HCLP programs by eliminating non-intuitive solutions. However, at the same time, it introduces nonmonotonic behaviour in HCLP programs which creates novel implementation problems.

In this paper we present an algorithm for efficient solving of constraint hierarchies using inter-hierarchy comparison within HCLP. The efficiency of the proposed algorithm is based on tight integration with the HCLP interpreter and on elimination of useless branches of computation. As the algorithm makes an extension of hierarchical constraint solvers which support global comparators, its efficiency can be further improved by harmonizing with underlying hierarchical constraint solver. We present such an extension that uses properties of our generalized approach to solving constraint hierarchies.

Keywords: constraint hierarchies, HCLP, inter-hierarchy comparison

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1 Introduction

Constraint hierarchies were introduced for describing over-constrained systems of constraints by specifying constraints with hierarchical strengths or preferences[2]. It allows one to specify declaratively not only the constraints that are required to hold, but also weaker, so called soft constraints at an arbitrary but finite number of strengths. Weakening the strength of constraints helps to find a solution of previously over-constrained system of constraints. Intuitively, the hierarchy does not permit to the weakest constraints to influence the result. Moreover, constraint hierarchies allow “relaxing” of constraints with the same strength by applying, e.g., weighted-sum, least-squares or similar comparators.

This constraint hierarchy scheme can be parameterized by a comparator *C* that allows one to compare different possible solutions to a single hierarchy and to select the best ones. Currently, there are three widely used groups of comparators, namely locally-better, regionally-better and globally-better comparators. While the locally-better comparators consider each constraint individually, the globally-better comparators combine errors of all constraints at a given level using some combining function. Thus, the globally-better comparators can be used for inter-hierarchy comparison [13], i.e., comparison of solutions to two or more constraint hierarchies. For a regionally-better comparator, each constraint at a given level is considered individually (as with a local comparator), but, unlike a local comparator, two solutions that are incomparable at strong levels may still be compared at weaker levels.

The constraint hierarchies are used in Hierarchical Constraint Logic Programming (HCLP) which is an extension of Constraint Logic Programming (CLP). In the original definition of HCLP [4,14] only alternate solutions to one constraint hierarchy are compared and the best solutions are returned. The later extension of HCLP [13] also enables comparison of solutions to more constraint hierarchies arising from different choices of rules in HCLP program. Such comparison, called inter-hierarchy comparison (opposite to more common intra-hierarchy comparison), rules out non-intuitive solutions and, in our opinion, it is closer to declarative character of HCLP programs. Then, the HCLP programs with inter-hierarchy comparison can be used as rule-based expert systems [2]. At the same time, the inter-hierarchy comparison introduce nonmonotonic behaviour in HCLP programs that creates novel implementation problems.

In this paper we present an efficient algorithm for solving constraint hierarchies using inter-hierarchy comparison. This algorithm is intended to be used within the HCLP framework and, thus, it can take advantage of tight integration with the HCLP interpreter. Opposite to CLP/HCLP approach that does not change the operational behaviour of logic programs, i.e., usage of depth-first search method, the proposed algorithm utilizes breadth-first search approach to solving HCLP programs. HCLP interpreter supporting inter-hierarchy comparison has to explore the whole computation tree of the program to collect all possible constraint hierarchies arising from different choices of rules in HCLP program. We propose to use breadth-first search to explore the computation tree combining with the evaluation of partial solutions. This enables us to eliminate useless branches (subtrees) of the computation tree and, thus, to dramatically improve efficiency of the algorithm. This approach is more efficient than other approaches which have to collect all constraint hierarchies first. Additionally, the proposed algorithm makes an extension of hierarchical constraint solvers which support global comparators. Therefore, the efficiency of the algorithm can be further improved by harmonizing with the underlying hierarchical constraint solver.

The paper is organized as follows. In Section 2, we give a practical motivation for studying hierarchical constraint solvers supporting inter-hierarchy comparison. This section is of a particular interest for a "practical application" audience as it emphasis on the methodological benefits of using constraint hierarchies and inter-hierarchy comparison. In Section 3, we give a

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[2] The other methods for describing over-constrained systems include the "partial CSP" model by Freuder & Wallace, the "fuzzy CSP" model by Fargier, and the "probabilistic CSP" model by Fargier & Lang.

background of our research and we outline the theory of constraint hierarchies and inter-hierarchy comparison. In Section 4, we outline the methodology and theoretical foundation of our approach. In Section 5, we describe the skeleton of proposed algorithm for solving constraint hierarchies using inter-hierarchy comparison. We concentrate on its integration with HCLP interpreter there. In Section 6, we give an example that illustrates the work of the proposed algorithm. Section 7 is dedicated to further improvements of the proposed algorithm via harmonizing with underlying hierarchical constraint solver. We sketch our generalized approach to solving constraint hierarchies there and then we describe how one may utilize its properties to improve efficiency of inter-hierarchy comparison. We conclude with some final remarks and description of future research.

2 Motivation and benefits

Although the concept of inter-hierarchy comparison is not new at all [13], it is used scarce in practical applications if at all. Even the original constraint hierarchies with intra-hierarchy comparison are used sparingly, mostly in areas of graphical user interfaces, geometric layout and document formatting. It sounds surprisingly because of the nice declarative description of over-constrained systems by constraint hierarchies.

