Problem 1. Consider the Knapsack problem with the following items.

id	1	2	3	4	5
weight w_i	21	11	51	26	30
price c_i	37	12	500	50	41

Find the optimal solution using the simplex method with Gomory-Chvátal cutting planes.

Problem 2. Prove that the linear programming

$$\begin{array}{lrcll} \text{Minimize} & \boldsymbol{cx} & & & \\ \text{subject to} & \delta^u \boldsymbol{x} & = & 1 & \text{for all } u \in V \\ & \delta^D \boldsymbol{x} & \geq & 1 & \text{for all } D \in \mathcal{C} \\ & \boldsymbol{x} & \geq & \mathbf{0} & \end{array}$$

is feasible if and only if G has a perfect matching (without using algorithms from the lecture). Also prove the convex hull of characteristic vectors of perfect matchings is exactly the set of all feasible solution this set of linear inequalities.

Problem 3 (Homework)**.** For every $n \geq 3$ find a connected graph on n vertices such that the relaxed linear programming problem for perfect matching has no feasible solution.

Problem 4. Let G be a bipartite graph and $\boldsymbol{x}$ be a relaxed feasible solution of the linear programming problem for perfect matching. If $\boldsymbol{x}$ is not integral, then prove that G contains a cycle C such that $\boldsymbol{x}_e$ is non-integral for every edge e on the cycle C. Find how the the cycle C can be used to reduce the number of non-integral values of $\boldsymbol{x}$. Is the assumption that G is bipartite necessary?

Problem 5 (Homework)**.** Slither is a two-person game played on a graph $G = (V, E)$. The players play alternatively. At each step the player whose turn it is chooses a previously unchosen edge. The only rule is that at every step the set of chosen edges forms a path. The loser is the player unable to extern the path. Prove that, if G has a perfect matching, then the first player has a winning strategy.