



# Foundations of Automated Planning

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Lecture 03: Graphplan and compilation-based techniques

## **Classical planning**

- **search nodes** correspond to **partial plans**
- a **solution plan** reachable from a given search node contains **all the actions** from the partial plan
- state-space planning
- plan-space planning

## **Neoclassical planning**

- **search nodes** correspond to **several partial plans**
- **not all actions** from the partial plans appear in the solution plan reachable from the search node
- planning-graph planning

## Problems of existing techniques

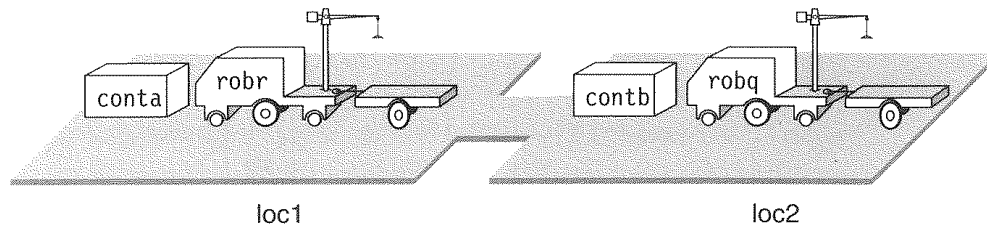
- **choice of bad action**, that is discovered late, causes exploration of large search space
- **large branching factor** is the source of such mistakes because it gives too many options

## How to discover “promising” actions for the plan?

- **solve a “relaxed” problem**, whose solution set contains all solutions of the original problem
  - **relaxed problem** = remove some constraints from the original problem (for example negative effects)
- use only the actions from the solution of the relaxed problem when selecting actions for the original problem

## What do we need?

- a compact representation of several plans such that not all the actions has to be used in the solution plan



$\text{move}(r, l, l')$  ;; robot  $r$  at location  $l$  moves to a connected location  $l'$   
precond:  $\text{at}(r, l), \text{adjacent}(l, l')$   
effects:  $\text{at}(r, l'), \neg \text{at}(r, l)$

$\text{load}(c, r, l)$  ;; robot  $r$  loads container  $c$  at location  $l$   
precond:  $\text{at}(r, l), \text{in}(c, l), \text{unloaded}(r)$   
effects:  $\text{loaded}(r, c), \neg \text{in}(c, l), \neg \text{unloaded}(r)$

$\text{unload}(c, r, l)$  ;; robot  $r$  unloads container  $c$  at location  $l$   
precond:  $\text{at}(r, l), \text{loaded}(r, c)$   
effects:  $\text{unloaded}(r), \text{in}(c, l), \neg \text{loaded}(r, c)$

## We will work with fully instantiated atoms and actions:

### Atoms:

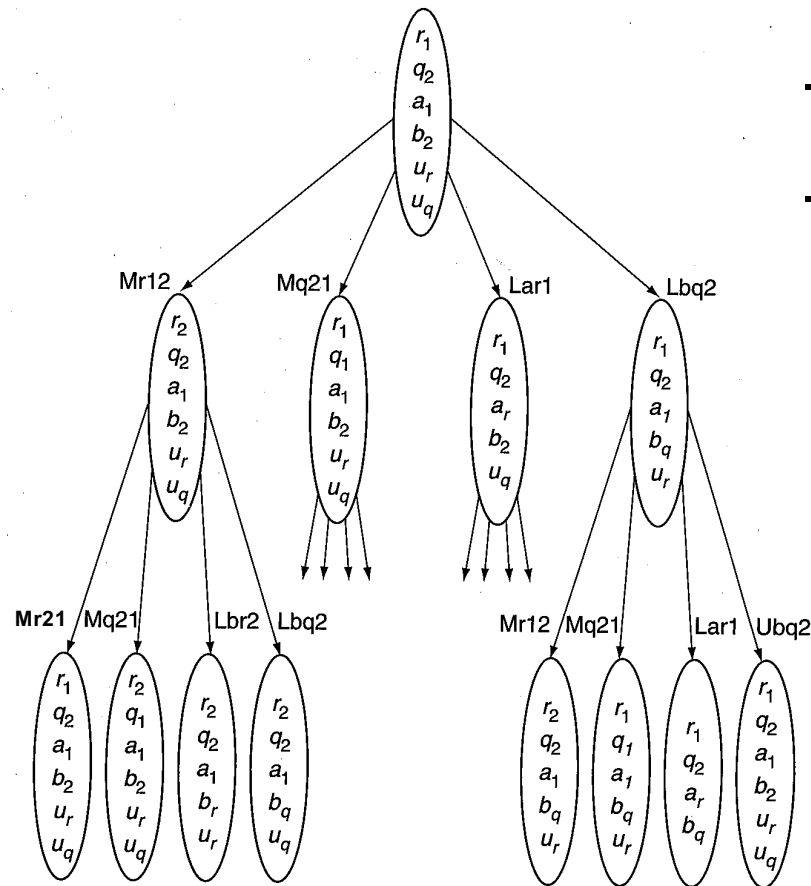
- $r_1, r_2, q_1, q_2$  – robot positions
- $a_1, a_2, a_r, a_q, b_1, b_2, b_r, b_q$  – container positions
- $u_r, u_q$  – robot is empty

The initial state is  $\{r_1, q_2, a_1, b_2, u_r, u_q\}$ .

### Actions:

- $\text{Mr}_{12}, \text{Mr}_{21}, \text{Mq}_{12}, \text{Mq}_{21}$  – robot moves
- $\text{Lar}_1, \text{Lar}_2, \text{Laq}_1, \text{Laq}_2, \text{Lbr}_1, \text{Lbr}_2, \text{Lbq}_1, \text{Lbq}_2$  – loading container to robot
- $\text{Uar}_1, \text{Uar}_2, \text{Uaq}_1, \text{Uaq}_2, \text{Ubr}_1, \text{Ubr}_2, \text{Ubq}_1, \text{Ubq}_2$  – unloading container from robot

Nodes correspond to states and arcs to transitions



- **root** – initial state  $s_0$
- Reachability tree of depth  $d$  with the root  $s_0$  contains **all solution plans** for problems, where the goal is reachable from  $s_0$  with at most  $d$  actions.

- **A plan exists if any goal state is in the reachability tree!**

## Problem:

The reachability tree contains  $O(k^d)$  nodes, where  $k$  is #applicable actions per state.

**Reachability tree gives sufficient and necessary conditions for a reachability of a given state.**

- a state is reachable if and only if it appears in the reachability tree

**Planning graph will give only the necessary condition of reachability.**

- if a state is reachable then it can be found in the planning graph
- however, not all the states in the planning graph are reachable

**How to exploit a planning graph?**

- If we can construct the planning graph fast with small memory consumption, it gives an estimate which actions are necessary to reach the goal state
- from the planning graph we still need to extract the solution plan

## **How to relax the reachability tree?**

- The nodes describe approximations of states at a given level of the reachability tree.