The problem is probably hidden in hierarchical constraint solvers which utilize different concepts than traditional constraint satisfaction. Additionally, most current hierarchical constraint solvers are constructed for locally-better comparators only [8,9,11,12], and thus they are not suitable for inter-hierarchy comparison. Also, if the constraint solvers with intra-hierarchy comparison are computational-hungry, the solvers supporting inter-hierarchy comparison are even more demanding for computational power.

Despite the current spare usage of constraint hierarchies we believe that many practical applications, particularly those with complicated relations among variables, can profit from constraint hierarchies and inter-hierarchy comparison. The potential benefit of constraint hierarchies and inter-hierarchy comparison is especially visible in the area of modeling, i.e., in expressing a real world problem in terms of constraints. Constraints labeled by strength can naturally and declaratively describe user's preferences and vision of importance of individual constraints. In addition, the possibility to utilize inter-hierarchy comparison extends further the expressive power of constraint hierarchies because the inter-hierarchy comparison supports naturally the disjunctive constraints. Actually, the inter-hierarchy comparison is more powerful than simple disjunctive constraints as following example demonstrates.

The following program [13] shows the advantages of constraint hierarchies and inter-hierarchy comparison in the area of scheduling. We utilize the HCLP (Hierarchical Constraint Logic Programming) framework here as it perfectly incorporates both constraint hierarchies and inter-hierarchy comparison (via different choices of rules). The objective of the program is to find time for meeting of two people. Notice the natural representation of the problem using labeled constraints and alternative rules.

```
can_meet(Person1,Person2,Day,Time,Length):-
     free(Person1,Day,StartTime1,EndTime1),
     free(Person2,Day,StartTime2,EndTime2),
     required StartTime1≤Time, required StartTime2≤Time,
     required Time+Length≤EndTime1, required Time+Length≤EndTime2.

free(judy,saturday,11,12).
free(judy,friday,13,14).
free(chris,saturday,10,13).
free(chris,friday,T,16):-
     required T≥12, prefer T≥14.
```

The other benefit of constraint hierarchies is the possibility to use very weak constraints to establish stability of solution that is crucial for many up-to-date applications of constraint technology. Currently, this feature is widely used in the construction of graphical user

interfaces [11,12]. By stability of the solution we mean "as small as possible difference" between the original solution and the tuned solution obtained by solving the system of constraints.

Constraint hierarchies and inter-hierarchy comparison can be succesfully applied to traditional applications, like geometric layout, physical simulations, user interface design, document formatting, design and analysis of mechanical devices, planning and scheduling, where constraints settle in. But this approach also opens some new areas of usage. In [2] we proposed to use HCLP with inter-hierarchy comparison to write rule-based expert systems where constraint hierarchies can naturally handle uncertain information. In such systems, the existence of algorithms supporting inter-hierarchy comparison becomes crucial.

3 Constraint hierarchies and inter-hierarchy comparison

The theory of constraint hierarchies was developed in [4]. It is based on the idea of labeling each constraint by preference that determines the strength of the constraint. The constraint hierarchy is then a finite set of labeled constraints. Intuitively, the stronger a constraint is, the more it influences the solution of the hierarchy. The solution of the constraint hierarchy H can be formally expressed in a following way:

$$S_{H,0} = \{\ \theta \mid \forall c \in H_0\ \ c\theta \text{ holds}\ \}$$
$$S_H = \{\ \theta \mid \theta \in S_0\ \&\ \forall \sigma \in S_0\ \neg\ \text{better}(\sigma,\theta,H)\ \},$$

where θ, σ are valuations of free variables in constraints from H, H_0 is a vector of required constraints in H (in some arbitrary order, with their labels removed) and *better* is a relation comparing two valuations according to the hierarchy H. The set $S_{H,0}$ contains valuations satisfying required constraints and S_H is a subset of $S_{H,0}$ containing all valuations from $S_{H,0}$ which are not worse than any other valuation from $S_{H,0}$. The set S_H is called a solution set for the hierarchy H.

There is a number of reasonable candidates for the predicate *better* which is called a *comparator* and it is a parameter to the constraint hierarchy scheme. Note that comparators should respect the hierarchy, i.e., the stronger constraints influence the solution more than the weaker constraints. Currently, three groups of comparators are used, namely locally-better, regionally-better and globally-better comparators (these notions will be explained below). The choice of comparator in a particular application depends on the way of handling constraints. If each constraint is considered individually then the locally-better or regionally-better comparators are used. If the set of constraints at a level is manipulated at once then the globally-better comparator is the choice.

