Predicate and Propositional Logic - Seminar 6

Nov 10, 2015

1. (*previous homework*) Assume that we have available MgO, H_2, O_2, C and we can perform the following chemical reactions.

$$\begin{array}{ll} (1) & MgO + H_2 \rightarrow Mg + H_2O \\ (2) & C + O_2 \rightarrow CO_2 \\ (3) & CO_2 + H_2O \rightarrow H_2CO_3 \end{array}$$

   (a) Represent the state of affairs as a proposition in a suitable language and transform it into a set representation.

   (b) Prove by (linear input) resolution that we can produce H_2CO_3.

2. Find the *tree of reductions* of a formula $S = \{\{p, r\}, \{q, \neg r\}, \{\neg q\}, \{\neg p, t\}, \{\neg s\}, \{s, \neg t\}\}$.

3. Show that in Hilbert's calculus the following is provable for every formulas φ, ψ, χ.

   (a) $\vdash_H \varphi \rightarrow \varphi$

   (b) $T \vdash_H \varphi \rightarrow \chi$ where $T = \{\varphi \rightarrow \psi,\ \psi \rightarrow \chi\}$

   (c) $T \vdash_H \psi \rightarrow \chi$ where $T = \{\varphi,\ \psi \rightarrow (\varphi \rightarrow \chi)\}$

4. Which of the variable occurrences are free/bound in the following formulas? Find variants of these formulas without variables that have both free and bound occurrence.

   (a) $(\exists x)(\forall y)P(y, z) \vee (y = 0)$

   (b) $(\exists x)(P(x) \wedge (\forall x)Q(x)) \vee (x = 0)$

   (c) $(\exists x)(x > y) \wedge (\exists y)(y > x)$

5. Let φ denote the formula $(\forall x)((x = z) \vee (\exists y)(f(x) = y) \vee (\forall z)(y = f(z)))$. Which of the following terms are substitutable into φ?

   (a) the term z for the variable x, the term y for the variable x,

   (b) the term z for the variable y, the term $2 * y$ for the variable y,

   (c) the term x for the variable z, the term y for the variable z,

6. Are the following formulas variants of the formula $(\forall x)(x < y \vee (\exists z)(z = y \wedge z \neq x))$?

   (a) $(\forall z)(z < y \vee (\exists z)(z = y \wedge z \neq z))$

   (b) $(\forall y)(y < y \vee (\exists z)(z = y \wedge z \neq y))$

   (c) $(\forall u)(u < y \vee (\exists z)(z = y \wedge z \neq u))$