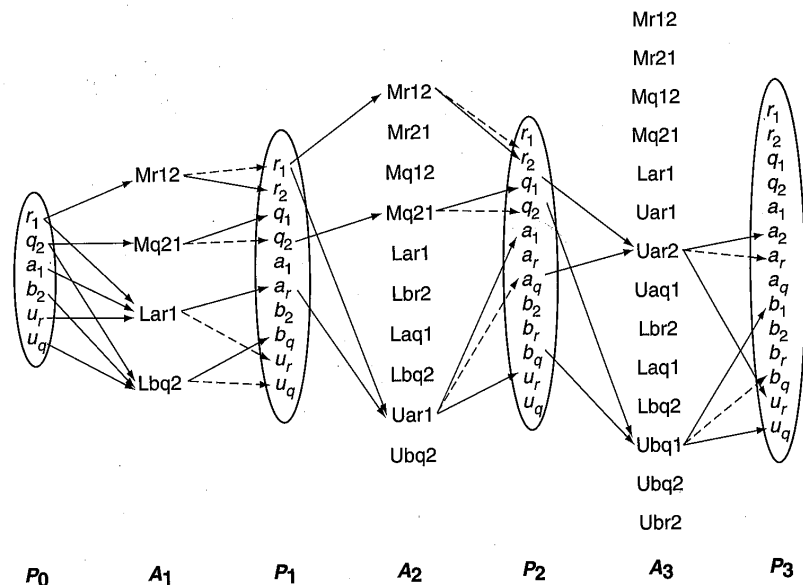
## **How to approximate the state?**

Recall that a state is represented as a set of propositions.

- we can approximate all states at the same level by union of propositions from these states
- **OK, but how to do it in practice?**
  - Apply all applicable actions in parallel to a given state and ignore negative effects of these actions.
  - Consequence: the number of propositions in approximated states will never decrease
  - Remember which actions provide a given proposition and which actions want to delete it

**Planning graph** is a directed layered graph, where

- each layer contains (exclusively)
  - instantiated propositions (a state layer) or
  - instantiated actions (an action layer)
- **state and action layers** interleave
  - the zero layer describes the initial state
  - the next layer describes all actions applicable to the initial state
  - the next layer contains positive effects of these actions
- an action layer with the following state layer forms a **level**
- **arcs** are between
  - propositions and actions that use them as preconditions
  - actions and propositions that are effects of the action
    - special arcs for negative effects
    - negative effects are not deleted!





## Where are the plans in the planning graph?

- Instead of a single sequence of actions we will use a sequence of sets of actions – **a layered plan**.
  - a set of actions at position  $j$  will be a subset of actions from  $j$ -th action layer
- The sequential plan is obtained from the layered plan by using any permutation of actions from each set.

### *Example:*

A layered plan  $\langle \{a_1, a_2\}, \{a_3, a_4\}, \{a_5, a_6, a_7\} \rangle$  gives  $2 \times 2 \times 6 = 24$  sequential plans.

## How to ensure that actions selected to a layered plan can be ordered without influencing the resulting state?

- Such actions must be independent!

## When is a pair of actions (a,b) dependent?

- When different orders of the actions give different states, in particular:
  - **a** deletes some precondition of **b** (hence **a** cannot be right before **b**)
  - **a** deletes some positive effect of **b** (hence the result depends on the order)
  - symmetrically for **b**

## A pair of actions (a,b) is independent if and only if:

- $\text{effects}^-(a) \cap (\text{precond}(b) \cup \text{effects}^+(b)) = \emptyset$
- $\text{effects}^-(b) \cap (\text{precond}(a) \cup \text{effects}^+(a)) = \emptyset$

### *Note:*

Independence of actions is given by the planning domain and it is not influenced by a particular planning problem!

Let  $\pi$  be a set of pairwise independent actions, then

- $\pi$  is applicable to a state  $s$  if and only if
  - preconditions of all actions from  $\pi$  are satisfied by  $s$
  - $\text{precond}(\pi) = \cup \{ \text{precond}(a) \mid \forall a \in \pi \} \subseteq s$
- The result of applying  $\pi$  to state  $s$  is a state
  - $\gamma(s, \pi) = (s - \text{effects}^-(\pi)) \cup \text{effects}^+(\pi)$ 
    - $\text{effects}^{+/-(\pi)}$  is a union of effects from  $\pi$

## **Claim:**

- If  $\pi$  consists of pairwise independent actions and it is applicable to state  $s$ , then for any permutation of actions  $\langle a_1, a_2, \dots, a_k \rangle$  from  $\pi$  we get  $\gamma(s, \pi) = \gamma(\dots \gamma(\gamma(s, a_1), a_2) \dots a_k)$ .

This claim makes using layered graphs practical!

A layered plan  $\Pi = \langle \pi_1, \pi_2, \dots, \pi_k \rangle$  is a **solution plan** for problem  $(O, s_0, g)$  if and only if:

- each set  $\pi_i$  consists of pairwise independent actions,
- a set of actions  $\pi_1$  is applicable to state  $s_0$ ,  $\pi_2$  is applicable to state  $\gamma(s_0, \pi_1)$  and so on,
- $g \subseteq \gamma(\dots \gamma(\gamma(s_0, \pi_1), \pi_2) \dots \pi_k)$ .

***Claim:***

If  $\Pi$  is solution plan for  $(O, s_0, g)$ , then any sequence of actions consisting of a permutation of actions from  $\pi_1$ , followed by action permutation from  $\pi_2$  and so on transfers the state  $s_0$  to a state satisfying  $g$ .

## How to do planning with the planning graph?

- **First build a planning graph** in such a way that the last propositional layer satisfies the goal condition.
  - More precisely, we will require that all the goal propositions can be used together in the last propositional layer.
- From the action layers **select subsets of independent actions** in such a way that they cover the goal propositions.
  - This is realised by a **backward run** from the last level, where actions giving the goal are selected and then, in the previous level, we select actions giving preconditions of actions selected from the last level etc.
  - Some goal proposition can be satisfied in the previous level (not the last one).

We ensure that each proposition is an effect of some action from the previous layer.

- Using a no-op action for each proposition:  $\alpha_p$  is a **no-op action for p**, iff  $\text{precond}(\alpha_p) = \text{effect}^+(\alpha_p) = \{p\}$ ,  $\text{effect}^-(\alpha_p) = \emptyset$

## How to find if two propositions can be together in the same layer?

- **All propositions** can be together at the **zero layer** (this is the initial state).
- **Two dependent actions** cannot be used together in the first action layer so their **positive effects cannot appear together** in the next state unless they are given by another pair of independent actions.
- **Two propositions** cannot be together if they are positive and negative effects of a single action (again, unless they are given by another pair of independent actions).
  - *No-op actions are treated as other actions in these conditions (if  $b$  deletes  $p$ , then  $\alpha_p$  and  $b$  are dependent).*
- An incompatible pair of propositions is called a **propositional mutex (mutual exclusion)**.

