Exam test

Feb 11, 2016

1. Let $T = \{p \to r \wedge q,\ (q \to r) \to \neg q\}$ be a theory over $\mathbb{P} = \{p, q, r\}$.
   (a) Using tableau method, determine all models of T. *(3b)*
   (b) Axiomatize $M^{\mathbb{P}}(T)$ by a proposition in DNF and by a proposition in CNF. *(2b)*
   (c) Is T an extension of a theory $S = \{\neg p \vee \neg q\}$ over $\{p, q\}$? Is T a conservative extension of S? Give an explanation. *(2b)*
   (d) Determine the number of mutually T-nonequivalent propositions over $\mathbb{P}$ that are independent in T. Provide at least two examples of such propositions. *(2b)*

2. Consider the following statements about following people on the network:
   (i) *Everyone is either followed by someone or is offline (but not both at the same time).*
   (ii) *Is a person is offline, (s)he follows noone.*

   Prove by resolution it follows that:

   (iii) *Everyone who is following someone, is also being followed (not necessarily by the same person).*

   More specifically:

   (a) Formalize the statements (i), (ii), (iii) as <u>sentences</u> φ_1, φ_2, φ_3 (respectively) of language $L = \langle F, O\rangle$ without equality where F is binary relation symbol, O is unary relation symbol and $F(x, y)$, $O(x)$ means (resp.) that *"a person x follows a person y"* and *"a person x is offline"*. *(2b)*
   (b) Applying skolemisation on the sentences from (a) or their negations find an open theory T (possibly in extended language) such that T is unsatisfiable if and only if $\{\varphi_1, \varphi_2\} \models \varphi_3$. *(2b)*
   (c) Transform the theory T to an equivalent theory T' that has only clauses as axioms. Write T' in clausal form. *(2b)*
   (d) Prove by resolution that T' is unsatisfiable. Present your resolution refutation as a resolution tree. For each step write down the unification used. *(3b)*
   (e) Find a conjunction of ground instances of axioms of T' that is unsatisfiable. *Hint: use unifications from (d).* *(2b)*

3. Let $T = \{R(x, x), R(x, y) \to R(y, x), R(x, y) \wedge R(y, z) \to R(x, z), \varphi\}$ be a theory of language $L = \langle R\rangle$ with equality where R is a binary relation symbol and the axiom φ says that "there are exactly 3 elements".
   (a) Determine the isomorphism spectrum of the theory T. *(2b)*
   (b) How many simple complete extensions the theory T has? Write at least two of them. *(2b)*
   (c) Is T equivalent to some open theory? Give an explanation. *(2b)*
   (d) Let $T' = T \cup \{R(c_1, c_2)\}$ be a theory of language $L = \langle R, c_1, c_2\rangle$ with equality where c_1, c_2 are new constant symbols. Is T' a conservative extension of T? Give an explanation. *(2b)*