The *locally-better* comparators consider each constraint individually. Thus, the valuation θ is locally-better than the valuation σ if the errors of all constraints till some level k-1 are equal after applying the respective valuations θ and σ, and the error of some constraint from level k is strictly less for the valuation θ than the error for the valuation σ and the errors of other constraints from level k are not greater for the valuation θ than for the valuation σ. The locally-better comparators are formally defined by the following way:

$$\textit{locally-better}(\theta,\sigma,H) \equiv_{\text{def}}$$
$$\exists k>0\ \forall i \in \{1,\ldots,k-1\}\ \forall c \in H_i\ \ e(c,\theta)=e(c,\sigma)\ \&$$
$$\forall c \in H_k\ e(c,\theta) \le e(c,\sigma)\ \&\ \exists c' \in H_k\ e(c',\theta)<e(c',\sigma),$$

where $e(c,\theta)$ is an *error function* that returns a non-negative real number indicating how closely a constraint *c* is satisfied for a valuation θ and H_i are levels of the hierarchy H defined in an obvious way, i.e., H_1 is a vector of strongest non-required constraints in H etc.

Because many valuations are not comparable using the locally-better comparator, the *regionally-better* comparator was introduced to extend the idea of the locally-better comparator. The regionally-better comparator uses a level-better comparator (will be defined below) which compares two valuations according to the set of equally prefered constraints (i.e., the level). The valuation θ is regionally-better than the valuation σ if both valuations are incomparable at levels 1,…,k-1 and the valuation θ is level-better than the valuation σ at some level k.

regionally-better$(\theta,\sigma,H) \equiv_{def}$

$\exists k>0\ \forall i\in\{1,\ldots,k-1\}\ \neg$ (*level-better*(θ,σ,H_i) & *level-better*(σ,θ,H_i)) &

level-better(θ,σ,H_k),

where

level-better$(\theta,\sigma,H_i) \equiv_{def}$

$\forall c\in H_i\ e(c,\theta)\leq e(c,\sigma)$ & $\exists c'\in H_i\ e(c',\theta)<e(c',\sigma)$.

By introduction of the level-better relation, the regionally-better comparator opens doors to a *globally-better* comparator that combines errors of all the constraints at given level H_i using a combining function *g*, and then it compares the combined errors. The scheme of globally-better comparator is defined as follows:

globally-better$(\theta,\sigma,H,g) \equiv_{def}$

$\exists k>0\ \forall i\in\{1,\ldots,k-1\}\ g(\theta,H_i)=g(\sigma,H_i)$ &

$g(\theta,H_k)<g(\sigma,H_k)$.

Note that the scheme of globally-better comparator is parameterized by the combining function *g* that combines errors of individual constraints via, e.g., weighted sum ($g(\theta,H_i)=\sum_{c\in H_i} w_c * e(c,\theta)$), worst case ($g(\theta,H_i)=\max\{w_c * e(c,\theta) | c\in H_i\}$) or least squares ($g(\theta,H_i)=\sum_{c\in H_i} w_c * e(c,\theta)^2$) methods. To justify soundness of our algorithm for inter-hierarchy comparison we require the combining function *g* to satisfy the *monotony* property that guarantees that the combined error does not decrease by adding a constraint to the hierarchy. The monotony property of the combining function *g* is formally defined by the following way:

$\forall H,c,\theta\ \forall i>0\ \ g(\theta,(H\cup\{c\})_i)\geq g(\theta,H_i)$, *(monotony property)*

where θ is a valuation, H is a constraint hierarchy and c is a labeled constraint ($(H\cup\{c\})_i$ is i-th level of the hierarchy ($H\cup\{c\}$)). Note also that the above examples of combining function satisfy the monotony property.

Constraint hierarchies extend the Constraint Logic Programming CLP(R) scheme parameterized by the domain R of constraints into a more general scheme Hierarchical Constraint Logic Programming HCLP(R,C) parameterized by the domain R and the comparator C. These languages provide both required constraints and default constraints of various strengths. The following program [13] is an example of HCLP program.

```
f(X):-strong X>3, g(X).
g(5).
g(1).
```

Given the goal `?-f(A)`, HCLP would first return the answer `A=5,` and on backtracking `A=1`. In reality, there are produced two constraint hierarchies arising from different choices of the rule `g`. One is hierarchy `required A=5, strong A>3` and the other is `required A=1, strong A>3`. The above definition of constraint hierarchies does not prefer neither the first nor the second answer as the definition allows intra-hierarchy comparison only.

To rule out unintuitive solutions described above, the extended definition of constraint hierarchies with inter-hierarchy comparison was proposed in [13]. It is a definition of the solution set for the set of constraint hierarchies Δ. Note that each valuation in sets $S_{\Delta,0}$, S_Δ is labeled by its respective constraint hierarchy.

$$S_{\Delta,0} = \{\ \theta_H \mid H \in \Delta\ \forall c \in H_0\ \ c\theta \text{ holds}\ \}$$

$$S_\Delta = \{\ \theta_H \mid \theta_H \in S_0\ \&\ \forall \sigma_J \in S_0\ \neg\ \text{better}(\sigma_J,\theta_H,\Delta)\ \}.$$

First, the valuations satisfying all required constraints of a hierarchy from Δ are collected and, then, those valuations satisfying best their respective hierarchies are selected. Because the locally-better and regionally-better comparators consider each constraint in the hierarchy individually and the comparator *better* compares valuations arising from more hierarchies now, neither the locally-better nor the regionally-better comparators are redefined to compare solutions to different hierarchies[3]. However, the definition of globally-better scheme can be naturally extended to compare valuations arising from different hierarchies.

$$\textit{globally-better}(\theta_H,\sigma_J,\Delta,g) \equiv_{def}$$
$$\exists k>0\ \forall i \in \{1,\ldots,k-1\}\ \ g(\theta_H,H_i)=g(\sigma_J,J_i)\ \ \&$$
$$g(\theta_H,H_k)<g(\sigma_J,J_k).$$

Again, the globally-better scheme is parameterized by some combining function *g*. Note that the combining function *g* is applied to the valuation and its respective constraint hierarchy now.

