Problem 1. Let G be a graph and $\boldsymbol{x}$ be a relaxed feasible solution of the linear programming problem for perfect matching. If $\boldsymbol{x}$ is not integral, then prove that G contains a cycle C such that $\boldsymbol{x}_e$ is non-integral for every edge e on the cycle C. Show the the cycle C can be used to reduce the number of non-integral values of $\boldsymbol{x}$. If a graph is bipartite, show that we can obtain a perfect matching in this way.

Problem 2 (Homework)**.** Let n be an integer and let S be the set of all non-negative $n \times n$ matrices such that the sum of elements in every row and column is one. Prove that S is an integral polytope. Which matrices are vertices of S? For formal purposes, consider an $n \times n$ matrix as a vector in $\mathbb{R}^{n^2}$.

Problem 3. Let $G = (V, E)$ be a graph with weights $c \in \mathbb{R}^E$ and let k be an integer. A k-matching in G is a matching of cardinality k. Using the algorithm for minimum-weight perfect matching find minimum-weight k-matching.

Problem 4. An edge cover of a graph $G = (V, E)$ without isolated vertices is a set of edge D such that vertex of G is incident with at least one edge of D. Prove that is size of maximum matching plus the size of minimum edge cover equals to the number of vertices. Find an algorithm for the minimum-weight edge cover problem.

Problem 5 (Homework)**.** Let A be totally unimodular $m \times n$ matrix with full row rank. Let B be a basis of A (that is, a regular $n \times n$ submatrix of A). Prove that $B^{-1}A$ is totally unimodular.