

Artificial Intelligence

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Knowledge Representation

How to effectively construct a **knowledge base**?
How should axioms look like?

- Representing **objects**
 - objects, categories, and ontologies
- Representing **time** and **actions**
 - situation calculus
 - frame problem



Let us notice that

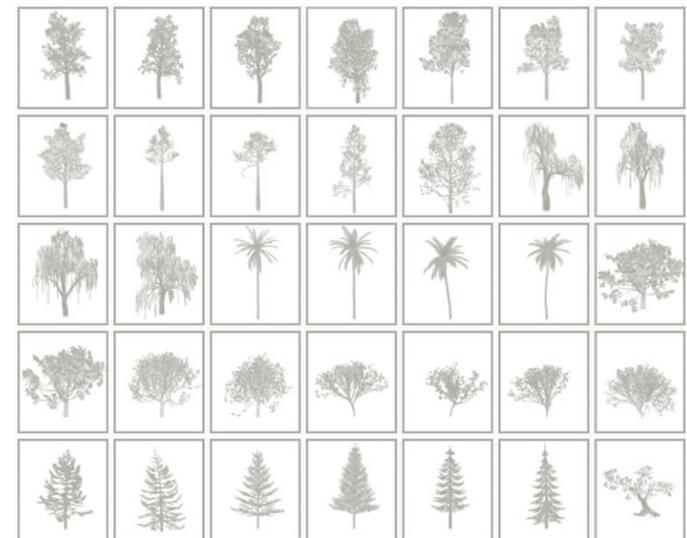
- agents **manipulate** with real **objects**
- but **reasoning** is done at the level of **categories**
- An agent uses observations to find properties of objects that are used to assign objects to categories. Reasoning on category then reveals useful information about the object itself.

Category

= a set of its members

= a complex object with relations

- MemberOf
- SubsetOf



How to represent a category in FOL?

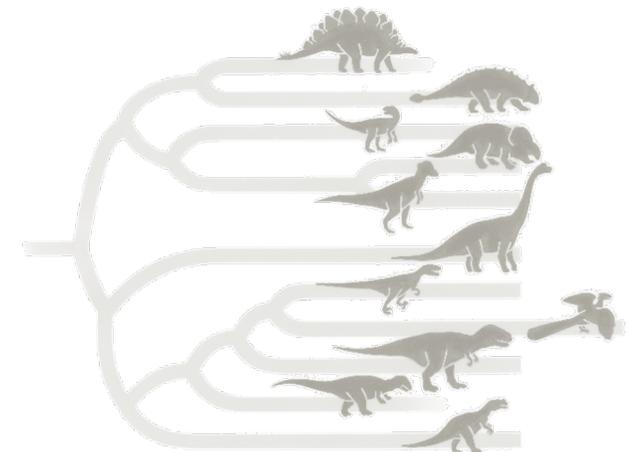
- an object is a **member** of a category
 - $\text{MemberOf}(\text{BB}_{12}, \text{Basketballs})$
- a category is **subset** of another category
 - $\text{SubsetOf}(\text{Basketballs}, \text{Balls})$
- all members of the category have some **property**
 - $\forall x (\text{MemberOf}(x, \text{Basketballs}) \Rightarrow \text{Round}(x))$
- all members of the category can be **recognized** using common properties
 - $\forall x (\text{Orange}(x) \wedge \text{Round}(x) \wedge \text{Diameter}(x)=9.5\text{in} \wedge \text{MemberOf}(x, \text{Balls}) \Rightarrow \text{MemberOf}(x, \text{Basketballs}))$
- category may also have some property
 - $\text{MemberOf}(\text{Dogs}, \text{DomesticatedSpecies})$

Categories organize and simplify knowledge base by using **inheritance of properties**.

- properties are defined for a category, but they are inherited to all members of the category
- food is eatable, fruits are food, apples are fruits, and hence apples are eatable

Subclasses organize categories to a **taxonomy**

- a hierarchical structure that is used to categorize objects
- originally proposed for classifying living organisms (alpha taxonomy)
- categories for all knowledge
 - Used in libraries
 - Dewey Decimal Classification
 - 330.94 European economy



So far we modelled a static world only.

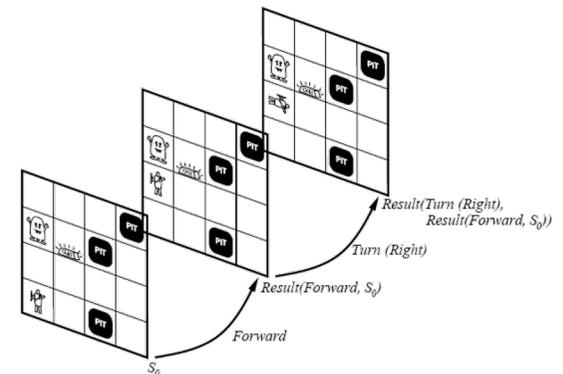
How to reason about actions and their effects in time?

