Predicate and Propositional Logic - Seminar 6

Nov 10, 2014

1. Let φ be the proposition $\neg(p \vee q) \to (\neg p \wedge \neg q)$.
   (a) Transform $\neg\varphi$ into CNF and into set representation (clausal form).
   (b) Find a resolution refutation of $\neg\varphi$; that is, a proof of φ.
2. Find resolution closures $\mathcal{R}(S)$ of the following formulas S.
   (a) $\{\{p, q\}, \{\neg p, \neg q\}, \{\neg p, q\}\}$
   (b) $\{\{p, q\}, \{p, \neg q\}, \{p, \neg q\}\}$
   (c) $\{\{p, \neg q, r\}, \{q, r\}, \{\neg p, r\}, \{q, \neg r\}, \{\neg q\}\}$
3. Find resolution refutations of the following propositions.
   (a) $(p \leftrightarrow (q \to r)) \wedge ((p \leftrightarrow q) \wedge (p \leftrightarrow \neg r))$
   (b) $\neg(((p \to q) \to \neg q) \to \neg q)$
4. Prove by resolution that s is valid in a theory $T = \{\neg p \to \neg q, \neg q \to \neg r, (r \to p) \to s\}$.
5. Show that if $S = \{C_1, C_2\}$ is satisfiable and C is a resolvent of C_1 and C_2, then C is satisfiable as well.
6. Find the *tree of reductions* of a formula $S = \{\{p, r\}, \{q, \neg r\}, \{\neg q\}, \{\neg p, t\}, \{\neg s\}, \{s, \neg t\}\}$.
7. Assume that we have available MgO, H_2, O_2, C and we can perform the following chemical reactions.

   (1) $\mathrm{MgO} + \mathrm{H_2} \;\to\; \mathrm{Mg} + \mathrm{H_2O}$
   (2) $\mathrm{C} + \mathrm{O_2} \;\to\; \mathrm{CO_2}$
   (3) $\mathrm{CO_2} + \mathrm{H_2O} \;\to\; \mathrm{H_2CO_3}$

   (a) Represent the state of affairs as a proposition in a suitable language and transform it into a set representation.
   (b) Prove by (linear input) resolution that we can produce $\mathrm{H_2CO_3}$.
8. Show that in Hilbert's calculus the following is provable for every formulas φ, ψ, χ.
   (a) $\vdash_H \varphi \to \varphi$
   (b) $T \vdash_H \varphi \to \chi$ where $T = \{\varphi \to \psi,\ \psi \to \chi\}$
   (c) $T \vdash_H \psi \to \chi$ where $T = \{\varphi,\ \psi \to (\varphi \to \chi)\}$