NAIL062 Propositional and predicate logic: Information about exams
Winter semester 2024/25

Exam dates

Exam dates will be available for registration in the Study Information System. The credit from the tutorial is required to register, with the exception of early exam dates (i.e., before the exam period starts, typically the beginning of January). Several exam dates will be scheduled after the end of the exam period, but their capacity will be limited. Therefore, we recommend at least two attempts in the Winter exam period. (We expect to open two exam dates during the teaching period of the Summer semester, and two more in the Summer exam period, each with capacity of approx. 20 students. The last exam date is planned for June.)

You can register for the exam, as well as cancel your registration, no later than 24 hours before it starts. Please check less than 24 hours before the exam to see if another student's cancellation has changed your allocated exam time.

Exam Requirements

The exam requirements correspond to the material covered in the lectures and described in the slides, with the exception of the following, which will not be required (the section numbers refer to the lecture notes):

- Section 2.5 Algebra of Propositions,
- Section 3.1 SAT Solvers,
- Section 3.4 The DPLL Algorithm,
- Section 4.8 Hilbert Calculus,
- Section 5.4 LI-resolution and Horn-SAT,
- Subsection 6.8.1 Database Queries,
- Section 7.6 Hilbert Calculus in Predicate Logic,
- Section 8.7 LI-resolution,
- Knowledge of proofs from Section 10.4 Gödel's Theorems will not be required.

Exam format and grading

The exam is oral, with written preparation phase (for which you will have 30 minutes). During the exam, you can be asked about any of the requirements, but the core of the exam will consist of an assignment that you will draw prior to the preparation phase. The assignment consists of three questions: "definition", "easy question", "hard question" from the lists below.

- **Definition:** Give a formal definition of the given concept, some non-trivial example (possibly also an example of an object that does not meet the given definition). Be prepared to answer questions about related properties.
- **Easy question:** If it is a mathematical statement, formulate and prove it. If it is an algorithm, formulate it, you can prepare a sample run of the algorithm, provide proof of correctness or other properties from the lecture.
- **Hard question:** Formulate the given mathematical theorem and give its proof, including auxiliary lemmata. If it is an immediate consequence of another theorem from the lecture, give the proof of that theorem as well. Be prepared to explain all the necessary notions from the formulation, and discuss the corollaries and applications of the theorem.

Tentative assessment

The following tentative assessment will be applied if no other deficiencies are uncovered. The evaluation can be even worse if during the exam we come across a fundamental lack of understanding of a concept or, for example, a serious misunderstanding of the proof written during the preparation phase.

- **Excellent:** Answer the *definition* and *hard question* including proof, answer the *easy question* if asked. Possible minor errors removed with just a little help from the examiner. Satisfactory answers to follow-up questions.
- **Very good:** Answering the *definition*, *easy question* (including proof, unless it is stated that the question is without proof) and formulation of the theorem from the *hard question* (without proof). Possible errors are corrected with the help of the examiner. Answering additional questions, possibly with hints.
- **Good:** Answering the *definition* and formulating the theorem from the *hard question*, there may be substantial errors, but the student must be able to correct them with help from the examiner. An answer to the *easy question* that is mostly correct, any errors or gaps in the proof removed with the help of the examiner. The ability to answer supplementary questions with the help of the examiner, the examination will not show a lack of understanding of basic concepts and context.
- **Fail:** In all other cases.

List of exam questions

Here is a list of the definitions (D), easy questions (E) and hard questions (H) that will be tested. If you draw a question that is the same as a student whose exam is taking place or will take place at the same time as yours, you may be asked to take a new draw. (Questions are randomly generated, but the probability distribution is secret and not necessarily uniform.) If the given definition, theorem, etc. was stated in both propositional and predicate logic, be prepared to answer the question in both propositional and predicate logic, unless specified otherwise.

List of definitions

(D1) Model in propositional logic, truth function of a proposition.

(D2) Semantic concepts (truth, falsity, independence, satisfiability) in logic, relative to a theory.

(D3) Equivalence of propositions or propositional theories, T-equivalence.

(D4) Semantic notions about a theory (contradictory, consistent, complete, satisfiable).

(D5) Extensions of theories (simple, conservative), the corresponding semantic criteria.

(D6) Tableau from a theory, tableau proof.

(D7) Congruence of a structure, quotient structure, axioms of equality.

(D8) CNF and DNF, Horn formulas. Set representation of CNF formulas, satisfying assignment.

(D9) Resolution rule, (most general) unification, resolution proof and refutation.

(D10) Signature and language of predicate logic, structure of the given language.

(D11) Atomic formula, formula, sentence, open formula.

(D12) Truth value of a formula in a structure with respect to an assignment, validity of a formula in a structure.

(D13) Complete theories in predicate logic, elementary equivalence.

(D14) Substructure, generated substructure, expansion and reduct.

(D15) Definability in a structure.

(D16) Extension by definitions.

(D17) Prenex normal form, Skolem variant.

(D18) Herbrand model, in comparison with canonical model.

(D19) Isomorphism of structures, ω-categorical theory.

(D20) Axiomatizability, finite axiomatizability, open axiomatizability.

(D21) Recursive axiomatization, recursive axiomatizability, recursively enumerable completion.

(D22) Decidable and partially decidable theory.

List of easy questions

(E1) Any set of models over a finite language can be axiomatized by a proposition in CNF, and by a proposition in DNF.

(E2) 2-SAT, the implication graph algorithm, its correctness.

(E3) Horn-SAT, the unit propagation algorithm, its correctness.

(E4) Properties of extension by definitions.

(E5) The relation between definable sets and automorphisms.

(E6) Tableau method in a language with equality.

(E7) The compactness theorem and its applications.

(E8) Soundness of resolution in propositional logic.

(E9) Soundness of resolution in predicate logic.

(E10) The tree of reductions and its connection to satisfiability of a CNF formula.

(E11) Non-standard model of natural numbers.

(E12) The existence of a countable algebraically closed field.

(E13) Fields of characteristic 0 are not finitely axiomatizable.

(E14) The criterion for open axiomatizability.

(E15) Recursively axiomatized theories are partially decidable, complete are decidable.

(E16) The theory of a finite structure in a finite language with equality is decidable.

(E17) Gödel's incompleteness theorems and their consequences (without proofs).

List of hard questions

(H1) Soundness of the tableau method in propositional logic.

(H2) Soundness of the tableau method in predicate logic.

(H3) Completeness of the tableau method in propositional logic.

(H4) Completeness of the tableau method in predicate logic.

(H5) Finiteness of contradiction, corollaries about finiteness and systematicity of proofs.

(H6) Completeness of resolution in propositional logic.

(H7) Completeness of resolution in predicate logic (the proof of the Lifting lemma is not required).

(H8) Skolem's theorem.

(H9) Herbrand's Theorem.

(H10) Löwenheim-Skolem theorem including its variant with equality, their consequences.

(H11) The relation between isomorphism and elementary equivalence.

(H12) The ω-categorical criterion for completeness.

(H13) Non-axiomatizability of finite models.

(H14) The theorem about finite axiomatizability.

(H15) Recursively axiomatized theory with recursively enumerable completion is decidable.