4 Theoretical basis

Original HCLP systems with intra-hierarchy comparison does not change the operational behaviour of CLP or Prolog respectively, i.e., they use the depth-first search to explore the computation tree. Consequently, the HCLP system can be built over the underlying CLP system. The approach of HCLP with inter-hierarchy comparison is a bit different as it requires to collect all possible constraint hierarchies arising from different choices of rules in HCLP program.

The algorithm for inter-hierarchy comparison presented in [6] uses the underlying HCLP system to collect constraint hierarchies and then it solves the resulting set of constraint hierarchies. The advantage of this approach is that it does not require to change the operational behaviour of HCLP, i.e., the depth-first search. Furthermore, this algorithm is not confined to HCLP framework, i.e., to the way of collecting constraint hierarchies, and thus it could be used in all applications where inter-hierarchy comparison is requested. However, we see also the disadvantage of this free binding and we assume that a closer relation between HCLP and the hierarchical constraint solver with inter-hierarchy comparison can improve dramatically the efficiency of the system. The reason of this hypothesis is the similarity of constraint hierarchies arising from different choices of rules in HCLP program. Consequently, such hierarchies, or at least their parts, can be solved in common. Additionally, the tight integration of HCLP with inter-hierarchy comparison can remove some infinite computations which lock the above approach (see Section 6).

We propose to utilize breadth-first search in HCLP with inter-hierarchy comparison. As we need to explore the whole computational tree, the computational complexity of breadth-first search is similar to the computational complexity of depth-first search. We understand that in general the memory consumptions of both methods are different, but again, all hierarchies appearing during the computation are collected and thus, the final memory consumption of both methods does not differ significantly. The advantage of breadth-first search is that it collects all

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[3] There exists a definition of constraint hierarchies which allows locally-better comparators in inter-hierarchy comparison.

finite constraint hierarchies even if infinite branches appear in the computational tree. The same does not hold for depth-first search.

As we discussed in the above paragraph, the elementary switch from depth-first to breadth-first search does not change the efficiency of the HCLP system, however it enables further improvements. Now, we show how to integrate the process of collecting constraint hierarchies with constraint hierarchical solver. In the following, we expect the existence of hierarchical constraint solver supporting globally-better comparators (for such systems look at [1,3,5,10]) and we extend this solver to support inter-hierarchy comparison.

The idea behind our algorithm uses the following observations. One can assign the error vector $EV_{\theta,H}=[g(\theta,H_1), g(\theta,H_2)\ldots]$ to each valuation θ of variables from the constraint hierarchy H (g is a combining function). Obviously, it is possible to order all error vectors lexicographically and to show that this ordering is a total ordering. The following example shows the way the error vector is obtained for a given constraint hierarchy and a given valuation.

Example:

combining function: (unsatisfied-count-better comparator)

$$g(\theta, H_i) = \sum_{c \in H_i} e(c,\theta) \quad \text{where } e(c,\theta) = \begin{cases} 0, \text{ if } c\theta \text{ holds} \\ 1, \text{ otherwise} \end{cases}$$

constraint hierarchy:

```
strong a+b=c, prefer a≥2, prefer c=3, weak a=3, weak b=1
```

error vector:

valuation	**error vector** [strong,prefer,weak]
{a/1,b/1,c/3}	[1,1,1]
{a/2,b/1,c/3}	[0,0,1]

Now, we can formulate propositions which justify soundness of the proposed algorithm for solving constraint hierarchies with inter-hierarchy comparison. In particular, these propositions provide methodology for efficient solving of constraint hierarchies by means of error vectors.

Proposition 1:

The solution set S_H of the hierarchy H contains only those valuations from the set $S_{H,0}$ which have the minimal error vector. Moreover, the error vectors of valuations from the solution set S_H are equal.

The proof of Proposition 1 follows directly from the definition of the solution set of hierarchy and the definition of the globally-better schema ($better(\sigma,\theta,H) \Leftrightarrow EV_{\sigma,H}<EV_{\theta,H}$).

Proposition 1 relates the notion of the solution set with the error vectors and it provides the correspondence between the comparator and the ordering of error vectors. As a consequence of Proposition 1 we can assign the error vector $EV_H=EV_{\theta,H}$, where θ is any valuation belonging to the solution set S_H of the hierarchy H, to each constraint hierarchy H. This error vector EV_H describes how nearly the constraints from the hierarchy H can be satisfied.

Proposition 2:

Let EV_H be the error vector of the hierarchy H and $EV_{H\cup\{c\}}$ be the error vector of the extended hierarchy $H\cup\{c\}$ (c is arbitrary constraint added to the hierarchy H). Then $EV_H \leq EV_{H\cup\{c\}}$ in the lexicographic ordering of error vectors.

