Hypercube problems

Homework Assignment 1

Due: Nov 9, 2020 — Updated: October 18, 2020

1. Determine the *total distance* of Q_n,
$$td(Q_n) = \sum_{\{x,y\}} d_H(x,y).$$
2. Show that the (vertex) connectivity and the edge-connectivity of the hypercube is $\kappa(Q_n) = \lambda(Q_n) = n$.
3. Let Γ be a group acting on a set V. Show that stabilizers of two points in the same orbit are conjugate. That is, $\Gamma_{g(x)} = g^{-1}\Gamma_x g$ for every $g \in \Gamma$, $x \in V$.
4. An s-arc is a sequence $(v_0, \ldots, v_s)$ such that consecutive vertices are adjacent and $v_{i-1} \neq v_{i+1}$ $(0 < i < s)$. A graph G is s-arc-transitive if $Aut(G)$ acts transitively on s-arcs. Determine maximal s such that Q_n is s-arc-transitive.
5. Let $X, Y \subseteq V(Q_n)$ with $|X| = |Y| = 3$. Find a necessary and sufficient condition for the existence of $g \in Aut(Q_n)$ such that $g(X) = Y$.
6. A cycle C in a graph G is *isometric* if $d_C(x,y) = d_G(x,y)$ for every $x, y \in C$. Show that every automorphism of isometric C_6 in Q_3 is induced by an automorphism of Q_3.
7. Show that every group Γ acting transitively on a finite V, $|V| > 1$, has an element with no fixed points.
8. Determine a smallest generator of $Aut(Q_n)$.