Level	Mutex elements
$A_1$	$\{Mr12\} \times \{Lar1\}$ $\{Mq21\} \times \{Lbq2\}$
$P_1$	$\{r_2\} \times \{r_1, a_r\}$ $\{q_1\} \times \{q_2, b_q\}$ $\{a_r\} \times \{a_1, u_r\}$ $\{b_q\} \times \{b_2, u_q\}$
$A_2$	$\{Mr12\} \times \{Mr21, Lar1, Uar1\}$ $\{Mr21\} \times \{Lbr2, Lar1^*, Uar1^*\}$ $\{Mq12\} \times \{Mq21, Laq1, Lbq2^*, Ubq2^*\}$ $\{Mq21\} \times \{Lbq2, Ubq2\}$ $\{Lar1\} \times \{Uar1, Laq1, Lbr2\}$ $\{Lbr2\} \times \{Ubr2, Lbq2, Uar1, Mr12^*\}$ $\{Laq1\} \times \{Uar1, Ubq2, Lbq2, Mq21^*\}$ $\{Lbq2\} \times \{Ubq2\}$
$P_2$	$\{b_r\} \times \{r_1, b_2, u_r, b_q, a_r\}$ $\{a_q\} \times \{q_2, a_1, u_q, b_q, a_r\}$ $\{r_1\} \times \{r_2\}$ $\{q_1\} \times \{q_2\}$ $\{a_r\} \times \{a_1, u_r\}$ $\{b_q\} \times \{b_2, u_q\}$
$A_3$	$\{Mr12\} \times \{Mr21, Lar1, Uar1, Lbr2^*, Uar2^*\}$ $\{Mr21\} \times \{Lbr2, Uar2, Ubr2\}$ $\{Mq12\} \times \{Mq21, Laq1, Uaq1, Ubq1, Ubq2^*\}$ $\{Mq21\} \times \{Lbq2, Ubq2, Laq1^*, Ubq1^*\}$ $\{Lar1\} \times \{Uar1, Uaq1, Laq1, Uar2, Ubr2, Lbr2, Mr21^*\}$ $\{Lbr2\} \times \{Ubr2, Ubq2, Lbq2, Uar1, Uar2, Ubq1^*\}$ $\{Laq1\} \times \{Uar1, Uaq1, Ubq1, Ubq2, Lbq2, Uar2^*\}$ $\{Lbq2\} \times \{Ubr2, Ubq2, Uaq1, Ubq1, Mq12^*\}$ $\{Uaq1\} \times \{Uar1, Uar2, Ubq1, Ubq2, Mq21^*\}$ $\{Ubr2\} \times \{Uar1, Uar2, Ubq1, Ubq2, Mr12^*\}$ $\{Uar1\} \times \{Uar2, Mr21^*\}$ $\{Ubq1\} \times \{Ubq2\}$
$P_3$	$\{a_2\} \times \{a_r, a_1, r_1, a_q, b_r\}$ $\{b_1\} \times \{b_q, b_2, q_2, a_q, b_r\}$ $\{a_r\} \times \{u_r, a_1, a_q, b_r\}$ $\{b_q\} \times \{u_q, b_2, a_q, b_r\}$ $\{a_q\} \times \{a_1, u_q\}$ $\{b_r\} \times \{b_2, u_r\}$ $\{r_1\} \times \{r_2\}$ $\{q_1\} \times \{q_2\}$

Propositional mutexes can give further incompatibilities between actions in addition to action dependence.

- **Two actions are mutex**, if some of their preconditions are mutex.
- An action whose preconditions are mutex can be removed immediately.



**Two actions  $a$  and  $b$  are mutex** at level  $A_i$ , if:

- $a$  and  $b$  are dependent, or
- precondition of  $a$  has a mutex with some precondition of  $b$  at level  $P_{i-1}$ .

*The set of action mutexes for level  $A_i$  is denoted  $\mu A_i$ .*

**Two propositions  $p$  and  $q$  are mutex** at level  $P_i$ , if:

- each action  $A_i$  giving  $p$  as its positive effect has a mutex with any action giving  $q$  as a positive effect, and
- there is no action in  $A_i$  having both  $p$  and  $q$  as positive effects.

*The set of propositional mutexes for level  $P_i$  is denoted  $\mu P_i$ .*



**Mutex** is a **symmetric relation**.

**Sets of mutexes** in the planning graph are **decreasing**:

- if  $p$  and  $q$  are from  $P_{i-1}$  and  $(p,q) \notin \mu P_{i-1}$ , then  $(p,q) \notin \mu P_i$
- if  $a$  and  $b$  are from  $A_{i-1}$  and  $(a,b) \notin \mu A_{i-1}$ , then  $(a,b) \notin \mu A_i$

*Proof:*

- If two propositions are not mutex then their no-op actions are not mutex and hence they give these propositions in the next layer.
- If two actions are not mutex then they are independent and their preconditions are not mutex. As the preconditions will not become mutex in the next layer (see above) the actions will not be mutex in the next layer.

*Note:*

**Sets of actions and propositions** in the planning graph are **increasing** ( $P_{i-1} \subseteq P_i$  and  $A_{i-1} \subseteq A_i$ ).

## **Graphplan is a planning system based on planning graph.**

- It repeats graph expansion and plan extraction until a solution plan is found.
- **Expansion:**
  - First construct a planning graph till the layer where there are all goal propositions and no pair of them is mutex (this is a necessary condition for plan existence).
  - If plan extraction fails, add a new level (stop if some final condition holds, then no plan exists).
- **Extraction:**
  - Extract a layered plan from the planning graph in such a way that the plan gives all the goal propositions.

*A technical restriction:*

we only assume actions with positive preconditions (can be ensured by modifying the planning domain)

Planning graph is seen as a sequence of levels and mutex sets

$$G = \langle P_0, A_1, \mu A_1, P_1, \mu P_1, \dots, A_i, \mu A_i, P_i, \mu P_i \rangle$$

Planning graph depends only on the planning operators  $O$  and on the initial state  $s_0$  (encoded in  $P_0$ ), but it does not depend on the goal  $g$ !

The procedure **Expand(G)** adds one level to the graph:

```

Expand( $\langle P_0, A_1, \mu A_1, P_1, \mu P_1, \dots, A_{i-1}, \mu A_{i-1}, P_{i-1}, \mu P_{i-1} \rangle$ )
   $A_i \leftarrow \{a \in A \mid \text{precond}(a) \subseteq P_{i-1} \text{ and } \text{precond}^2(a) \cap \mu P_{i-1} = \emptyset\}$ 
   $P_i \leftarrow \{p \mid \exists a \in A_i : p \in \text{effects}^+(a)\}$ 
   $\mu A_i \leftarrow \{(a, b) \in A_i^2, a \neq b \mid \text{effects}^-(a) \cap [\text{precond}(b) \cup \text{effects}^+(b)] \neq \emptyset$ 
    or  $\text{effects}^-(b) \cap [\text{precond}(a) \cup \text{effects}^+(a)] \neq \emptyset$ 
    or  $\exists (p, q) \in \mu P_{i-1} : p \in \text{precond}(a), q \in \text{precond}(b)\}$ 
   $\mu P_i \leftarrow \{(p, q) \in P_i^2, p \neq q \mid \forall a, b \in A_i, a \neq b :$ 
     $p \in \text{effects}^+(a), q \in \text{effects}^+(b) \Rightarrow (a, b) \in \mu A_i\}$ 
  for each  $a \in A_i$  do: link  $a$  with precondition arcs to  $\text{precond}(a)$  in  $P_{i-1}$ 
    positive arcs to  $\text{effects}^+(a)$  and negative arcs to  $\text{effects}^-(a)$  in  $P_i$ 
  return( $\langle P_0, A_1, \mu A_1, \dots, P_{i-1}, \mu P_{i-1}, A_i, \mu A_i, P_i, \mu P_i \rangle$ )
end

```

The number of different levels in the planning graph is restricted. Starting with some level, the graph is not changing – a fixed-point level.

**A fixed-point level** in the planning graph  $G$  is such a level  $k$  that  $\forall i, i > k$  levels  $i$  are identical to it, i.e.,  $P_i = P_k$ ,  $\mu P_i = \mu P_k$ ,  $A_i = A_k$ ,  $\mu A_i = \mu A_k$ .

- *Each planning graph  $G$  has a fixed-point level  $k$ , where  $k$  is the smallest number such that  $|P_{k-1}| = |P_k|$  and  $|\mu P_{k-1}| = |\mu P_k|$ .*

*Proof* is based on monotony and finiteness of the levels and mutex sets.

- This claim also gives an efficient method how to detect the fixed-point level!

**Plan extraction** is done in the **backward** direction from level  $P_i$  containing all the non-mutex goal propositions ( $g^2 \cap \mu P_i = \emptyset$ ).

