

Predicate and Propositional Logic - Seminar 5

Nov 3, 2014

1. Applying tableau method prove the following propositions or find counterexamples

- (a) $\{\neg q, p \vee q\} \models p$,
- (b) $\{q \rightarrow p, r \rightarrow q, (r \rightarrow p) \rightarrow s\} \models s$,
- (c) $\{p \rightarrow r, p \vee q, \neg s \rightarrow \neg q\} \models r \rightarrow s$.

2. Applying tableau method determine all models of the following theories.

- (a) $\{(\neg p \vee q) \rightarrow (\neg q \wedge r)\}$
- (b) $\{\neg q \rightarrow (\neg p \vee q), \neg p \rightarrow q, r \rightarrow q\}$
- (c) $\{q \rightarrow p, r \rightarrow q, (r \rightarrow p) \rightarrow s\}$

3. Prove directly (by tableau transformations) the deduction theorem, i.e. for every theory T and propositions φ, ψ ,

$$T \vdash \varphi \rightarrow \psi \text{ if and only if } T, \varphi \vdash \psi.$$

4. Show that every atomic tableau τ is *sound*, i.e. if an assignment v agrees with the root entry of τ , then it agrees with some branch in τ .

5. In the proof of the lemma on completeness during the lecture we verified that if an assignment v agrees with every entry on a finished branch V up to the depth i of formation trees, then it agrees also with every entry in the form $T(\varphi \wedge \psi)$ or $F(\varphi \wedge \psi)$ on V where $\varphi \wedge \psi$ has depth $i + 1$. Show that the same holds also for entries for other logical connectives.

6. Let \mathcal{S} be a countable nonempty family of nonempty finite sets. We say that \mathcal{S} has a *selector* if there exists an injective $f: \mathcal{S} \rightarrow \bigcup \mathcal{S}$ such that $f(S) \in S$ for every $S \in \mathcal{S}$. Prove that \mathcal{S} has a selector if and only if every nonempty finite part of \mathcal{S} has a selector.

7. Let φ be the proposition $\neg(p \vee q) \rightarrow (\neg p \wedge \neg q)$.

- (a) Transform $\neg\varphi$ into CNF and into set representation (clausal form).
- (b) Find a resolution refutation of $\neg\varphi$; that is, a proof of φ .

8. Find resolution closures $\mathcal{R}(S)$ of the following formulas S .

- (a) $\{\{p, q\}, \{\neg p, \neg q\}, \{\neg p, q\}\}$
- (b) $\{\{p, q\}, \{p, \neg q\}, \{p, \neg q\}\}$
- (c) $\{\{p, \neg q, r\}, \{q, r\}, \{\neg p, r\}, \{q, \neg r\}, \{\neg q\}\}$

9. Find resolution refutations of the following propositions.

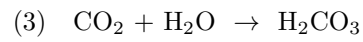
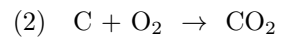
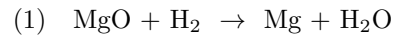
- (a) $(p \leftrightarrow (q \rightarrow r)) \wedge ((p \leftrightarrow q) \wedge (p \leftrightarrow \neg r))$
- (b) $\neg(((p \rightarrow q) \rightarrow \neg q) \rightarrow \neg q)$

10. Prove by resolution that s is valid in a theory $T = \{\neg p \rightarrow \neg q, \neg q \rightarrow \neg r, (r \rightarrow p) \rightarrow s\}$.

11. Show that if $S = \{C_1, C_2\}$ is satisfiable and C is a resolvent of C_1 and C_2 , then C is satisfiable as well.

12. Find the *tree of reductions* of a formula $S = \{\{p, r\}, \{q, \neg r\}, \{\neg q\}, \{\neg p, t\}, \{\neg s\}, \{s, \neg t\}\}$.

13. Assume that we have available MgO, H₂, O₂, C and we can perform the following chemical reactions.



- (a) Represent the state of affairs as a proposition in a suitable language and transform it into a set representation.
- (b) Prove by (linear input) resolution that we can produce H₂CO₃.