

# Constraint Programming

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Modelling Planning Problems

## Example Problem

### State Variables

$rloc \in \{loc1, loc2\}$  ;; robot's location  
 $cpos \in \{loc1, loc2, r\}$  ;; container's position

### Actions

**move(r, loc1, loc2)** ;; robot r at location loc1 moves to location loc2  
 Precond:  $rloc = loc1$   
 Effects:  $rloc \leftarrow loc2$

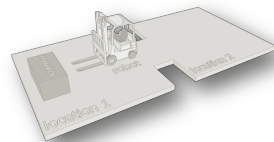
**move(r, loc2, loc1)** ;; robot r at location loc2 moves to location loc1  
 Precond:  $rloc = loc2$   
 Effects:  $rloc \leftarrow loc1$

**load(r, c, loc1)** ;; robot r loads container c at location loc1  
 Precond:  $rloc = loc1, cpos = loc1$   
 Effects:  $cpos \leftarrow r$

**load(r, c, loc2)** ;; robot r loads container c at location loc2  
 Precond:  $rloc = loc2, cpos = loc2$   
 Effects:  $cpos \leftarrow r$

**unload(r, c, loc1)** ;; robot r unloads container c at location loc1  
 Precond:  $rloc = loc1, cpos = r$   
 Effects:  $cpos \leftarrow loc1$

**unload(r, c, loc2)** ;; robot r unloads container c at location loc2  
 Precond:  $rloc = loc2, cpos = r$   
 Effects:  $cpos \leftarrow loc2$



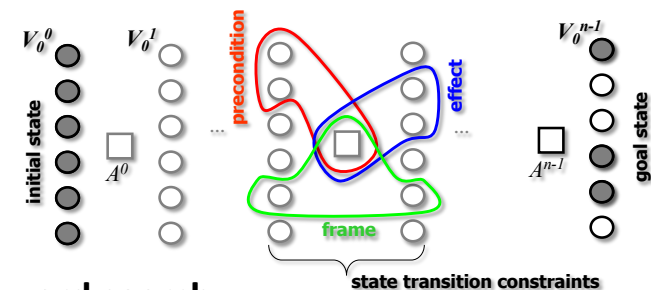
## We will deal with classical AI planning

- looking for the (shortest) sequence of actions (a **plan**) transferring the initial state of the world to a state satisfying some goal condition
- **state** is described using a set of **multi-valued variables**
- (grounded) **action** is specified by:
  - **precondition** (required values of certain state variables before action execution)
  - **effect** (changed values of certain state variables after action execution)

## Core Modeling Approach

### Iterative extension of the plan length

Formulating the problem of finding a plan of a given length as a CSP



### Backward search

- instantiation of action variables
- only actions relevant to the (sub)goal are tried

# Straightforward Model

## original formulation

- action constraints

$$A^s = act \rightarrow Pre(act)^s, \forall act \in Dom(A^s)$$

$$A^s = act \rightarrow Eff(act)^{s+1}, \forall act \in Dom(A^s)$$

- frame constraint

$$A^s \in NonAffAct(V_i) \rightarrow V_i^s = V_i^{s+1}, \forall i \in \langle 0, v-1 \rangle$$

$$A^s = move21 \rightarrow rloc^s = loc2$$

$$A^s = move21 \rightarrow rloc^{s+1} = loc1$$

$$A^s = move21 \rightarrow cpos^s = cpos^{s+1}$$

## problems

- disjunctive constraints do not propagate well

↳ do not prune well the search space

- a huge number of constraints (depend on the number of actions)

↳ the propagation loop takes a lot of time

# Model Reformulation



## idea

- encapsulate the logical constraints into a table constraint describing allowed tuples of values
- be careful about the size of the table!

## reformulated straightforward model

- action constraint = a single table

A <sup>s</sup>	rloc <sup>s</sup>	cpo <sup>s</sup>	rloc <sup>s+1</sup>	cpo <sup>s+1</sup>
move21	loc2		loc1	
move12	loc1		loc2	
load1	loc1	loc1		r
...				

- frame constraint

$$A^s \in NonAffAct(V_i) \rightarrow V_i^s = V_i^{s+1}, \forall i \in \langle 0, v-1 \rangle$$

# CSP-PLAN



## idea

- focus on modeling the reason for the value of a state variable (effect and frame constraints are merged)

## original model

- precondition constraint

$$A^s = act \rightarrow Pre(act)^s, \forall act \in Dom(A^s)$$

- successor state constraint

$$V_i^s = val \leftrightarrow A^{s-1} \in C(i, val) \vee (V_i^{s-1} = val \wedge A^{s-1} \in N(i))$$

- $C(i, val)$  = the set of actions containing  $V_i \leftarrow val$  among their effects
- $N(i)$  =  $NonAffAct(V_i)$

## reformulated model

- use a single table constraint to describe preconditions
- use ternary table constraints to describe successor state constraints (one table per state variable)

