Predicate and Propositional Logic - Seminar 3

Oct 20, 2014

1. Applying the implication graph determine whether the following proposition in 2-CNF is satisfiable or not; and if yes, find a satisfying assignment.

$$(p_0 \vee p_2) \wedge (p_0 \vee \neg p_3) \wedge (p_1 \vee \neg p_3) \wedge (p_1 \vee \neg p_4) \wedge (p_2 \vee \neg p_4) \wedge (p_0 \vee \neg p_5) \wedge \\ (p_1 \vee \neg p_5) \wedge (p_2 \vee \neg p_5) \wedge (\neg p_1 \vee \neg p_6) \wedge (p_4 \vee p_6) \wedge (p_5 \vee p_6) \wedge p_1$$

2. Applying unit propagation determine whether the following Horn formula is satisfiable; and if yes, find a satisfying assignment.

$$(\neg p_1 \vee \neg p_3 \vee p_2) \wedge (\neg p_1 \vee p_2) \wedge p_1 \wedge (\neg p_1 \vee \neg p_2 \vee p_3) \wedge \\ (\neg p_2 \vee \neg p_4 \vee p_1) \wedge (p_4 \vee \neg p_3 \vee \neg p_2) \wedge (\neg p_4 \vee p_5)$$

3. Consider a theory $T = \{\neg q \to (\neg p \vee q),\ \neg p \to q,\ r \to q\}$. Which of the following propositions are valid, contradictory, independent, satisfiable, equivalent in T?
   (a) p, q, r, s
   (b) $p \vee q$, $p \vee r$, $p \vee s$, $q \vee s$
   (c) $p \wedge q$, $q \wedge s$, $p \to q$, $s \to q$
4. Consider an infinite theory $T = \{p_i \to (p_{i+1} \vee q_{i+1}), q_i \to (p_{i+1} \vee q_{i+1}) \mid i \in \mathbb{N}\}$ over var(T).
   (a) Which propositions in the form $p_i \to p_j$ are logical consequences of T?
   (b) Which propositions in the form $p_i \to (p_j \vee q_j)$ are logical consequences of T?
   (c) Determine all models of the theory T.
5. Prove or disprove (or find the correct relation) that for every theory T and propositions φ, ψ over $\mathbb{P}$ it holds
   (a) $T \models \varphi$, if and only if $T \not\models \neg\varphi$
   (b) $T \models \varphi$ and $T \models \psi$, if and only if $T \models \varphi \wedge \psi$
   (c) $T \models \varphi$ or $T \models \psi$, if and only if $T \models \varphi \vee \psi$
   (d) $T \models \varphi \to \psi$ and $T \models \psi \to \chi$, if and only if $T \models \varphi \to \chi$
6. Prove or disprove (or find the correct relation). For every theories T and S over $\mathbb{P}$
   (a) $S \subseteq T \Rightarrow \theta^{\mathbb{P}}(T) \subseteq \theta^{\mathbb{P}}(S)$
   (b) $\theta^{\mathbb{P}}(S \cup T) = \theta^{\mathbb{P}}(S) \cup \theta^{\mathbb{P}}(T)$
   (c) $\theta^{\mathbb{P}}(S \cap T) = \theta^{\mathbb{P}}(S) \cap \theta^{\mathbb{P}}(T)$
7. Let $|\mathbb{P}| = n$ and $\varphi \in \mathrm{VF}_{\mathbb{P}}$ with $|M(\varphi)| = m$.
   (a) What is the number of nonequivalent propositions ψ such that $\varphi \models \psi$ or $\psi \models \varphi$?
   (b) What is the number of nonequivalent theories over $\mathbb{P}$ in which φ is valid? What is the number of nonequivalent *complete* theories over $\mathbb{P}$ in which φ is valid?
   (c) What is the number of nonequivalent theories T over $\mathbb{P}$ such that $T \cup \{\varphi\}$ is satisfiable?
   (d) Let, moreover, $\{\varphi, \psi\}$ be an unsatisfiable theory with $|M(\psi)| = p$. What is the number of nonequivalent propositions χ such that $\varphi \vee \psi \models \chi$? What is the number of nonequivalent theories in which $\varphi \vee \psi$ is valid?
8. Find tableau proofs of the following tautologies.

(a) $(p \to (q \to q))$

(b) $p \leftrightarrow \neg\neg p$

(c) $\neg(p \lor q) \leftrightarrow (\neg p \land \neg q)$

(d) $(p \to q) \leftrightarrow (\neg q \to \neg p)$

(e) $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$