

Hypercube problems

Homework Assignment 1

Due: Nov 11, 2021

Updated: October 20, 2021

1. Determine the *total distance* of Q_n ,

$$td(Q_n) = \sum_{\{x,y\}} d_H(x,y).$$

2. Show that the (vertex) connectivity and the edge-connectivity of the hypercube is $\kappa(Q_n) = \lambda(Q_n) = n$.
3. Let Γ be a group acting on a set V . Show that stabilizers of two points in the same orbit are conjugate. That is, $\Gamma_{g(x)} = g^{-1}\Gamma_xg$ for every $g \in \Gamma$, $x \in V$.
4. An s -arc is a sequence (v_0, \dots, v_s) such that consecutive vertices are adjacent and $v_{i-1} \neq v_{i+1}$ ($0 < i < s$). A graph G is s -arc-transitive if $Aut(G)$ acts transitively on s -arcs. Determine maximal s such that Q_n is s -arc-transitive.
5. Let $X, Y \subseteq V(Q_n)$ with $|X| = |Y| = 3$. Find a necessary and sufficient condition for the existence of $g \in Aut(Q_n)$ such that $g(X) = Y$.
6. A cycle C in a graph G is *isometric* if $d_C(x,y) = d_G(x,y)$ for every $x, y \in C$. Show that every automorphism of isometric C_6 in Q_3 is induced by an automorphism of Q_3 .
7. Show that every group Γ acting transitively on a finite V , $|V| > 1$, has an element with no fixed points.
8. Determine a smallest generator of $Aut(Q_n)$.