Predicate and Propositional Logic - Seminar 12

Jan 5, 2016

1. *(Previous homework)* Show that the following set of clauses if resolution refutable. Describe the resolution refutation by a resolution tree. In each resolution step write down the unification used and underline resolved literals.
   (a) $\{P(a, x, f(y)), P(a, z, f(h(b))), \neg Q(y, z)\}$
   (b) $\{\neg Q(h(b), w), H(w, a)\}$
   (c) $\{\neg P(a, w, f(h(b))), H(x, a)\}$
   (d) $\{P(a, u, f(h(u))), H(u, a), Q(h(b), b)\}$
   (e) $\{\neg H(v, a)\}$
2. *(Previous homework)* We know that
   (a) If a brick is on (another) brick, then it is not on the ground.
   (b) Every brick is on (another) brick or on the ground.
   (c) No brick is on a brick that is on (another) brick.

   Express these facts in a first-order language and prove by resolution that if a brick is on another brick, the lower brick is on the ground.
3. We know that
   (a) Every barber shaves all who do not shave themselves.
   (b) No barber shaves someone who shaves himself.

   Express these facts in a first-order language and prove by resolution that no barber exists.
4. Consider structures $\langle \mathbb{N}, \leq \rangle$, $\langle \mathbb{Z}, \leq \rangle$, $\langle \mathbb{Q}, \leq \rangle$ with standard orderings.
   (a) Which of them are elementarily non-equivalent?
   (b) Which of them are non-isomorphic?
   (c) Determine how many automorphisms these structures have.
5. Let $L = \langle U \rangle$ be with equality, where U is a unary relation symbol.
   (a) For a given $n \in \mathbb{N}^+$ find a formula φ that says *"$U(x)$ holds for exactly n elements x"*.
   (b) Is the theory $T = \{\varphi\}$ complete?
   (c) Is T ω-categorical?
   (d) Determine the isomorphism spectrum of T (up to countable cardinality).
   (e) Find some simple complete extension of T.
   (f) Find all (up to equivalence) simple complete extensions of T.
6. Let T be an extension of the theory $DeLO$ (i.e. of dense linear orders without ends) by a new constant symbol c (and no additional axioms).
   (a) Is T ω-categorical?
   (b) Is T complete?
   (c) Does the same hold for theory $DeLO^+$ (of dense linear orders with maximal element and without minimal element) instead of $DeLO$?