In **propositional logic** we need a copy of each action for each time (situation):

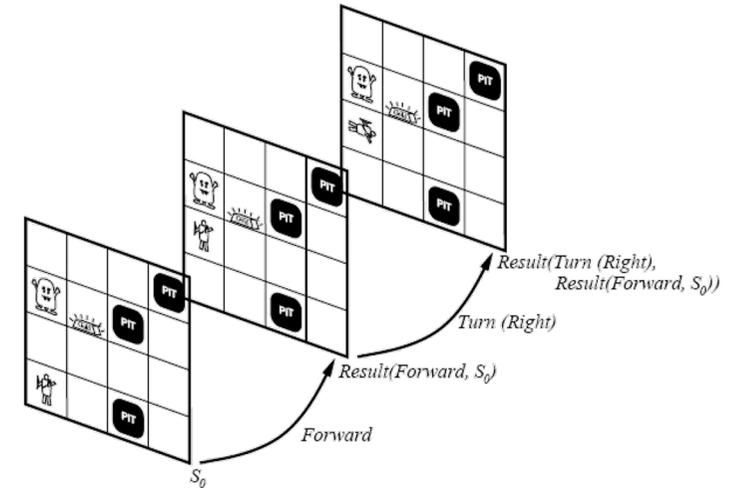
- $L_{x,y}^t \wedge \text{FacingRight}^t \wedge \text{Forward}^t \Rightarrow L_{x+1,y}^{t+1}$
- We need an upper bound for the number of steps to reach a goal but this will lead to a huge number of formulas.

Can we do it better in **first order logic**?

- We do not need copies of axioms describing state changes; this can be implemented using a universal quantifier for time (situation)
- $\forall t$ P is the result of action A in time t+1



- **actions** are represented by terms
 - $\text{Go}(x,y)$
 - $\text{Grab}(g)$
 - $\text{Release}(g)$
- **situation** is also a term
 - initial situation: S_0
 - situation after applying action a to state s : $\text{Result}(a,s)$
- **fluent** is a predicate changing with time
 - the situation is in the last argument of that term
 - $\text{Holding}(G, S_0)$
- **rigid (eternal) predicates**
 - $\text{Gold}(G)$
 - $\text{Adjacent}(x,y)$

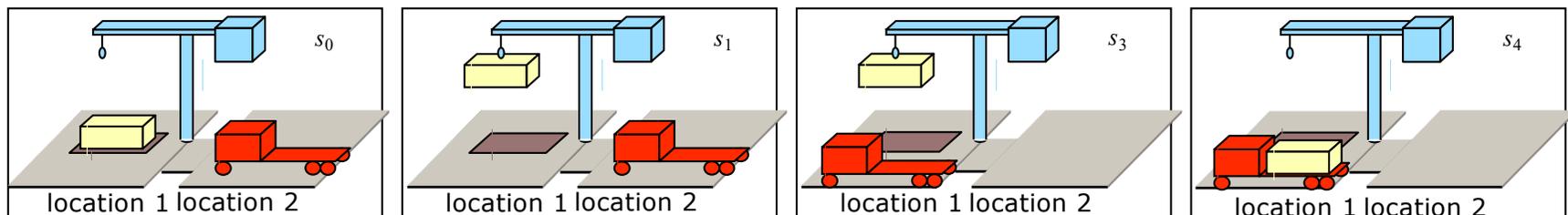


We need to reason about sequences of actions – about **plans**.

- $\text{Result}([],s) = s$
- $\text{Result}([a | \text{seq}],s) = \text{Result}(\text{seq}, \text{Result}(a,s))$

What are typical tasks related to plans?

- **projection task** – what is the state/situation after applying a given sequence of actions?
 - $\text{At}(\text{Agent}, [1,1], S_0) \wedge \text{At}(G, [1,2], S_0) \wedge \neg \text{Holding}(o, S_0)$
 - $\text{At}(G, [1,1], \text{Result}([\text{Go}([1,1],[1,2]), \text{Grab}(G), \text{Go}([1,2],[1,1])], S_0))$
- **planning task** – which sequence of actions reaches a given state/situation?
 - $\exists \text{seq } \text{At}(G, [1,1], \text{Result}(\text{seq}, S_0))$



Each **action** can be described using two axioms:

