Bio-Inspired SAT

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Outline

● EA and SAT
● General ideas for EA
● Belief propagation
● GPU
Evolutionary algorithm (EA)

- Pseudo-code for a genetic algorithm:

  Initialize the population
  Evaluate initial population
  Repeat
    Perform competitive selection
    Apply genetic operators to generate new solutions
    Evaluate solutions in the population
  Until some convergence criteria is satisfied
EA and SAT - Representation

- Bit String Representation
- Floating Point Representation
  - transforming SAT into a continuous optimization problem
    - replace literals $x_j$ and $\neg x_j$ by $(x_j-1)^2$ and $(x_j+1)^2$
    - substitute $\wedge$ and $\vee$ by $*$ and $+$
    - minimization
- Path Representation
- Graphs:
  - Node: variables, edge: appear at the same clause
  - Node: clause, edge: share the same variable
## EA and SAT - Operators and Fitness

- **Operators**
  - Crossover
  - Mutation
- **Local search => Memetic algorithms (MA)**
- **Fitness**
  - \( f_{\text{MAX}} = c_1(x) + \ldots + c_m(x) \)
  - \( f_{\text{SAW}}(x) = w_1 \cdot c_1(x) + \ldots + w_m \cdot c_m(x) \)
    - stepwise adaptation of weights (SAW) principle
    - \( w_i \leftarrow w_i + 1 - c_i(x^*) \), \( x^* \) is the current fittest individual
  - \( f_{\text{REF}}(x) = c_1(x) + \ldots + c_m(x) + \alpha \cdot r(x), \ \alpha \in [0, 1) \)
    - refining function
  - Multi-objective fitness
General ideas for EA

- **Structured-population EA**
  - Standard GA = Panmictic model of GA
- **Multi-agent EA**
- **PSO and its variants**
Island GA

- Populate islands
- While (not :-))
  - Separate island evolution
  - Replacement of individuals
Cellular EA

L5  L9  C9

C25  D13
Patchwork GA

Figure 1: A grid interconnection topology in classical diffusion models (a) and in the PATCHWORK model (b).
Terrain-Based GA

Figure 2: Mutation rate Spread Along the X-axis of a CGA. Number of Crossover pts Spread Along the Y-axis.

Figure 3: Mutation Rate (along the X-axis) and Number of Crossover Points (along the Y-axis), after Sifting.
Religion-based GA

- Initialization
- While( not :-) )
  - Random Walk
  - Conversion
  - Mating
Multi-national GA
Multi-agent EA

Fig. 1 The model of the agent lattice
Hypercube Tree

- Initialization
- While( not :-) )
  - Selection and mating in leaf and with neighbours
  - Split leaf in hypercubes?
  - Join leaves?
Particle Swarm Optimization (PSO)

- Inspired by the social behavior of bird flocking
- \( x_i(t) = x_i(t-1) + v_i(t) \)
- \( v_i(t) = \omega v_i(t-1) + \varphi_1 r_1 (p_i - x_i(t-1)) + \varphi_2 r_2 (p_g - x_i(t-1)) \)

Parameter selection:
- maximum velocity
- acceleration constants
- inertia constant
PSO - Variants

- **Topology:**
  - global best
  - local best

- **Binary PSO**
  - $s(v_i(t)) = 1 / (1 + \exp(-v_i(t))$ (sigmoid function)
  - $x_i(t) = 1$, if rand < $s(v_i(t))$
  - $x_i(t) = 0$, otherwise

- **Memetic approach (local search)**

- **Hybrid PSO**
  - Genetic Algorithm and PSO
  - Evolutionary Programming and PSO
  - Differential Evolution and PSO

- **PSO in Complex Environment**
  - Multi-objective Particle Swarm Optimization
  - Constraint Handling in PSO

- **Other Variants of PSO**
  - Gaussian PSO
  - Dissipative PSO
  - Cooperative PSO

- ...
Belief propagation

“I (variable x) think that you (variable y) belong in these states with various likelihoods…”

- Converges for trees
- Empiric proof for general graph: $O(\#\text{vars} * \exp(k))$
  - where $k=3, 4, 5, \ldots$?

\[
m_{ij}^{\text{new}}(x_j) = \sum_{x_i} f_{ij}(x_i, x_j) g_i(x_i) \prod_{k \in \text{Nbd}(i) \setminus j} m_{ki}^{\text{old}}(x_i) h(x_i)
\]

\[
m_{ij}^{\text{new}}(x_j) = \sum_{x_i} f_{ij}(x_i, x_j) h(x_i)
\]
SAT & GPU

- Poor results with WalkSat
- Survey SAT: 20x faster
- MiniSat: 2x faster
- DPLL: 10x faster
Q & A
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