Proof:

The idea of the proof is to show that $EV_H > EV_{H\cup\{c\}}$ evokes conflict and, thus, $EV_H \leq EV_{H\cup\{c\}}$ holds.

Clearly, $\exists\theta \ EV_{H\cup\{c\}}=EV_{\theta,H\cup\{c\}}$. Now, it is easy to show that $EV_{\theta,H\cup\{c\}} \geq EV_{\theta,H}$:

Let $(H\cup\{c\})_i$ denotes i-th level of the hierarchy $H\cup\{c\}$ and *k* be the number indicating the preference (level) of the added constraint c. Then:

1) $\forall i\neq k \;\; (H\cup\{c\})_i=H_i$, i.e., $\forall i\neq k \;\; g(\theta,(H\cup\{c\})_i)=g(\theta,H_i)$

2) $g(\theta,(H\cup\{c\})_k)\geq g(\theta,H_k)$ (monotony of the combining function).

Consequently,

$[g(\theta,(H\cup\{c\})_1),\ldots,g(\theta,(H\cup\{c\})_k),\ldots] \geq [g(\theta,H_1),\ldots,g(\theta,H_k),\ldots]$, i.e.,

$EV_{\theta,H\cup\{c\}} \geq EV_{\theta,H}$

Now,

$(\exists\theta\ EV_H>EV_{H\cup\{c\}} \;\&\; EV_{H\cup\{c\}}=EV_{\theta,H\cup\{c\}} \;\&\; EV_{\theta,H\cup\{c\}}\geq EV_{\theta,H}) \Rightarrow \exists\theta\ EV_H>EV_{\theta,H}$

but $\exists\theta\ EV_H>EV_{\theta,H}$ is in conflict with the definition of EV_H. Thus $EV_H\leq EV_{H\cup\{c\}}$ holds.

⊗

Proposition 2 says that adding arbitrary constraint to the hierarchy does not "improve" the error vector of the hierarchy. This feature of error vectors can be exploited by incremental algorithms for solving constraint hierarchies. Now, we can naturally extend Proposition 2 to the inter-hierarchy comparison.

Proposition 3:

Let EV_H be the error vector of a hierarchy H and EV_J be the error vector of a hierarchy J. If $EV_H<EV_J$ in lexicographic ordering of error vectors, then $EV_H<EV_{J\cup\{c\}}$ for arbitrary constraint *c*. Consequently $S_{\{H\}}=S_{\{H,J\}} \Rightarrow S_{\{H\}}=S_{\{H,J\cup\{c\}\}}$ (note that valuations are labeled by the corresponding hierarchy and thus $\theta_H\neq\theta_J$).

Proposition 3 is a direct consequence of Proposition 2 because of $EV_H<EV_J \;\&\; EV_J\leq EV_{J\cup\{c\}} \Rightarrow EV_H<EV_{J\cup\{c\}}$. Proposition 3 enables one to eliminate some partial constraint hierarchies, i.e., branches (subtrees) of the computational tree if any complete constraint hierarchy is found during the breadth-first search (see next section). In some cases, it even enables elimination of an infinite branch of the computation tree (see example in Section 6). However, we should note that the algorithm proposed in the next section do not guarantee to eliminate all the infinite branches because of obvious reason (the halting problem, i.e., the discovery of all infinite branches of the computation, is not algorithmically solvable).

Note also, that the above propositions are not in conflict with the nonmonotonic and disorderly aspects of inter-hierarchy comparison presented in [13]. A comparator is *orderly* if $S_H\supseteq S_{H\cup\{c\}}$, where S_H is a solution set for hierarchy H and $S_{H\cup\{c\}}$ is a solution set for hierarchy $H\cup\{c\}$. A comparator that is not orderly is *disorderly*. Unfortunately, each comparator that respects the hierarchy is disorderly [13] which means that adding a constraint could in general require us to change the previous solution in a significant way. Confining ourselves to the error vector of hierarchy instead of to the solution set of hierarchy we weaken the orderly aspect and Proposition 2 guarantees us that adding a constraint does not improve the error vector of the hierarchy.

