

## Test 2

Jan 5, 2015

*Remark: Each of the following four problems is valued max. 2 pts. Totally max. 8 pts.*

1. Let  $T = \{\{\neg p, q\}, \{p, \neg q\}, \{\neg p, \neg q\}, \{p, q, r\}\}$  be a theory in clausal form. Prove by resolution that  $T \models \neg r \rightarrow s$ .
2. Determine whether the following terms are substitutable into the following formula.

$$(\exists x)(P(x) \rightarrow (P(y) \vee (\exists y)(P(y) \wedge P(z))))).$$

Write the instance obtained by the substitution or explain why it is not possible to substitute.

- (a) the term  $y + z$  for the variable  $y$ .
  - (b) the term  $x + y$  for the variable  $z$ .
3. Give examples of structures for the language  $L = \langle P, R \rangle$  where  $P, R$  are unary relation symbols in which the following sentence is a) valid, b) contradictory.

$$((\forall y)P(y) \rightarrow (\forall z)R(z)) \rightarrow (\forall x)(P(x) \rightarrow R(x))$$

4. Let  $\varphi$  be a formula in a free variable  $x$  and  $\psi$  be a sentence. Prove by tableau method that

$$\models (\exists x)(\varphi(x) \rightarrow \psi) \rightarrow ((\forall x)\varphi(x) \rightarrow \psi).$$