Predicate and Propositional Logic - Seminar 5

Nov 3, 2015

1. Prove directly (by tableau tranformations) the deduction theorem, i.e. for every theory T and propositions φ, ψ,
$$T \vdash \varphi \to \psi \text{ if and only if } T, \varphi \vdash \psi.$$

2. Show that every atomic tableau τ is *sound*, i.e. if an assignment v agrees with the root entry of τ, then it agrees with some branch in τ.

3. In the proof of the lemma on completeness during the lecture we verified that if an assignment v agrees with every entry on a finished branch V up to the depth i of formation trees, then it agrees also with every entry in the form $T(\varphi \wedge \psi)$ or $F(\varphi \wedge \psi)$ on V where $\varphi \wedge \psi$ has depth $i + 1$. Show that the same holds also for entries for other logical connectives.

4. Let $\mathcal{S}$ be a countable nonempty family of nonempty finite sets. We say that $\mathcal{S}$ has a *selector* if there exists an injective $f\colon \mathcal{S} \to \bigcup \mathcal{S}$ such that $f(S) \in S$ for every $S \in \mathcal{S}$. Prove that $\mathcal{S}$ has a selector if and only if every nonempty finite part of $\mathcal{S}$ has a selector.

5. Let φ be the proposition $\neg(p \vee q) \to (\neg p \wedge \neg q)$.
   (a) Transform $\neg\varphi$ into CNF and into set representation (clausal form).
   (b) Find a resolution refutation of $\neg\varphi$; that is, a proof of φ.

6. Find resolution closures $\mathcal{R}(S)$ of the following formulas S.
   (a) $\{\{p, q\}, \{\neg p, \neg q\}, \{\neg p, q\}\}$
   (b) $\{\{p, q\}, \{p, \neg q\}, \{p, \neg q\}\}$
   (c) $\{\{p, \neg q, r\}, \{q, r\}, \{\neg p, r\}, \{q, \neg r\}, \{\neg q\}\}$

7. Find resolution refutations of the following propositions.
   (a) $(p \leftrightarrow (q \to r)) \wedge ((p \leftrightarrow q) \wedge (p \leftrightarrow \neg r))$
   (b) $\neg(((p \to q) \to \neg q) \to \neg q)$

8. Prove by resolution that s is valid in a theory $T = \{\neg p \to \neg q, \neg q \to \neg r, (r \to p) \to s\}$.

9. Show that if $S = \{C_1, C_2\}$ is satisfiable and C is a resolvent of C_1 and C_2, then C is satisfiable as well.

10. Find the *tree of reductions* of a formula $S = \{\{p, r\}, \{q, \neg r\}, \{\neg q\}, \{\neg p, t\}, \{\neg s\}, \{s, \neg t\}\}$.

11. Assume that we have available MgO, H_2, O_2, C and we can perform the following chemical reactions.
$$\begin{array}{lll} (1) & MgO + H_2 & \to\ Mg + H_2O \\ (2) & C + O_2 & \to\ CO_2 \\ (3) & CO_2 + H_2O & \to\ H_2CO_3 \end{array}$$
   (a) Represent the state of affairs as a proposition in a suitable language and transform it into a set representation.
   (b) Prove by (linear input) resolution that we can produce H_2CO_3.