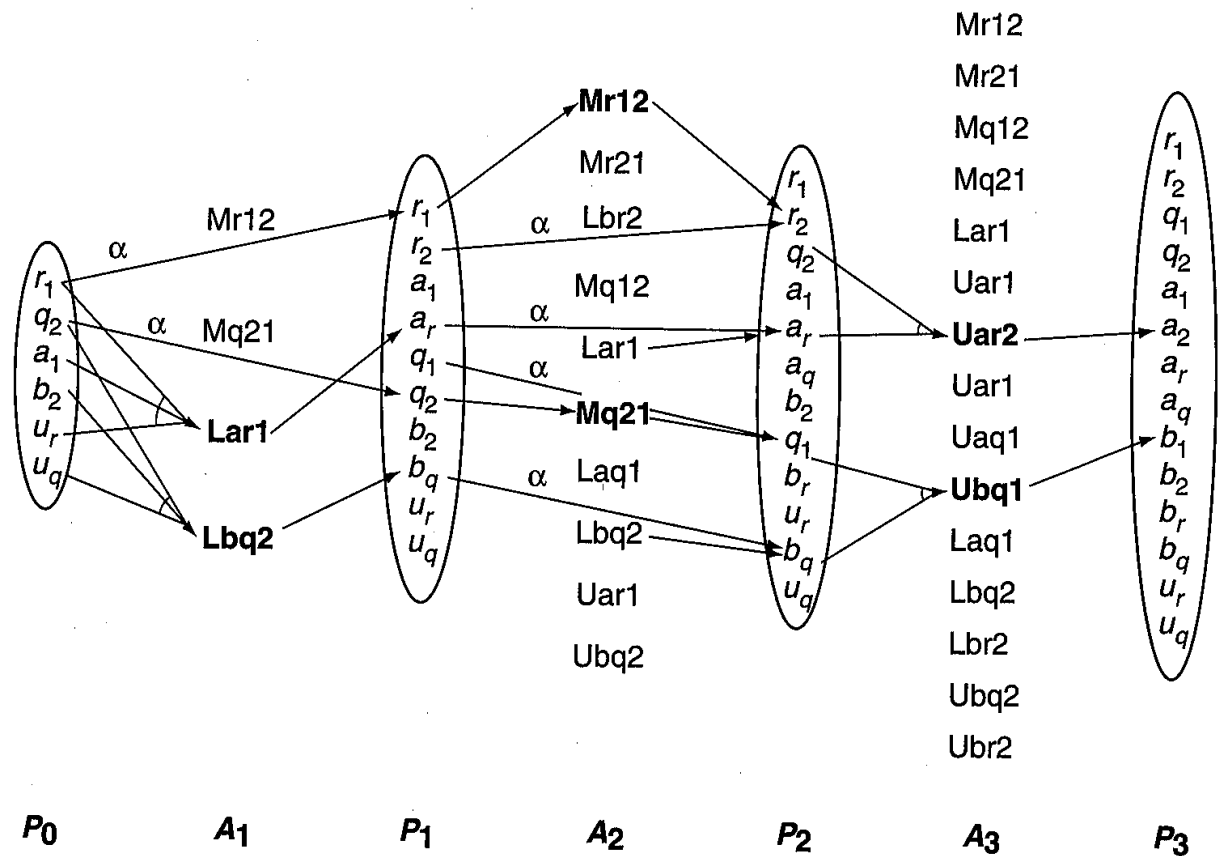
- First, find a set of non-mutex actions  $\pi_i \subseteq A_i$  that give all the goal propositions.
- Preconditions of actions from  $\pi_i$  form a new goal for the previous level  $P_{i-1}$ .
- If the goal for a level  $P_j$  cannot be covered by actions then go back to level  $j+1$  and explore an alternative set  $\pi_{j+1}$ .
- If level 0 is reached, the sequence  $\langle \pi_1, \pi_2, \dots, \pi_k \rangle$  is a solution.

This is basically **AND/OR search**:

- **OR branches** represent alternative actions giving the goal proposition
- **AND branches** connect the preconditions with actions

# Plan extraction – an example

Level	Mutex elements
A <sub>1</sub>	{Mr12} × {Lar1}
P <sub>1</sub>	{Mq21} × {Lbq2}
A <sub>2</sub>	{r <sub>2</sub> } × {r <sub>1</sub> , a <sub>r</sub> }
	{q <sub>1</sub> } × {q <sub>2</sub> , b <sub>q</sub> }
	{a <sub>r</sub> } × {a <sub>1</sub> , u <sub>r</sub> }
	{b <sub>q</sub> } × {b <sub>2</sub> , u <sub>q</sub> }
A <sub>2</sub>	{Mr12} × {Mr21, Lar1, Uar1}
	{Mr21} × {Lbr2, Lar1*, Uar1*}
	{Mq12} × {Mq21, Laq1, Lbq2*, Ubq2*}
	{Mq21} × {Lbq2, Ubq2}
	{Lar1} × {Uar1, Laq1, Lbr2}
	{Lbr2} × {Ubq2, Lbq2, Uar1, Mr12*}
	{Laq1} × {Uar1, Ubq2, Lbq2, Mq21*}
	{Lbq2} × {Ubq2}
P <sub>2</sub>	{b <sub>r</sub> } × {r <sub>1</sub> , b <sub>2</sub> , u <sub>r</sub> , b <sub>q</sub> , a <sub>r</sub> }
	{a <sub>q</sub> } × {q <sub>2</sub> , a <sub>1</sub> , u <sub>q</sub> , b <sub>q</sub> , a <sub>r</sub> }
	{r <sub>1</sub> } × {r <sub>2</sub> }
	{q <sub>1</sub> } × {q <sub>2</sub> }
	{a <sub>r</sub> } × {a <sub>1</sub> , u <sub>r</sub> }
	{b <sub>q</sub> } × {b <sub>2</sub> , u <sub>q</sub> }
A <sub>3</sub>	{Mr12} × {Mr21, Lar1, Uar1, Lbr2*, Uar2*}
	{Mr21} × {Lbr2, Uar2, Ubr2}
	{Mq12} × {Mq21, Laq1, Uaq1, Ubq1, Ubq2*}
	{Mq21} × {Lbq2, Ubq2, Laq1*, Ubq1*}
	{Lar1} × {Uar1, Uaq1, Laq1, Uar2, Ubr2, Lbr2, Mr21*}
	{Lbr2} × {Ubr2, Ubq2, Lbq2, Uar1, Uar2, Ubq1*}
	{Laq1} × {Uar1, Uaq1, Ubq1, Ubq2, Lbq2, Uar2*}
	{Lbq2} × {Ubr2, Ubq2, Uaq1, Ubq1, Mq12*}
	{Uaq1} × {Uar1, Uar2, Ubq1, Ubq2, Mq21*}
	{Ubr2} × {Uar1, Uar2, Ubq1, Ubq2, Mr12*}
	{Uar1} × {Uar2, Mr21*}
	{Ubq1} × {Ubq2}
P <sub>3</sub>	{a <sub>2</sub> } × {a <sub>r</sub> , a <sub>1</sub> , r <sub>1</sub> , a <sub>q</sub> , b <sub>r</sub> }
	{b <sub>1</sub> } × {b <sub>q</sub> , b <sub>2</sub> , q <sub>2</sub> , a <sub>q</sub> , b <sub>r</sub> }
	{a <sub>r</sub> } × {u <sub>r</sub> , a <sub>1</sub> , a <sub>q</sub> , b <sub>r</sub> }
	{b <sub>q</sub> } × {u <sub>q</sub> , b <sub>2</sub> , a <sub>q</sub> , b <sub>r</sub> }
	{a <sub>q</sub> } × {a <sub>1</sub> , u <sub>q</sub> }
	{b <sub>r</sub> } × {b <sub>2</sub> , u <sub>r</sub> }
	{r <sub>1</sub> } × {r <sub>2</sub> }
	{q <sub>1</sub> } × {q <sub>2</sub> }



- let {a<sub>2</sub>, b<sub>1</sub>} be the goal atoms
- **bold actions** are selected to the layered plan
- We get the layered plan  
 $\langle \{\text{Lar1, Lbq2}\}, \{\text{Mr12, Mq21}\}, \{\text{Uar2, Ubq1}\} \rangle$

**Mutex** can capture **incompatible pairs**.

We may find that a given goal cannot be satisfied at a given level. Then the set of propositions forming the goal is together **incompatible**.