# CSP-PLAN Constraints

Table for precondition constraint

A <sup>s</sup>	rloc <sup>s</sup>	cpo <sup>s</sup>
1: move(r, loc1, loc2)	loc1	{...}
2: move(r, loc2, loc1)	loc2	{...}
3: load(r, c, loc1)	loc1	loc1
4: load(r, c, loc2)	loc2	loc2
5: unload(r, c, loc1)	loc1	r
6: unload(r, c, loc2)	loc2	r

Tables for successor state constraints

A <sup>s</sup>	rloc <sup>s</sup>	rloc <sup>s+1</sup>
2	{...}	loc1
1	{...}	loc2
{3,4,5,6}	loc1	loc1
{3,4,5,6}	loc2	loc2

A <sup>s</sup>	cpo <sup>s</sup>	cpo <sup>s+1</sup>
5	{...}	loc1
6	{...}	loc2
{3,4}	{...}	r
{1,2}	loc1	loc1
{1,2}	loc2	loc2
{1,2}	r	r

# Model Comparison

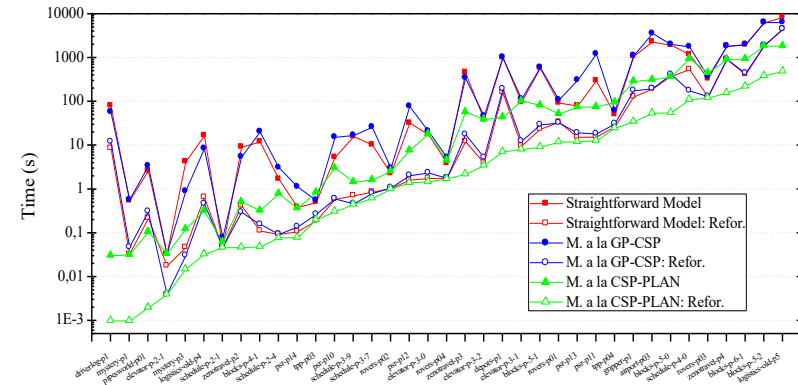
The total number of constraints

	original	reformulated
straightforward	$n(ap+ae+v)$	$n(1+v)$
GP-CSP	$n(ap+ae+3v)$	$n(1+3v)$
CSP-Plan	$n(ap+vd)$	$n(1+v)$

n - number of actions in the plan  
 a - number of grounded actions in the problem  
 v - number of multi-valued variables  
 p - average number of preconditions per action  
 e - average number of effects per action

# Model Comparison

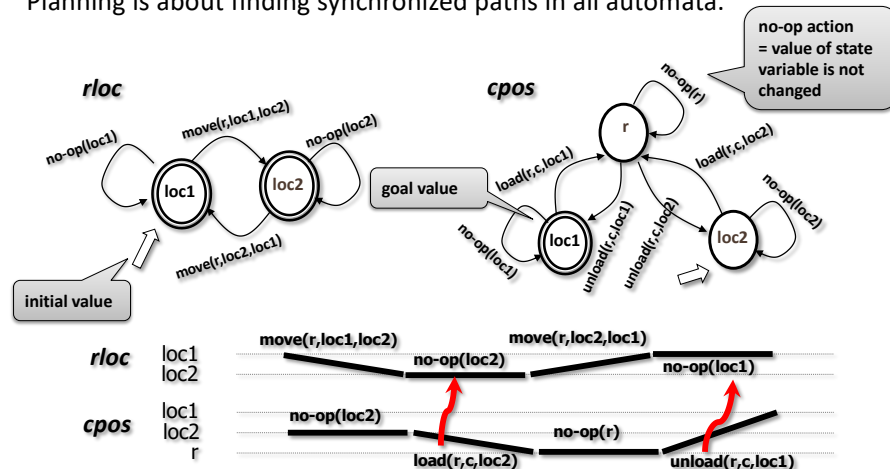
The runtime to solve selected problems from IPC 1-5 (logarithmic scale)



Barták (2011)

## Timelines

Planning can also be seen as **synchronized changes of state variables**. Evolution of each variable is described using finite state automaton. Planning is about finding synchronized paths in all automata.

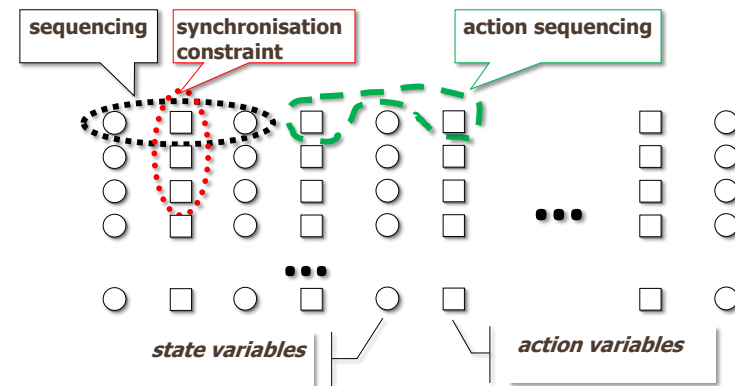


Barták (2011)

## PaP: Constraint Model

### timeline model

state and action variables organized to „layers“



## Search Strategy



a more or less standard CP labeling procedure  
instantiating (by the search algorithm) only the  
**action variables**

