

Problem 1. Consider the polyhedron P given by the following conditions.

$$\begin{aligned} x_1 - x_2 &\leq -1 \\ -x_1 - x_2 &\leq -3 \\ 2x_1 - x_2 &\leq 2 \\ x_1, x_2 &\geq 0. \end{aligned}$$

1. Draw the polyhedron P .
2. Using the Simplex methods find all vertices of P .
3. Find the optimal solution of the problem $\min \{x_1 + 2x_2; x \in P\}$.
4. Find the optimal solution of the problem $\max \{3x_1 + x_2; x \in P\}$.

Problem 2. Solve the following problem

$$\begin{aligned} \text{Maximize} \quad & 3x_1 + x_2 \\ \text{subject to} \quad & x_1 - x_2 \leq -1 \\ & -x_1 - x_2 \leq -3 \\ & 2x_1 + x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Problem 3. Solve the following problem

$$\begin{aligned} \text{Maximize} \quad & x_1 + 2x_2 \\ \text{subject to} \quad & x_1 + 3x_2 + x_3 = 4 \\ & 2x_2 + x_3 = 2 \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

Problem 4. Solve the following problem

$$\begin{aligned} \text{Maximize} \quad & x_2 \\ \text{subject to} \quad & -x_1 + x_2 \leq 0 \\ & x_1 \leq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Problem 5 (Homework – 2 points). Solve the following problem

$$\begin{aligned} \text{Maximize} \quad & 10x_1 - 57x_2 - 9x_3 - 24x_4 \\ \text{subject to} \quad & x_5 = -0,5x_1 + 5,5x_2 + 2,5x_3 - 9x_4 \\ & x_6 = -0,5x_1 + 1,5x_2 + 0,5x_3 - x_4 \\ & x_7 = 1 - x_1 \\ & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0 \end{aligned}$$

First, try to use the pivot rule “largest coefficient”. Then, solve the problem using “Bland rule”.

Problem 6 (Homework – 2 points). Find all optimal vertices of the following problem.

$$\begin{aligned} \text{Maximize} \quad & 2x_1 + 3x_2 + 5x_3 + 4x_4 \\ \text{subject to} \quad & x_1 + 2x_2 + 3x_3 + x_4 \leq 5 \\ & x_1 + x_2 + 2x_3 + 3x_4 \leq 3 \\ & x_1 + x_2 + 2x_3 + 7x_4 \geq 3 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$