- If we explore the same level later with the same (or larger) goal, then we already know that such a goal cannot be satisfied and the algorithm can immediately backtrack.

Unsatisfied goals can be remembered for each level in the form of a **nogood** table  $\nabla$ .

- A goal that is nogood immediately fails and the algorithm can backtrack.

```

Extract( $G, g, i$ )
  if  $i = 0$  then return ( $\langle \rangle$ )
  if  $g \in \nabla(i)$  then return(failure)
   $\pi_i \leftarrow \text{GP-Search}(G, g, \emptyset, i)$ 
  if  $\pi_i \neq \text{failure}$  then return( $\pi_i$ )
   $\nabla(i) \leftarrow \nabla(i) \cup \{g\}$ 
  return(failure)
end
    
```

**Extract** tries to cover a goal at level  $i$ .

- uses and updates the nogood table  $\nabla$

**GPSearch** looks for a set  $\pi_i$  of actions covering the goal.

- incrementally adds actions to  $\pi_i$  such that the actions are non-mutex
- after covering the goal continues to the previous level

```

GP-Search( $G, g, \pi_i, i$ )
  if  $g = \emptyset$  then do
     $\Pi \leftarrow \text{Extract}(G, \bigcup \{\text{precond}(a) \mid \forall a \in \pi_i, i - 1\})$ 
    if  $\Pi = \text{failure}$  then return(failure)
    return( $\Pi, \langle \pi_i \rangle$ )
  else do
    select any  $p \in g$ 
     $\text{resolvers} \leftarrow \{a \in A_i \mid p \in \text{effects}^+(a) \text{ and } \forall b \in \pi_i : (a, b) \notin \mu A_i\}$ 
    if  $\text{resolvers} = \emptyset$  then return(failure)
    nondeterministically choose  $a \in \text{resolvers}$ 
    return( $\text{GP-Search}(G, g - \text{effects}^+(a), \pi_i \cup \{a\}, i)$ )
end
    
```



Graphplan( $A, s_0, g$ )

$i \leftarrow 0, \nabla \leftarrow \emptyset, P_0 \leftarrow s_0$

$G \leftarrow \langle P_0 \rangle$

until [ $g \subseteq P_i$  and  $g^2 \cap \mu P_i = \emptyset$ ] or Fixedpoint( $G$ ) do

$i \leftarrow i + 1$

$G \leftarrow \text{Expand}(G)$

if  $g \not\subseteq P_i$  or  $g^2 \cap \mu P_i \neq \emptyset$  then return(failure)

$\Pi \leftarrow \text{Extract}(G, g, i)$

if Fixedpoint( $G$ ) then  $\eta \leftarrow |\nabla(\kappa)|$

else  $\eta \leftarrow 0$

while  $\Pi = \text{failure}$  do

$i \leftarrow i + 1$

$G \leftarrow \text{Expand}(G)$

$\Pi \leftarrow \text{Extract}(G, g, i)$

if  $\Pi = \text{failure}$  and Fixedpoint( $G$ ) then

if  $\eta = |\nabla(\kappa)|$  then return(failure)

$\eta \leftarrow |\nabla(\kappa)|$

return( $\Pi$ )

end

- first find a planning graph satisfying goal condition  $g$
- if it does not exist then stop (no solution)
- otherwise extract a layered plan
- if not successful
- add one more layer
- and again extract a layered plan
- stop when plan is found or no plan exists.

## Graphplan is sound, complete, and always finishes.

### *Proof:*

- if the algorithm returns a plan, then it is a solution plan
- if the algorithm fails, then no plan exists
  - $\text{Fixedpoint}(G)$  and  $(g \notin P_i \text{ or } g^2 \cap \mu P_i \neq \emptyset)$ 
    - If there is no level satisfying the goal before reaching the fixed point then no other level can cover the goal
  - $|\nabla_{i-1}(k)| = |\nabla_i(k)|$ 
    - nogood tables only increase – so we get  $\nabla_{i-1}(k) = \nabla_i(k)$
    - If the nogood table in the fixed point does not change then the goals from the fixed point cannot be ever satisfied – the nogood table just propagates to next levels in next steps  $\nabla_{i-1}(k) = \nabla_i(k+1)$
- The algorithm always stops thanks to monotony and a finite number of atoms and actions (some layers start to repeat and the nogood tables become full).

## Classical approach to problem solving



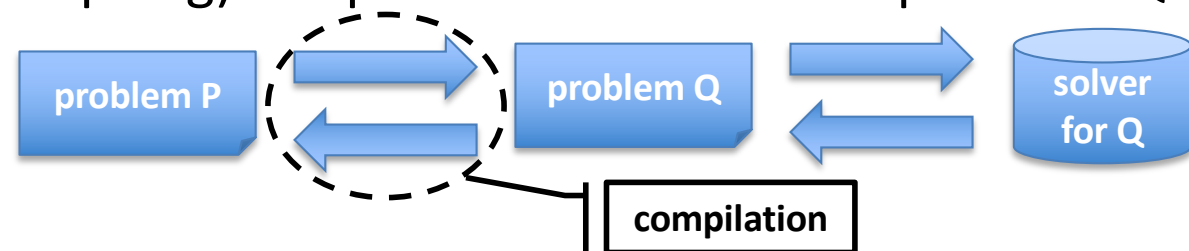
## Compilation-based approach to problem solving

*What is the idea?*

**exploit knowledge of others** for solving own problems

*How to do it?*

by **translating** (compiling) the problem P to another problem Q



*Why is it useful?*

if anybody improves the solver for Q then we get an improved solver for P for free

*Other names:*

reduction techniques, re-formulation techniques



## Constraint satisfaction problem consists of:

- a finite set of **variables**
  - describe some features of the world state that we are looking for, for example position of queens at a chessboard
- **domains** – finite sets of values for each variable
  - describe “options” that are available, for example the rows for queens
- a finite set of **constraints**
  - a constraint is a **relation** over a subset of variables; constraint can be defined in extension (a set of tuples satisfying the constraint) or using a formula ( $\text{rowA} \neq \text{rowB}$ )

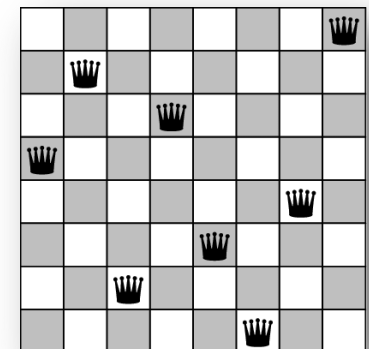
## Example (N-queens problem):

the core decision: each queen is pre-allocated to its own column  
and we are looking for its row

