Hypercube problems

Homework Assignment 2

Due: November 30, 2020 Updated: November 8, 2020

1. A mapping $f\colon V(G) \to \{1, \ldots, k\}$ is a *distinguishing k-labeling* of a graph G if the only automorphism of G that preserves f is the identity. Find a distinguishing 3-labeling of Q_3 and show that Q_3 has no distinguishing 2-labeling.
2. Let F_n be the subgraph of Q_n induced by all vertices without two consecutive 1's. Determine $|V(F_n)|$ and show that F_n is a partial cube.
3. Show that in a bipartite graph, $(uv)\Theta(u'v')$ if and only if [1]
$$d(u, u') + d(v, v') \neq d(u, v') + d(u', v).$$
4. Show that every convex set in Q_n induces a subcube.
5. Show that every interval of a Hamming graph induces a hypercube. A *Hamming graph* is a Cartesian product of (several) complete graphs.
6. Show that a Cartesian product of two median graphs is a median graph.
7. Show that in a bipartite graph, every isometric path is a retract.
8. Show that for three pairs (x_i, y_i), $i = 1, 2, 3$, of vertices in Q_n there is a nonexpansive map f on Q_n with $f(x_i) = y_i$ for all $i = 1, 2, 3$ if and only if
$$d_H(y_i, y_j) \leq d_H(x_i, x_j) \quad \text{for every } 1 \leq i < j \leq 3.$$

[1] Winkler's definition of the relation Θ.