Test 2

January 3, 2023

Let $T = \{(\forall x)(\exists y)P(x,y), (\forall u)(\forall v)(\exists x)(\forall y)(P(x,y) \rightarrow R(u,v)), (\exists x)\neg R(x,x)\}$ be a theory of the language $L = \langle P, R \rangle$ without equality, where P, R are binary relation symbols.

1. Using Skolemization, find a theory T' (over a suitably extended language) equisatisfiable with T and axiomatized only by universal sentences. *(20p)*

2. Use the tableau method or resolution to prove that T' is unsatisfiable. In case of resolution, draw the resolution tree and in each step, write the unification used. *(40p)*

3. Let T'' be a theory consisting of open matrices of all the axioms of T'. Find an unsatisfiable conjunction of ground instances of axioms of T''. *Hint: use the proof from (b). (20p)*

4. Is the sentence $(\forall x)R(x,x)$ true / contradictory / independent in the theory T? And in the theory $S = \{(\forall x)(\exists y)P(x,y), (\forall u)(\forall v)(\exists x)(\forall y)(P(x,y) \rightarrow R(u,v))\}$? Justify your answers. *(20p)*