Midterm test

Nov 15, 2022

(In Shakespeare's play The Merchant of Venice) those who wish to win a beautiful Portia's hand in marriage need to find out (by resolution) which of the three caskets, made of gold, silver, and lead, hides Portia's portrait. We know that

(i) Portia's portrait is in exactly one casket.

(ii) At most one of the inscriptions on the caskets is true.

(iii) The inscription on the golden casket says: *"The portrait is not in this casket."*

(iv) The inscription on the silver casket says: *"If the inscription on the golden casket is true, then the portrait is in the leaden casket."*

(v) The inscription on the leaden casket says: *"The inscription on the golden casket is false."*

Let the propositional letters g, s, l represent (respectively) that *"the portrait is in golden / silver / leaden casket"* and letters t_g, t_s, t_l represent (respectively) that *"the inscription on golden / silver / leaden casket is true."* Furthermore, let $\mathbb{P}' = \{g, s, l, t_g, t_s, t_l\}$.

1. Write propositions φ_1, φ_2 over $\mathbb{P}'$ expressing the statements (i), (ii), and propositions (in form of equivalences) φ_3, φ_4, φ_5 over $\mathbb{P}'$ representing (respectively) our knowledge from (iii), (iv), (v). *(20p)*

2. Applying T'-equivalent transformations (replacing a subformula with a T'-equivalent formula) find a theory T over $\mathbb{P} = \{g, s, l\}$ such that the theory $T' = \{\varphi_1, \ldots, \varphi_5\}$ over $\mathbb{P}'$ is a conservative extension of T. *(20p)*.

3. Find out by resolution from T which casket contains Porcia's portrait. *(40p)*

4. What is the number of (mutually nonequivalent) extensions of the theory T' over $\mathbb{P}'$? How many of them are complete? Give an explanation. *(20p)*