**variables:** N variables  $r(i)$  with the domain  $\{1, \dots, N\}$

**constraints:** no two queens attack each other

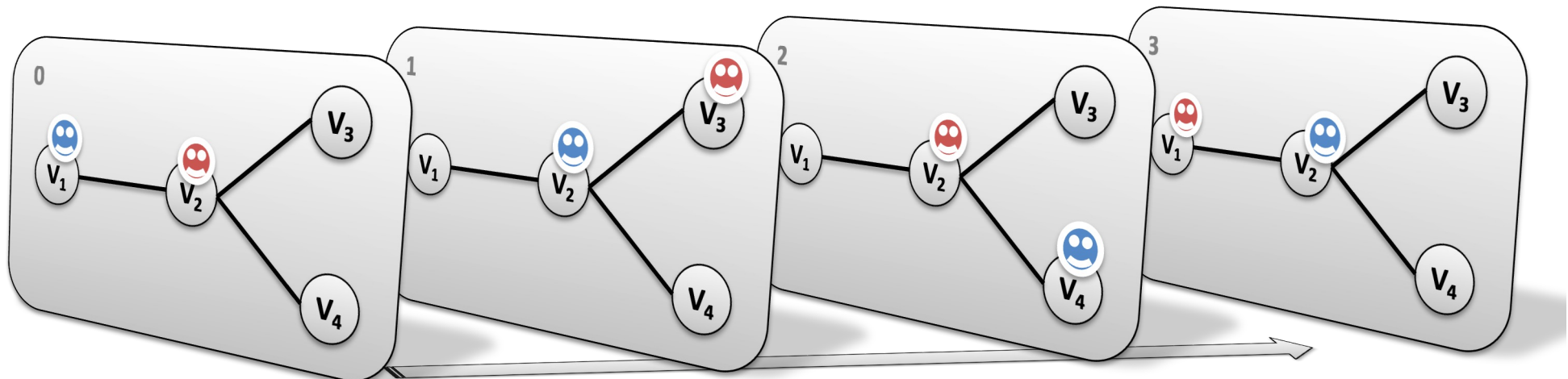
$$\forall i \neq j \quad r(i) \neq r(j) \wedge |i-j| \neq |r(i)-r(j)|$$



In planning, we do not know the lengths of plans!

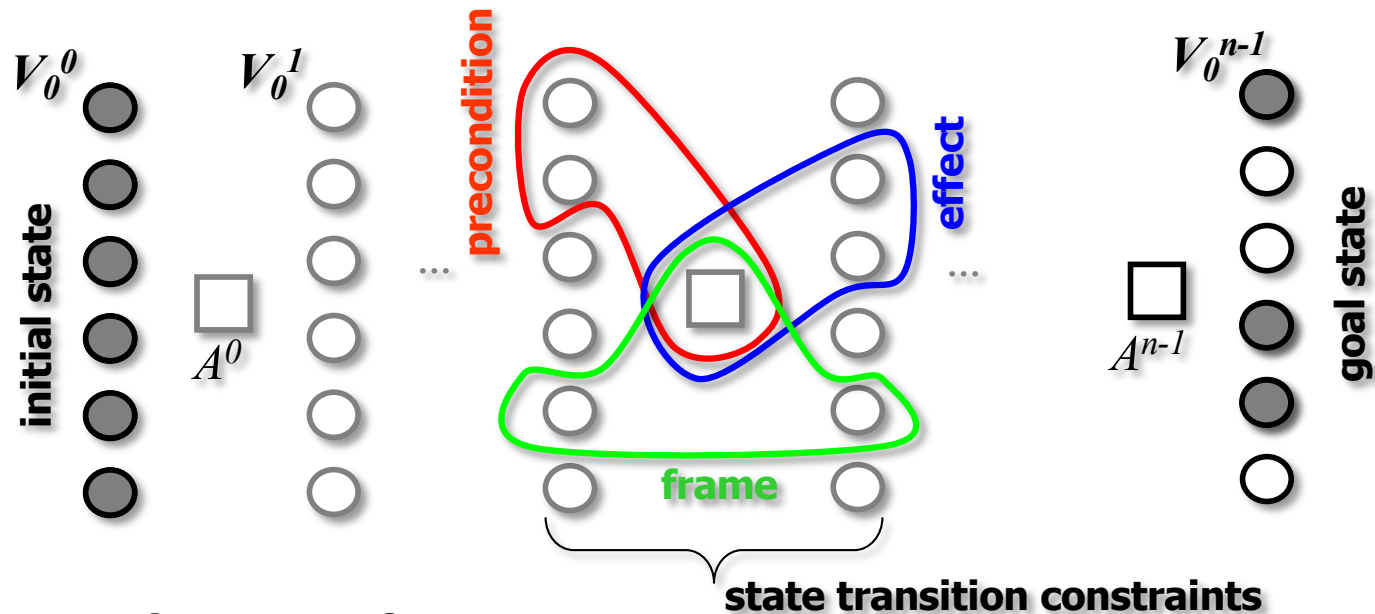
We can encode plans of a known length using a **layered graph** (temporally extended graph).

- each layer corresponds to one state
- transition between layers corresponds to application of (one) action



## Iterative extension of the plan length

Formulating the problem of finding a plan of a given length as a CSP



## Backward search

- instantiation of action variables
- only actions relevant to the (sub)goal are tried

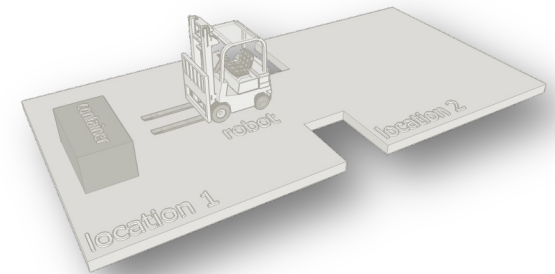
In classical representation, state is a set of logical atoms that are true in a given state.

Can be modelled as a vector of true/false values indicating which proposition is true in the state.

**State-variable representation** models a state as a vector of values of state variables.

### Example (state variables):

- $rloc(r) \in \{l1, l2, l3\}, r \in \{r1, r2\}$
- $rload(r) \in \{c1, c2, c3, nil\}, r \in \{r1, r2\}$
- $cpos(c) \in \{l1, l2, l3, r1, r2\}, c \in \{c1, c2, c3\}$



Operators describe how values of certain variables change.

### Example (operator):

$load(r, c, loc)$  ;; robot  $r$  loads container  $c$  at location  $loc$

Precond:  $rloc(r) = loc, cpos(c) = loc$

Effects:  $cpos(c) \leftarrow r$

## constraint model

- action constraints

$$A^s = act \rightarrow \text{Pre}(act)^s, \forall act \in \text{Dom}(A^s)$$

$$A^s = act \rightarrow \text{Eff}(act)^{s+1}, \forall act \in \text{Dom}(A^s)$$

- frame constraint

$$A^s \in \text{NonAffAct}(V_i) \rightarrow V_i^s = V_i^{s+1}, \forall i \in \langle 0, v-1 \rangle$$

$$A^s = \text{move21} \rightarrow \text{rloc}^s = \text{loc2}$$

$$A^s = \text{move21} \rightarrow \text{rloc}^{s+1} = \text{loc1}$$

$$A^s = \text{move21} \rightarrow \text{cpos}^s = \text{cpos}^{s+1}$$

## problems

- disjunctive constraints do not propagate well

↳ do not prune well the search space

- a huge number of constraints (depend on the number of actions)

↳ the propagation loop takes a lot of time



for each state variable  $V_i^s$  there is a supporting action variable  $S_i^s$  describing the action which sets the state variable (no-op action if the variable is not changed)