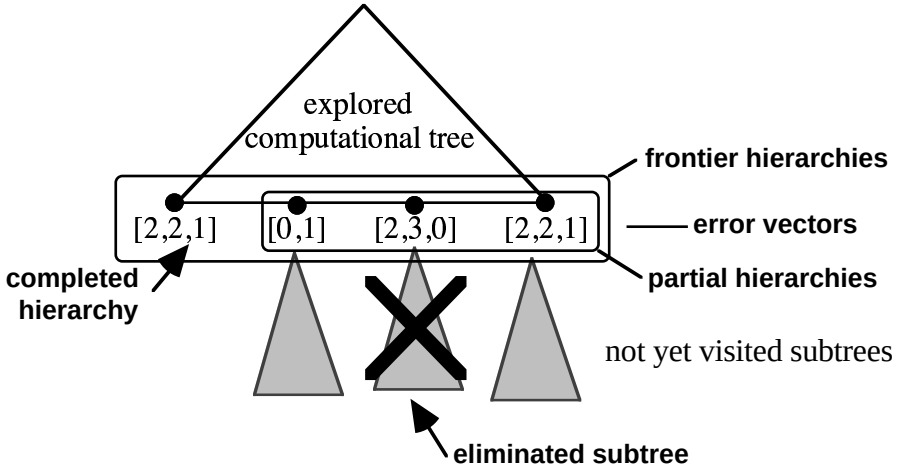
Similarly, a comparator is *monotonic* if $S_\Delta\subseteq S_{\Delta\cup\Gamma}$, where S_Δ is a solution set for the set Δ of hierarchies and $S_{\Delta\cup\Gamma}$ is a solution set for the set $\Delta\cup\Gamma$ of hierarchies. A comparator that is not monotonic is *nonmonotonic*. Again, each comparator that respects the hierarchy is nonmonotonic [13] which means that adding a hierarchy to the set of hierarchies (e.g., by using alternative rule in HCLP) could in general require us to change the previous solution in a significant way. Nevertheless, Propositions 2 and 3 enables us to restrict ourselves to solving smaller set Δ of hierarchies in case of the error vectors of all hierarchies from Γ are worse.

5 The algorithm

As mentioned in above sections, we utilize breadth-first search in our framework for solving HCLP programs using inter-hierarchy comparison. The HCLP goal is resolved in an obvious way. The required (hard) constraints are processed immediately and the soft (preferential) constraints are collected. As soon as the goal is completely reduced in any branch of the computational tree, the computation is interrupted temporarily and the elimination phase starts.

To simplify description of the elimination phase, we classify the collected constraint hierarchies into three categories first. The collected constraint hierarchies which are attached to the leaves of computational branches are called *frontier hierarchies*. We call the constraint hierarchy attached to the leave of the branch terminated with completely reduced goal a *completed hierarchy*. The hierarchies attached to leaves of other branches which are not reduced completely yet are called *partial hierarchies*.

During the elimination phase, the frontier hierarchies are solved using hierarchical constraint solver supporting globally-better comparator. Also the error vectors of all frontier hierarchies are computed. Next, the algorithm eliminates all branches labeled by partial hierarchies which error vectors are worse than the error vector of the completed hierarchy. According to Proposition 3 we know that the computation in the eliminated branch will not bring better solution (better error vector) than the solution of the completed hierarchy. Thus, the elimination does not influence the final solution. The error vector of the completed hierarchy can be seen as the top estimate of the final solution. Note also that this elimination can cut some infinite branches (see Section 6), but the algorithm does not guarantee to eliminate all the infinite branches.



After finishing the elimination, the exploration of the computational tree is restarted. Note, that the solution of all completed hierarchies must be considered in subsequent elimination phases. This means that, if a completed hierarchy H is computed and its solution is better than the solution of previously found completed hierarchies, then the solution of the hierarchy H displaces the previously found solution. Obversely, if the solution of the hierarchy H is worse than the previously found solution, then the solution of the hierarchy H is not included in the current solution (see below the `case` structure of the procedure `distribute_list`).

The algorithm terminates as soon as there is no goal to reduce, i.e., all goals were completely reduced or eliminated respectively. Because of obvious reasons (halting problem), we can not guarantee the termination of the algorithm in case of occurrence of infinite branches. Nevertheless, the algorithm terminates if there are no infinite branches and it can terminate even if there are infinite branches (for discussion see above paragraphs; for example see Section 6).

To simplify the following algorithm scheme we expect that the goal, current valuation of variables and collected soft constraints are encapsulated into one data structure. We also assume that the same data structure with the goal reduced represents the solution.

Algorithm Scheme:

```
solve_goal(Goal,Program)
%% main procedure that solves Goal using HCLP Program
%% returns list of solutions, i.e., valuations of variables

  GoalsToSolve:=append_to_list(new_list,Goal)
    % initialize the list of unsolved goals
  SolutionOfCompletedHier:=new_list
    % prepare (empty) list for collecting solutions

  while not empty(GoalsToSolve) % repeat while there is any unsolved goal
    GoalToReduce:=delete_first(GoalsToSolve)
      % select (remove) first goal from the list of unsolved goals
    GoalList:=reduce(GoalToReduce,Program)
      % reduce the selected goal using all possible rules from the Program
    distribute_list(GoalList,GoalsToSolve,SolutionOfCompletedHier)
      % update the list of unsolved goals and the list of solutions
  end while

  return SolutionOfCompletedHier
end solve_goal


distribute_list(ListOfGoals,GoalsToSolve,CurrentSolution)
%% update the list of unsolved goals by adding new goals; if any new goal
   is reduced then do elimination and update the list of solutions

  AnyReduction:=false  % clear the indicator of reduction occurrence

  while not empty(ListOfGoals)  % repeat while there is any unfilled goal
    Goal:=delete_first(ListOfGoals)
      % select (remove from the list) first unfilled goal
    if reduced(Goal) then    % the Goal is completely reduced
      AnyReduction:=true    % set the indicator of reduction occurrence
      case of
        :better(Goal,best_of(CurrentSolution))
          % solution of Goal is better than the previously found solution
          CurrentSolution:=append_to_list(new_list,Goal)
                % displace the previous solution by the new solution

        :not better(best_of(CurrentSolution),Goal)
          % & not better(Goal,best_of(CurrentSolution))
          % the solution of the Goal is as good as the previous solution
          CurrentSolution:=append_to_list(CurrentSolution,Goal)
                % add the new solution to the list of solutions

        :otherwise
          % the solution of the Goal is worse than the previous solution
          % the Goal is immediately eliminated, i.e., it is not placed
            neither to the list of unsolved goals nor to the list of
            solutions
      end case
    else   % Goal is not reduced yet
      GoalsToSolve:=append_to_list(GoalsToSolve,Goal)
        % add the Goal to the list of unsolved goals
    end if
  end while

  if AnyReduction then % any goal is completely reduced
    GoalsToSolve:=delete_worse(best_of(CurrentSolution),GoalsToSolve)
      % do elimination, i.e., remove unsolved goals which are known to
        produce solutions worse than the current solution
  end if
end distribute_list
```

