Predicate and Propositional Logic - Tutorial 4

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1. Applying the implication graph determine whether the following proposition in 2-CNF is satisfiable or not; and if yes, find a satisfying assignment.

$$(p_0 \lor p_2) \land (p_0 \lor \neg p_3) \land (p_1 \lor \neg p_3) \land (p_1 \lor \neg p_4) \land (p_2 \lor \neg p_4) \land (p_0 \lor \neg p_5) \land$$
$$(p_1 \lor \neg p_5) \land (p_2 \lor \neg p_5) \land (\neg p_1 \lor \neg p_6) \land (p_4 \lor p_6) \land (p_5 \lor p_6) \land p_1$$

2. Applying unit propagation determine whether the following Horn formula is satisfiable; and if yes, find a satisfying assignment.

$$(\neg p_1 \lor \neg p_3 \lor p_2) \land (\neg p_1 \lor p_2) \land p_1 \land (\neg p_1 \lor \neg p_2 \lor p_3) \land$$
$$(\neg p_2 \lor \neg p_4 \lor p_1) \land (p_4 \lor \neg p_3 \lor \neg p_2) \land (\neg p_4 \lor p_5)$$

3. Find tableau proofs of the following tautologies.
   (a) $(p \to (q \to q))$
   (b) $p \leftrightarrow \neg\neg p$
   (c) $\neg(p \lor q) \leftrightarrow (\neg p \land \neg q)$
   (d) $(p \to q) \leftrightarrow (\neg q \to \neg p)$
   (e) $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$

   Are the constructed tableaux systematic?

4. Applying tableau method prove the following propositions or find counterexamples
   (a) $\{\neg q,\ p \lor q\} \models p$,
   (b) $\{q \to p,\ r \to q,\ (r \to p) \to s\} \models s$,
   (c) $\{p \to r,\ p \lor q,\ \neg s \to \neg q\} \models r \to s$.

5. Applying tableau method determine all models of the following theories.
   (a) $\{(\neg p \lor q) \to (\neg q \land r)\}$
   (b) $\{\neg q \to (\neg p \lor q),\ \neg p \to q,\ r \to q\}$
   (c) $\{q \to p,\ r \to q,\ (r \to p) \to s\}$

6. Propose suitable atomic tableaux for Peirce arrow $\downarrow$ (NOR) and for Sheffer stroke $\uparrow$ (NAND).

7. Prove directly (by tableau tranformations) the deduction theorem, i.e. for every theory T and propositions φ, ψ,

$$T \vdash \varphi \to \psi \text{ if and only if } T, \varphi \vdash \psi.$$

8. Let $\mathcal{S}$ be a countable nonempty family of nonempty finite sets. We say that $\mathcal{S}$ has a *selector* if there exists an injective $f\colon \mathcal{S} \to \bigcup \mathcal{S}$ such that $f(S) \in S$ for every $S \in \mathcal{S}$. Prove that $\mathcal{S}$ has a selector if and only if every nonempty finite part of $\mathcal{S}$ has a selector.