### constraint model

- action constraints

$$A^s = act \rightarrow \text{Pre}(act)^s, \forall act \in \text{Dom}(A^s)$$

$$S_i^s = act \rightarrow V_i^s = val, \forall act \in \text{Dom}(S_i^s)$$

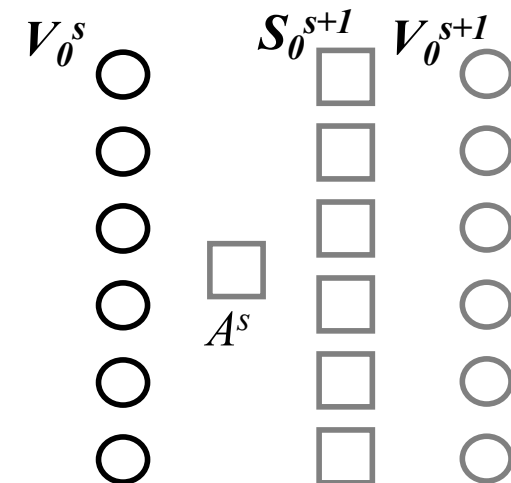
- frame constraint

$$S_i^{s+1} = no-op \rightarrow V_i^s = V_i^{s+1}.$$

- channeling constraint

$$A^s \in \text{AffAct}(V_i) \leftrightarrow S_i^{s+1} = A^s, \text{ and}$$

$$A^s \in \text{NonAffAct}(V_i) \leftrightarrow S_i^{s+1} = no-op$$





## idea

- focus on modeling the reason for the value of a state variable (effect and frame constraints are merged)

## constraint model

- precondition constraint

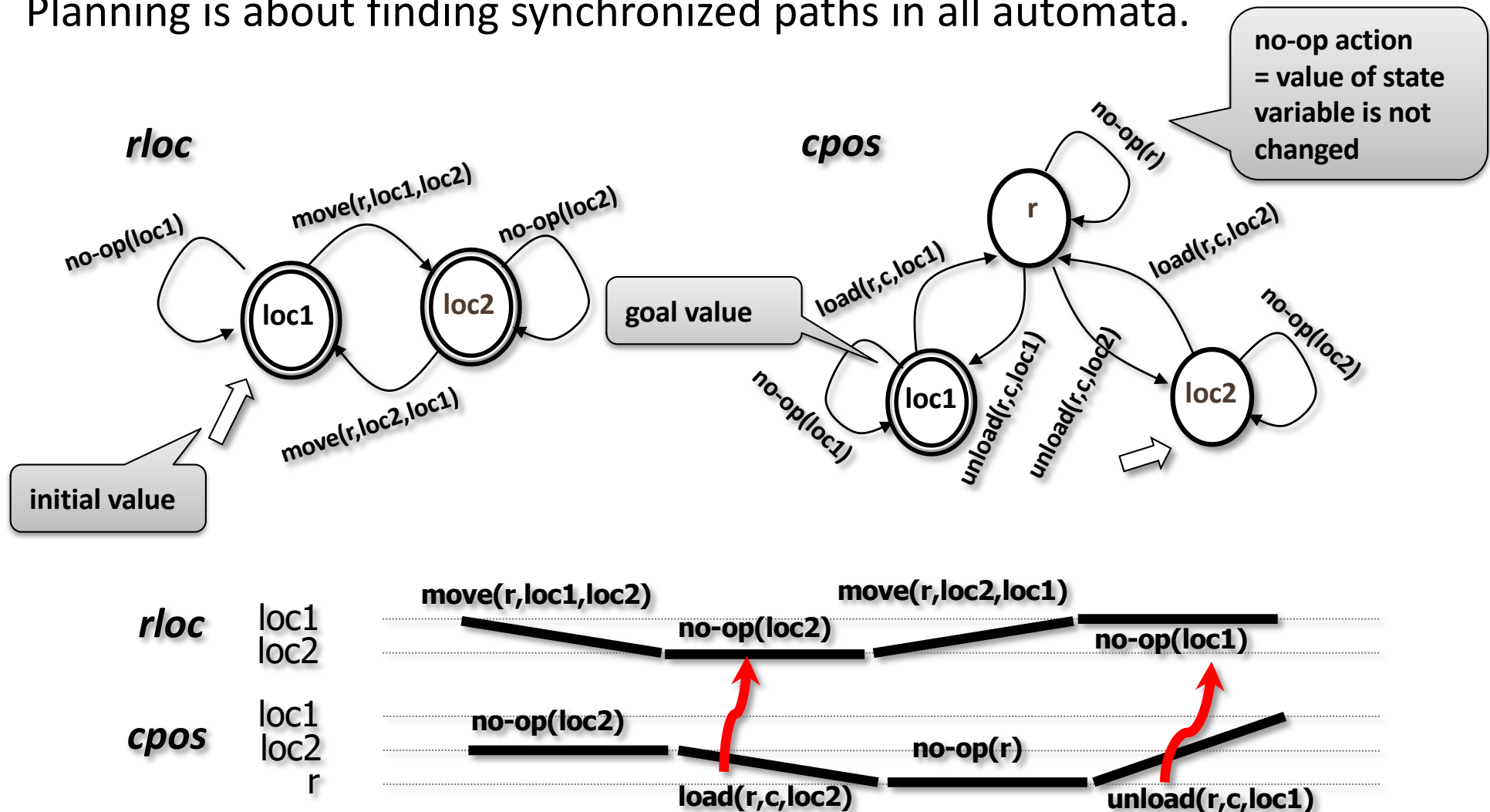
$$A^s = act \rightarrow Pre(act)^s, \forall act \in Dom(A^s)$$

- successor state constraint

$$V_i^s = val \leftrightarrow A^{s-1} \in C(i, val) \vee (V_i^{s-1} = val \wedge A^{s-1} \in N(i))$$

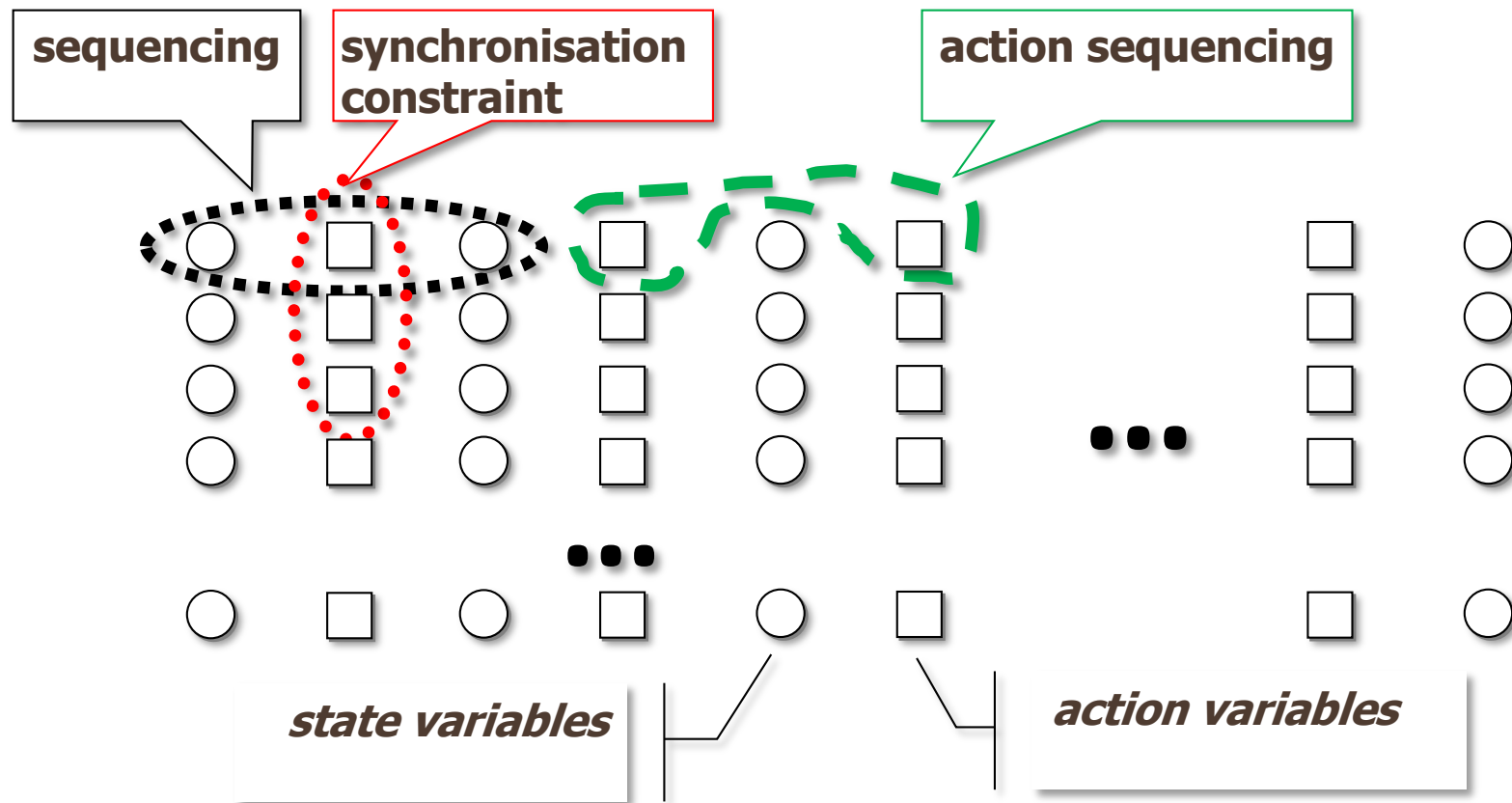
- $C(i, val)$  = the set of actions containing  $V_i \leftarrow val$  among their effects
- $N(i) = NonAffAct(V_i)$