- **possibility axiom:** $\text{Preconditions} \Leftrightarrow \text{Poss}(a,s)$
 - $\text{At}(\text{Agent},x,s) \wedge \text{Adjacent}(x,y) \Leftrightarrow \text{Poss}(\text{Go}(x,y),s)$
 - $\text{Gold}(g) \wedge \text{At}(\text{Agent},x,s) \wedge \text{At}(g,x,s) \Leftrightarrow \text{Poss}(\text{Grab}(g),s)$
 - $\text{Holding}(g,s) \Leftrightarrow \text{Poss}(\text{Release}(g),s)$
- **effect axiom:** $\text{Poss}(a,s) \Rightarrow \text{Changes}$
 - $\text{Poss}(\text{Go}(x,y),s) \Rightarrow \text{At}(\text{Agent},y,\text{Result}(\text{Go}(x,y),s))$
 - $\text{Poss}(\text{Grab}(g),s) \Rightarrow \text{Holding}(g,\text{Result}(\text{Grab}(g),s))$
 - $\text{Poss}(\text{Release}(g),s) \Rightarrow \neg \text{Holding}(g,\text{Result}(\text{Release}(g),s))$

Beware! This is not enough to deduce that a plan reaches a given goal.

We can deduce $\text{At}(\text{Agent}, [1,2], \text{Result}(\text{Go}([1,1],[1,2]), S_0))$
but we **cannot deduce** $\text{At}(G, [1,2], \text{Result}(\text{Go}([1,1],[1,2]), S_0))$

Effect axioms describe what has been changed in the world but they say nothing about the property that everything else is not changed!

This is a so called **frame problem**.

We need to represent properties that are not changed by actions.

A simple **frame axiom** says what is not changed:

$$\text{At}(o,x,s) \wedge o \neq \text{Agent} \wedge \neg \text{Holding}(o,s) \Rightarrow \\ \text{At}(o,x,\text{Result}(\text{Go}(y,z),s))$$

- for F fluents and A actions we need $O(FA)$ frame axioms
- This is a lot especially taking in account that most predicates are not changed.



Can we use less axioms to model the frame problem?

- **successor-state axiom**

$\text{Poss}(a,s) \Rightarrow$
 $(\text{fluent holds in Result}(a,s) \Leftrightarrow$
 $\text{fluent is effect of } a \vee (\text{fluent holds in } s \wedge a \text{ does not change fluent}))$

– We get F axioms (F is the number of fluents) with $O(AE)$ literals in total (A is the number of actions, E is the number of effects).

Examples:

$\text{Poss}(a,s) \Rightarrow$
 $(\text{At}(\text{Agent},y,\text{Result}(a,s)) \Leftrightarrow a=\text{Go}(x,y) \vee (\text{At}(\text{Agent},y,s) \wedge a \neq \text{Go}(y,z)))$

$\text{Poss}(a,s) \Rightarrow$
 $(\text{Holding}(g,\text{Result}(a,s)) \Leftrightarrow a=\text{Grab}(g) \vee (\text{Holding}(g,s) \wedge a \neq \text{Release}(g)))$

Beware of implicit effects!

- If an agent holds some object and the agent moves then the object also moves.
- This is called a **ramification problem**.

$\text{Poss}(a,s) \Rightarrow$
 $(\text{At}(o,y,\text{Result}(a,s)) \Leftrightarrow$
 $(a=\text{Go}(x,y) \wedge (o=\text{Agent} \vee \text{Holding}(o,s))) \vee$
 $(\text{At}(o,y,s) \wedge \neg \exists z (y \neq z \wedge a=\text{Go}(y,z) \wedge (o=\text{Agent} \vee \text{Holding}(o,s))))))$



Successor-state axiom is still too big with $O(AE/F)$ literals in average.

- To solve the projection task with t actions, the time complexity depends on the total number of actions – $O(AEt)$ – rather than on the actions in plan.
- If we know each action, cannot we do it better, say $O(Et)$?

classical successor-state axiom:

$$\text{Poss}(a,s) \Rightarrow (F_i(\text{Result}(a,s)) \Leftrightarrow (a=A_1 \vee a=A_2 \vee \dots) \vee (F_i(s) \wedge a \neq A_3 \wedge a \neq A_4 \dots))$$

actions having F_i among effects

actions having $\neg F_i$ among effects

We can introduce **positive** and **negative effects** of actions:

- **PosEffect(a, F_i)** action a causes F_i to become true
- **NegEffect(a, F_i)** action a causes F_i to become false

modified successor-state axiom:

$$\text{Poss}(a,s) \Rightarrow (F_i(\text{Result}(a,s)) \Leftrightarrow \text{PosEffect}(a, F_i) \vee (F_i(s) \wedge \neg \text{NegEffect}(a, F_i)))$$

PosEffect(A_1 , F_i)

PosEffect(A_2 , F_i)

NegEffect(A_3 , F_i)

NegEffect(A_4 , F_i)





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