The call to underlying hierarchical constraint solver is hidden in the test `better` where the error vectors are used to compare solutions. The elimination of not yet reduced goals, which are known to bring worse solution, is done within the procedure `delete_worse`.

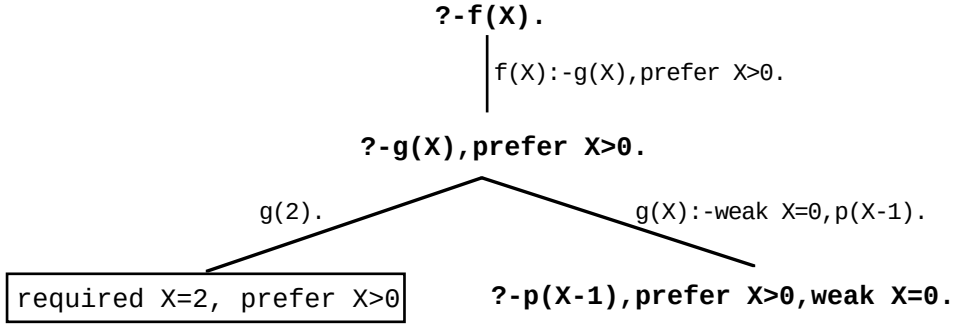
```
delete_worse(BestCompleted,ListOfPartial)

     Goals:=new_list % prepare list for unsolved goals which pass the test

     while not empty(ListOfPartial)
      % repeat while there is any unfilled goal
          G:=delete_first(ListOfPartial)
            % remove first goal from the list of unfilled goals
          if not better(BestCompleted,G)
            % solve the partial hierarchy attached to the goal G
            % compare the partial solution with current best solution
            Goals:=append_to_list(Goals,G)
                % if the partial solution is not worse than the
                  previous solution then update the list of goals
          end if
     end while

     return Goals
end delete_worse
```

6 Example

The following example illustrates the computation of the above presented algorithm. We use the domain of integers, the weighted-sum-better comparator (all weights are equal to 1) and the following HCLP program.

```
f(X):-g(X), prefer X>0.
g(2).
g(X):-weak X=0, p(X-1).
p(1).
p(X):-p(X-1).
```

Given the goal `?-f(X)`, the following figure shows the schema of partial computational tree when the first elimination phase occurs, i.e., when a goal in one branch is completely reduced:



The goal in the left branch of the tree is completely reduced and we get the complete hierarchy `required X=2,prefer X>0`. The solution of this hierarchy is `X=2` with the error vector `[0]` (only non-required levels are included in the error vector). The goal `p(X-1)` in the right branch is not reduced yet, and thus we get the partial hierarchy `prefer X>0,weak X=0` with the solution[4] `X=1` and the error vector `[0,1]`. Because the error vector of the partial hierarchy is "worse" than the error vector of the completed hierarchy, the algorithm eliminates

[4] Note, that the hierarchy has no solution if we work with the domain of reals.

the right branch. As there are no other goals to reduce, the algorithm stops and returns the currently found solution `X=2`.

Notice that the right subtree, which was eliminated by the algorithm, contains an infinite branch. In this particular case the algorithm eliminates the infinite branch but note again that, in general, it is not possible to detect all infinite branches. In fact, the presented algorithm does not deal with infinity of branches, it rather concerns with quality of the partial solution and eliminates bad partial solutions. Nevertheless, remind that the traditional approach, which collects all constraint hierarchies first, does not terminate at all in case of occurence of an infinite branch.

7 Further improvements

The scheme of the algorithm presented in Section 5 offers a lot of opportunities for further improvements. In fact, the presented algorithm corresponds to the well-known branch and bound method. We use breadth-first search to branch the tree of computation. The bound phase called elimination is evoked when the goal in any branch is reduced completely.

It is clear that finding a "good" initial solution can eliminate larger part of the computation tree. However, the breadth-first search is a method of blind search which is not influenced by the "quality" of the (partial) solution. Thus, we propose to use best-first search which is a method of knowledge-driven search. The only change in the algorithm from Section 5 is the replacement of the call `delete_first` in the procedure `solve_goal` by the call `delete_best`, which selects the most promising goal to reduce.

