Safety Verification of Hybrid Systems

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Hybrid System

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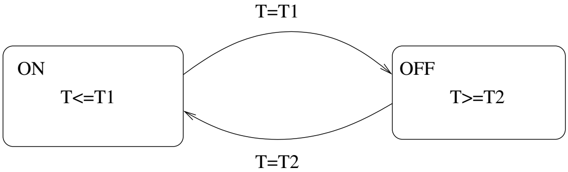
- Example: Thermostat



$$\dot{T}_{on} = c_1(T - T_0) + c_2$$
$$\dot{T}_{off} = c_1(T - T_0)$$

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- Jump: Relation on State pairs. If two states are in jump relation, there is possible discrete transition between them

What can we model with Hybrid Systems?

- Various traffic protocols within project AVACS `http://avacs.org`
  - Aircraft collision avoidance protocol
  - ETCS European Train Control System
- Embedded systems
  - Consumer and household products
  - Office, telecommunications devices
  - Medicine measurement and treatment devices
- Networking and locking protocols
- Physical systems with impact

Verification of Hybrid System

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- In Jump
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Correctness of algorithm should not be hampered by floating-point rounding errors (reliable computing)

Basic Verification Algorithm

- We split continuous statespace into multidimensional intervals - boxes
- Box and Mode form abstract state of the Hybrid System. We explore possible transitions between abstract states. This involves solving quantified constraints.
- If there is a route from initial to unsafe abstract state, we refine abstraction by splitting one of the boxes

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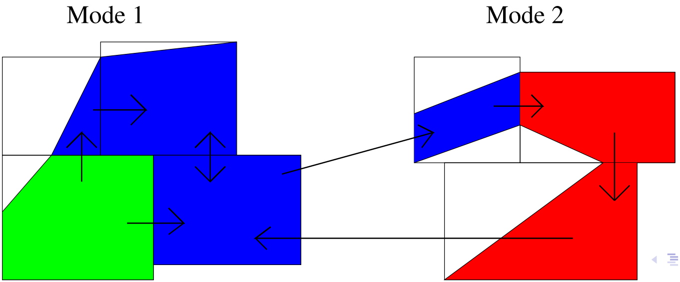
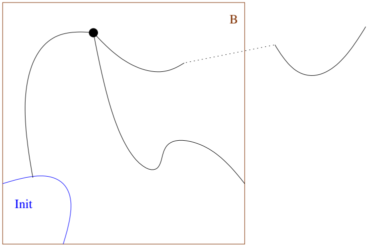


Reach Set Constraint

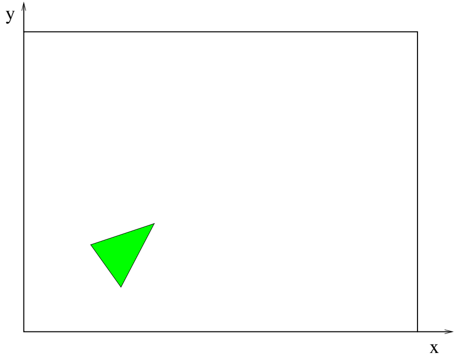
A point in a box B can be reachable

- from the initial set via a flow in B
- from a jump via a flow in B
- from a neighboring box via a flow in B



Reachability Analysis Using Constraints

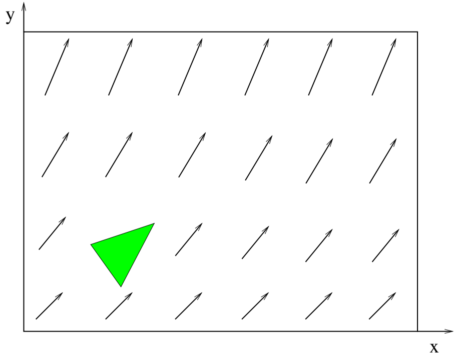
Single box and initial states



$Init(x_0, y_0)$

Reachability Analysis Using Constraints

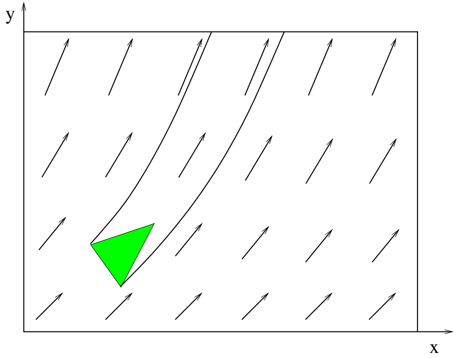
Vector field for derivatives



$Init(x_0, y_0) \wedge Flow(x_d, y_d, x_a, y_a)$

Reachability Analysis Using Constraints

Reachable states for this example



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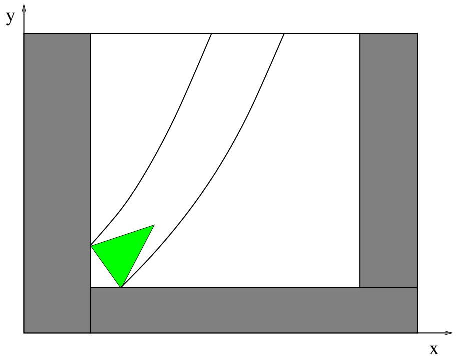
Reachability Analysis Using Constraints

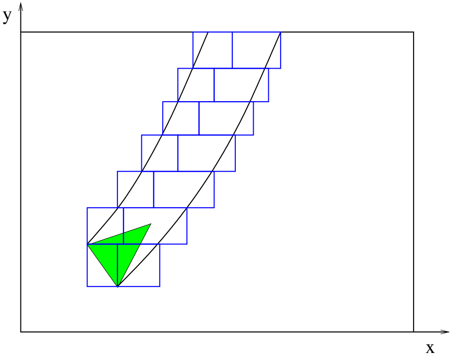
We create a constraint that overapproximates the reachable set and solve it, using interval methods



$\exists x_0, y_0, x_a, y_a, t :$
$Init(x_0, y_0) \land Flow(x_d, y_d, x_a, y_a) \land$
$x = x_0 + t * x_d \land y = y_0 + t * y_d$

Reachability Analysis Using Constraints

Splitting reduces overapproximation



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$Init(x_0, y_0) \wedge Flow(x_d, y_d, x_a, y_a) \wedge$
$x = x_0 + t * x_d \wedge y = y_0 + t * y_d$

Method Properties

Advantages of this method:

- Possible non-linear `Init`, `Flow`, `Unsafe` and `Jump` constraint
- Speed of one solving step
- Simple hyper-rectangle representation
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Disadvantage of the method:

- Potentially big amount of overapproximation

Tools We Use for Verification

RSolver - algorithm for solving quantified inequality constraints

- Based on constraint propagation

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All computations have to be rounding-safe

- Use of interval arithmetic library

Future Work

Improving abstraction: Use polyhedrons or Taylor Models instead of boxes

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Improving verification algorithm

- Exact solution for certain types of ODEs
- Polyhedral Quantifier Elimination
- Reduce Wrapping Effect
- Increase degree of Taylor constraint
- SAT modulo ODE
- LP Solver on linear jumps/flows etc.

References

- T. Dzetkulič, S. Ratschan: How to Capture Hybrid Systems Evolution into Slices of Parallel Hyperplanes, ADHS 2009
- S. Ratschan and Z. She: Safety verification of hybrid systems by constraint propagation based abstraction refinement, HSCC 2005
- `http://hsolver.sourceforge.net/`