Predicate and Propositional Logic - Seminar 4

Oct 27, 2015

1. (*previous homework*) Let $|\mathbb{P}| = n$ and $\varphi \in \mathrm{VF}_{\mathbb{P}}$ with $|M(\varphi)| = m$.
   (a) What is the number of nonequivalent propositions ψ such that $\varphi \models \psi$ or $\psi \models \varphi$?
   (b) What is the number of nonequivalent theories over $\mathbb{P}$ in which φ is valid? What is the number of nonequivalent *complete* theories over $\mathbb{P}$ in which φ is valid?
   (c) What is the number of nonequivalent theories T over $\mathbb{P}$ such that $T \cup \{\varphi\}$ is satisfiable?
   (d) Let, moreover, $\{\varphi, \psi\}$ be an unsatisfiable theory with $|M(\psi)| = p$. What is the number of nonequivalent propositions χ such that $\varphi \vee \psi \models \chi$? What is the number of nonequivalent theories in which $\varphi \vee \psi$ is valid?

2. Prove or disprove (or find the correct relation). For every theories T and S over $\mathbb{P}$
   (a) $S \subseteq T \Rightarrow \theta^{\mathbb{P}}(T) \subseteq \theta^{\mathbb{P}}(S)$
   (b) $\theta^{\mathbb{P}}(S \cup T) = \theta^{\mathbb{P}}(S) \cup \theta^{\mathbb{P}}(T)$
   (c) $\theta^{\mathbb{P}}(S \cap T) = \theta^{\mathbb{P}}(S) \cap \theta^{\mathbb{P}}(T)$

3. Find tableau proofs of the following tautologies.
   (a) $(p \to (q \to q))$
   (b) $p \leftrightarrow \neg\neg p$
   (c) $\neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$
   (d) $(p \to q) \leftrightarrow (\neg q \to \neg p)$
   (e) $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$

   Are the constructed tableaux systematic?

4. Applying tableau method prove the following propositions or find counterexamples
   (a) $\{\neg q,\ p \vee q\} \models p$,
   (b) $\{q \to p,\ r \to q,\ (r \to p) \to s\} \models s$,
   (c) $\{p \to r,\ p \vee q,\ \neg s \to \neg q\} \models r \to s$.

5. Applying tableau method determine all models of the following theories.
   (a) $\{(\neg p \vee q) \to (\neg q \wedge r)\}$
   (b) $\{\neg q \to (\neg p \vee q),\ \neg p \to q,\ r \to q\}$
   (c) $\{q \to p,\ r \to q,\ (r \to p) \to s\}$

6. Propose suitable atomic tableaux for Peirce arrow $\downarrow$ (NOR) and for Sheffer stroke $\uparrow$ (NAND).

7. Prove directly (by tableau tranformations) the deduction theorem, i.e. for every theory T and propositions φ, ψ,
$$T \vdash \varphi \to \psi \ \text{ if and only if } \ T, \varphi \vdash \psi.$$

8. Show that every atomic tableau τ is *sound*, i.e. if an assignment v agrees with the root entry of τ, then it agrees with some branch in τ.

9. In the proof of the lemma on completeness during the lecture we verified that if an assignment v agrees with every entry on a finished branch V up to the depth i of formation trees, then it agrees also with every entry in the form $T(\varphi \wedge \psi)$ or $F(\varphi \wedge \psi)$ on V where $\varphi \wedge \psi$ has depth $i + 1$. Show that the same holds also for entries for other logical connectives.