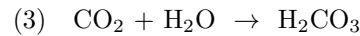
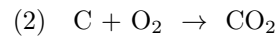
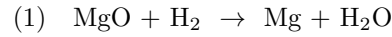


Predicate and Propositional Logic - Seminar 6

Nov 10, 2015

1. (*previous homework*) Assume that we have available MgO, H₂, O₂, C and we can perform the following chemical reactions.



- (a) Represent the state of affairs as a proposition in a suitable language and transform it into a set representation.
 - (b) Prove by (linear input) resolution that we can produce H₂CO₃.
2. Find the *tree of reductions* of a formula $S = \{\{p, r\}, \{q, \neg r\}, \{\neg q\}, \{\neg p, t\}, \{\neg s\}, \{s, \neg t\}\}$.
 3. Show that in Hilbert's calculus the following is provable for every formulas φ, ψ, χ .

$$(a) \vdash_H \varphi \rightarrow \varphi$$

$$(b) T \vdash_H \varphi \rightarrow \chi \text{ where } T = \{\varphi \rightarrow \psi, \psi \rightarrow \chi\}$$

$$(c) T \vdash_H \psi \rightarrow \chi \text{ where } T = \{\varphi, \psi \rightarrow (\varphi \rightarrow \chi)\}$$

4. Which of the variable occurrences are free/bound in the following formulas? Find variants of these formulas without variables that have both free and bound occurrence.

$$(a) (\exists x)(\forall y)P(y, z) \vee (y = 0)$$

$$(b) (\exists x)(P(x) \wedge (\forall x)Q(x)) \vee (x = 0)$$

$$(c) (\exists x)(x > y) \wedge (\exists y)(y > x)$$

5. Let φ denote the formula $(\forall x)((x = z) \vee (\exists y)(f(x) = y) \vee (\forall z)(y = f(z)))$. Which of the following terms are substitutable into φ ?

$$(a) \text{ the term } z \text{ for the variable } x, \text{ the term } y \text{ for the variable } x,$$

$$(b) \text{ the term } z \text{ for the variable } y, \text{ the term } 2 * y \text{ for the variable } y,$$

$$(c) \text{ the term } x \text{ for the variable } z, \text{ the term } y \text{ for the variable } z,$$

6. Are the following formulas variants of the formula $(\forall x)(x < y \vee (\exists z)(z = y \wedge z \neq x))$?

$$(a) (\forall z)(z < y \vee (\exists z)(z = y \wedge z \neq z))$$

$$(b) (\forall y)(y < y \vee (\exists z)(z = y \wedge z \neq y))$$

$$(c) (\forall u)(u < y \vee (\exists z)(z = y \wedge z \neq u))$$