

Test 2

Jan 5, 2016

Remark: Each of the following four problems is valued max. 2 pts. Totally max. 8 pts.

Let φ denote the following formula where P, R are binary relation symbols.

$$(\exists x)((\exists y)P(x, y) \rightarrow R(y, z)) \rightarrow ((\forall x)(\exists y)P(x, y) \rightarrow R(y, z))$$

1. Underline free occurrences of variables in φ and determine whether the following terms are substitutable. Write the instance obtained by the substitution or explain why it is not possible to substitute.
 - (a) the term $y + z$ for the variable z .
 - (b) the term $x + z$ for the variable y .
2. Let φ' denote a general closure of the formula φ . Prove by tableau method that

$$\models \varphi'$$

3. Convert φ' into prenex normal form.
4. Write a Skolem variant of the obtained formula in PNF and based on this, write an open formula that is equisatisfiable with φ .