– the state variables are instantiated by inference

- **variable selection**

– dom heuristic (only variables with real action in their domain are assumed)

- **value selection** (in two steps)

– split the domain into no-op actions (explored first) and real actions

– domains with real actions only are enumerated then

## Summary Results (#solved problems)

planning domain	SeP	PaP
airport (15)	4	<b>6</b>
blocks (16)	<b>7</b>	<b>7</b>
depots (10)	<b>2</b>	<b>2</b>
driverlog (15)	4	<b>12</b>
elevator (30)	<b>30</b>	27
freecell (10)	1	<b>3</b>
openstacks (7)	<b>5</b>	0
rovers (10)	4	<b>6</b>
tpp (15)	4	<b>8</b>
zenotravel (15)	6	<b>11</b>

problems from International Planning Competition, runtime limit 30 minutes

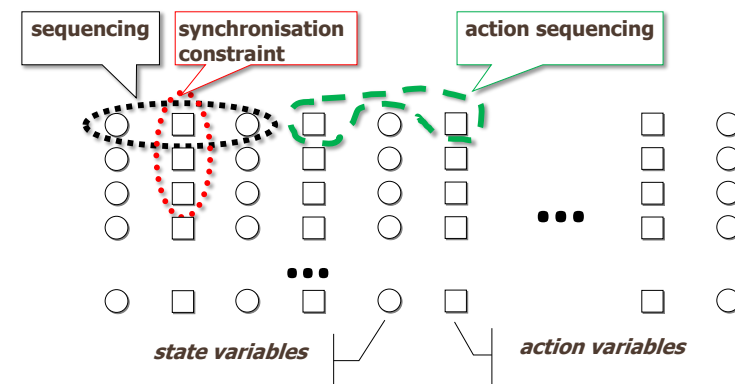
## Detailed Results (runtime)

problem	plan length			runtime (ms)	
	SeP	PaP		SeP	PaP
		par	seq		
zenotravel-p01	1	1	1	<b>10</b>	20
zenotravel-p02	6	5	6	60	<b>50</b>
zenotravel-p03	6	5	9	300	<b>130</b>
zenotravel-p04	8	5	11	970	<b>130</b>
zenotravel-p05	11	5	14	153 990	<b>240</b>
zenotravel-p06	11	5	12	530 390	<b>510</b>
zenotravel-p07	≥12	6	16	-	<b>560</b>
zenotravel-p08	≥10	5	15	-	<b>1 690</b>
zenotravel-p09	≥11	6	24	-	<b>145 760</b>
zenotravel-p10	≥12	6	24	-	<b>252 040</b>
zenotravel-p11	≥9	6	16	-	<b>41 780</b>

## PaP: Constraint Model

### timeline model

state and action variables organized to „layers“



## Summary Results (#solved problems)

planning domain	SeP	PaP	PaP-2
airport (15)	4	6	<b>8</b>
depots (10)	<b>2</b>	<b>2</b>	<b>2</b>
driverlog (15)	4	12	<b>13</b>
elevator (30)	<b>30</b>	27	<b>30</b>
freecell (10)	1	<b>3</b>	<b>3</b>
openstacks (7)	<b>5</b>	0	0
rovers (10)	4	6	<b>7</b>
tpp (15)	4	<b>8</b>	<b>8</b>
zenotravel (15)	6	11	<b>12</b>

## Detailed Results (runtime)

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		par	seq			
zenotravel-p01	1	1	1	<b>10</b>	20	30
zenotravel-p02	6	5	6	60	<b>50</b>	60
zenotravel-p03	6	5	9	300	<b>130</b>	140
zenotravel-p04	8	5	11	970	<b>130</b>	160
zenotravel-p05	11	5	14	153 990	<b>240</b>	320
zenotravel-p06	11	5	12	530 390	<b>510</b>	<b>350</b>
zenotravel-p07	≥12	6	16	-	<b>560</b>	<b>440</b>
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zenotravel-p10	≥12	6	24	-	<b>252 040</b>	<b>8 400</b>
zenotravel-p11	≥9	6	16	-	<b>41 780</b>	<b>3 250</b>
zenotravel-p12	≥11	6	24	-	-	<b>5 930</b>

## Conclusions

### Take away messages:

- constraint modeling is critical for efficiency
- models that prune more values are (usually) better
- search strategies can describe specific solving procedures

### Can we do even better?

- Yes, definitely (for example see paper “Transition Constraints for Parallel Planning”, AAAI 15)

### Constraints elsewhere in planning?

- solving specific sub-problems
- temporal and resource reasoning



## Course summary

Constraint satisfaction is a technology for **declarative solving combinatorial (optimization) problems**.

### Constraint modeling

- describing problems as constraint satisfaction problems (variables, domains, constraints)

### Constraint satisfaction

- local search techniques
- combination of depth-first search with inference (constraint propagation/consistency techniques)
- ad-hoc algorithms encoded in global constraints
- soft constraints to express preferences

**It is easy to model problems in terms of a CSP ... but it is complicated to design solvable models.**