Planning can also be seen as **synchronized changes of state variables**. Evolution of each variable is described using finite state automaton. Planning is about finding synchronized paths in all automata.



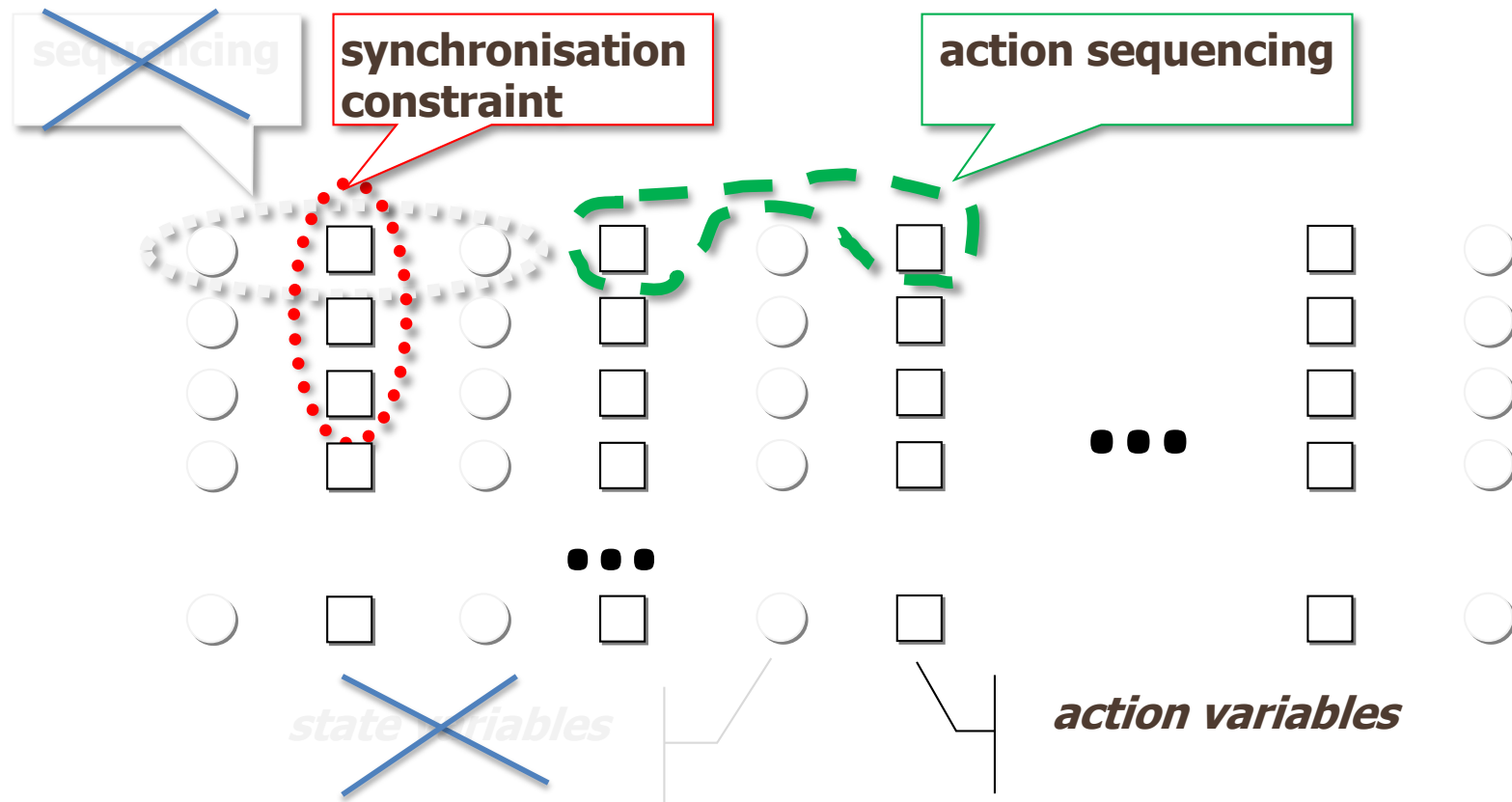
## timeline model

state and action variables organized to „layers“



# timeline model

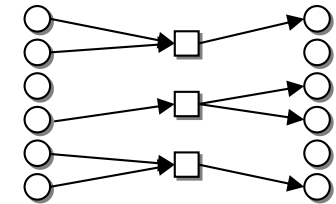
with action variables only



Which actions can be used in parallel in a single step?

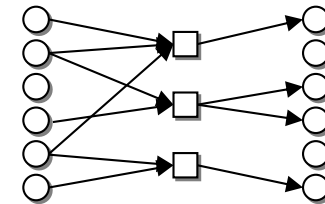
## $\forall$ -step encoding

- only actions that do not share state variables in preconditions and effects (independent actions)
- any order of actions can be used to reach the same state



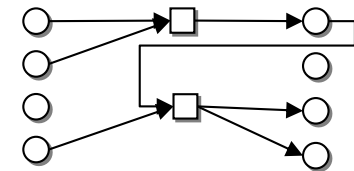
## $\exists$ -step encoding [Rintanen et al. 2006]

- action preconditions are satisfied in the previous layer
- actions effects do not destroy preconditions and effects of other actions in the step



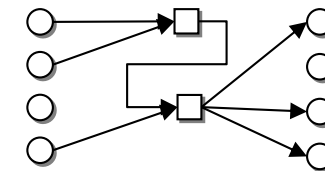
## Relaxed $\exists$ -step encoding [Wehrle and Rintanen 2007]

- like  $\exists$ -step encoding but other actions in the step may provide preconditions



## Relaxed<sup>2</sup> $\exists$ -step encoding [Balyo 2013]

- like relaxed  $\exists$ -step encoding but effects of some actions might be destroyed within the parallel step (if not needed in next layer)





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