The fourth homework are Problems 5 and 6. You can use only the following problems for reductions: SAT, 3-SAT, Hamiltonian cycle, Vertex cover, 3-Dimensional matching, Subset-sum, Partition (see Problem 1), 3-partition.

Problem 1. Prove that the following Partition problem

Instance: Positive integers $a_1, \dots, a_k$.
Question: Can integers $a_1, \dots, a_k$ be split into two groups of the same sum?

is NP-complete if all integers are binary coded and polynomial if all integers are unary coded.

Problem 2. In CNF, a clause is a disjunction of literals and a formula is a conjunction of clauses. In DNF (Disjunctive normal form), a clause is a conjunction of literals and a formula is a disjunction of clauses. Is the problem deciding whether a given DNF formula is satisfiable (called DNF-SAT) polynomial or NP-complete?

Problem 3. For the sake of contradiction assume that $\mathsf{P} = \mathsf{NP}$. Then SAT $\in \mathsf{P}$, so there exists k such that SAT $\in \mathsf{TIME}(n^k)$ where n is the length of SAT formula. Since every language in NP is polynomially reducible to SAT (i.e. in time $\mathcal{O}(n^l)$ for some l), every language in NP is deterministically solvable in time $\mathcal{O}(n^{kl})$ which implies $\mathsf{NP} \subseteq \mathsf{TIME}(n^{kl})$. However, time hierarchy theorem implies that there exists a language $A \in \mathsf{TIME}(n^{kl+1}) \setminus \mathsf{TIME}(n^{kl})$ which gives a contraction $\mathsf{P} \subseteq \mathsf{NP} \subseteq \mathsf{TIME}(n^{kl}) \subsetneq \mathsf{TIME}(n^{kl+1}) \subseteq P$.

Every CNF formula can be easily transformed to an equivalent DNF formula using the distribution law. Then, we can decide whether the obtained DNF formula is satisfiable in polynomial time.

The first paragraph claims that $\mathsf{P} \neq \mathsf{NP}$ while the second one claims that $\mathsf{P} = \mathsf{NP}$. Is it correct?

Problem 4. Decide whether the following problem is polynomial or NP-complete.
Instance: A complete graph $G = (V, E)$ with weights on edges $w : E(G) \to \mathbb{N}$ satisfying triangular inequality and a limit d.
Question: Does G contain a Hamiltonian cycle with total weight at most d?

Problem 5. Consider the following variant of partition problem.

Instance: Positive integers $a_1, \dots, a_{3k}$.
Question: Can integers $a_1, \dots, a_{3k}$ be split into 3 groups so that the sum of integers in each group is the same (i.e. $\frac{1}{3}\sum_{i=1}^{3k} a_i$)?

Is this problem polynomial or NP-complete if all integers are binary coded? Is this problem polynomial or NP-complete if all integers are unary coded?

Problem 6. Decide whether the following problem is polynomial or NP-complete.
Instance: An undirected graph $G = (V, E)$.
Question: Does G contain a path throught all vertices?

Problem 7. Decide whether the following problem is polynomial or NP-complete.
Instance: A set of tasks U, processing time $d(u) \in \mathbb{N}$ associated with every task $u \in U$, number of processors m, deadline $D \in \mathbb{N}$
Question: Is it possible to assign all tasks to processors so that the (parallel) processing time is at most D?