Predicate and Propositional Logic - Tutorial 4

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1. Find tableau proofs of the following tautologies.
   (a) $(p \to (q \to q))$
   (b) $p \leftrightarrow \neg\neg p$
   (c) $\neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$
   (d) $(p \to q) \leftrightarrow (\neg q \to \neg p)$
   (e) $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$

   Are the constructed tableaux systematic?

2. Applying tableau method prove the following propositions or find counterexamples
   (a) $\{\neg q,\ p \vee q\} \models p$,
   (b) $\{q \to p,\ r \to q,\ (r \to p) \to s\} \models s$,
   (c) $\{p \to r,\ p \vee q,\ \neg s \to \neg q\} \models r \to s$.

3. Applying tableau method determine all models of the following theories.
   (a) $\{(\neg p \vee q) \to (\neg q \wedge r)\}$
   (b) $\{\neg q \to (\neg p \vee q),\ \neg p \to q,\ r \to q\}$
   (c) $\{q \to p,\ r \to q,\ (r \to p) \to s\}$

4. Propose suitable atomic tableaux for Peirce arrow $\downarrow$ (NOR) and for Sheffer stroke $\uparrow$ (NAND).

5. Prove directly (by tableau tranformations) the deduction theorem, i.e. for every theory T and propositions φ, ψ,
$$T \vdash \varphi \to \psi \text{ if and only if } T, \varphi \vdash \psi.$$

6. Let $\mathcal{S}$ be a countable nonempty family of nonempty finite sets. We say that $\mathcal{S}$ has a *selector* if there exists an injective $f\colon \mathcal{S} \to \bigcup \mathcal{S}$ such that $f(S) \in S$ for every $S \in \mathcal{S}$. Prove that $\mathcal{S}$ has a selector if and only if every nonempty finite part of $\mathcal{S}$ has a selector.