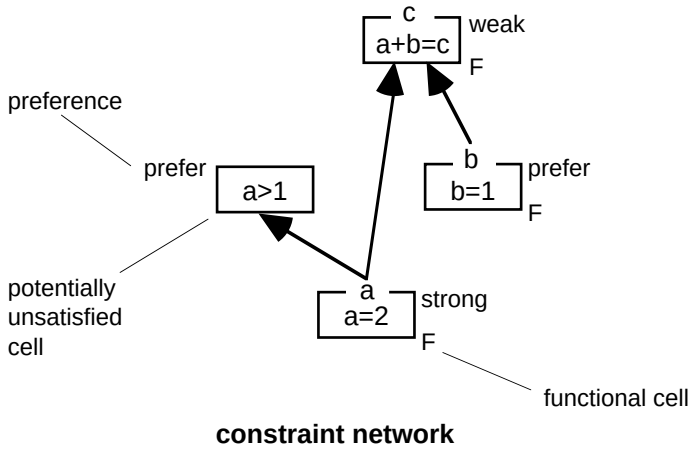
The error vector of the hierarchy is a natural quality indicator for the best-first search. The problem is how to obtain efficiently the error vector of the partial hierarchy. The straightforward approach, which uses the underlying hierarchical constraint solver to solve the constraint hierarchy entirely, i.e., to compute the error vector, does not seem to be reasonable because the evocation of this solver is still time consuming process. So, we suggest to use an estimate of the error vector.

As the computation of the estimate of the error vector requires close co-operation with the underlying constraint hierarchy solver, we briefly sketch the algorithm for solving constraint hierarchies that we developed in our previous works [1,2,3]. This hierarchical constraint solver consists of two phases: planning and propagation. During the planning phase, the constraint network is constructed, while the valuation of variables is computed within the propagation phase. In our view of HCLP system, the planning algorithm is used to collect the labeled constraints successively, i.e., to construct incrementally the constraint network, whereas the propagation phase is delayed till the complete constraint hierarchy is collected, i.e., till the original goal is completely reduced.

The constraint network is a directed acyclic graph whose nodes are labeled by constraint cells. The constraint cell is a set of equally preferred constraints. In [1,2,3] we distinguish two classes of constraint cells: functional cells and potentially unsatisfied cells. The algorithm guarantees that the constraints in functional cells are satisfied during the propagation phase which computes the solution by propagating sets of values through the constraint network.

The planning phase, whose features we exploit here, is well known from hierarchical constraint solvers like DeltaBlue [12] and SkyBlue [11]. In [3] we generalized the planning phase in such a way that all types of constraints and comparators, including globally-better, are supported. Now, we propose to further exploit the planning algorithm in such a way that it also computes the top estimate of the error vector. We compute the top estimate of the error vector using the assumption that the constraints from the potentially unsatisfied cells are not satisfied while the constraints from the functional cells are satisfied (therefore top estimate). As the planning phase runs anyway, there is no time penalty of computing the top estimate of the error vector.

Example:



constraint network

If one uses unsatisfied-count-better comparator, which minimizes the number of unsatisfied constraints at each level, then the top estimate of the error vector is [0,1,0] as there is exactly one potentially unsatisfied constraint (i.e., a>1) at the "prefer" level. Note that the real error vector for this hierarchy is [0,0,0].

Unsatisfied-count-better comparator corresponds to the weighted-sum-better comparator with weights equal to 1 and the predicate error function e (returns 0, if the constraint is satisfied, and 1 otherwise).

The enhancements proposed in this section can improve the efficiency of the algorithm for inter-hierarchy comparison. However, we should also mention that the expenses of computing additional information has to be balanced with the total gain of usage such information.

8 Conclusions and future research

In this paper, we present a basic framework for solving constraint hierarchies using inter-hierarchy comparison within HCLP. The proposed algorithm is based on tight integration with the HCLP interpreter and on elimination of useless branches of computation. The algorithm is built on top of a hierarchical constraint solver supporting globally-better comparators. The elimination of useless branches reduces the space of computation but, on the other hand, this approach demands more attention from the underlying hierarchical constraint solver which is applied to complete constraint hierarchies as well as to partial constraint hierarchies.

In comparison with the algorithm from [6] our approach presents the advantage of early elimination of unuseful branches of the computation tree which speeds up the computation and reduces the memory consumption. The open architecture of our algorithm, that can use arbitrary hierarchical constraint solver supporting globally-better comparators, is another benefit of our approach (the algorithm from [6] also presents this advantage).

There are still a lot of opportunities for further improvements, we show the direction for some of them. Especially adding (semi) incrementality feature to the underlying hierarchical constraint solver can dramatically improve the efficiency of the proposed algorithm because the subsequent calls to the solver can exploit the results of previous calls. Also, the area of empirical investigation of the proposed algorithm and its refinements is still open.

Hierarchical Constraint Logic Programming with inter-hierarchy comparison provides framework for natural representation of real-world problems. We believe that the practical application potential of constraint hierarchies and HCLP, in particular, can be further exploited by using the open algorithm for inter-hierarchy comparison within HCLP, that is presented in this paper.

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