Predicate and Propositional Logic - Tutorial 10

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1. Assume that
   (a) all guilty persons are liars,
   (b) at least one of the accused is also a witness,
   (c) no witness lies.

   Prove by tableau method that not all accused are guilty.

2. Let $L(x, y)$ represent that *"there is a flight from x to y"* and let $S(x, y)$ represent that *"there is a connection from x to y"*. Assume that
   (a) From Prague you can flight to Bratislava, London and New York, and from New York to Paris,
   (b) $(\forall x)(\forall y)(L(x, y) \rightarrow L(y, x))$,
   (c) $(\forall x)(\forall y)(L(x, y) \rightarrow S(x, y))$,
   (d) $(\forall x)(\forall y)(\forall z)(S(x, y) \wedge L(y, z) \rightarrow S(x, z))$.

   Prove by tableau method that there is a connection from Bratislava to Paris.

3. Let φ, ψ be sentences or formulas in a free variable x, denoted by $\varphi(x)$, $\psi(x)$. Find tableau proofs of the following formulas.
   (a) $(\exists x)(\varphi(x) \vee \psi(x)) \leftrightarrow (\exists x)\varphi(x) \vee (\exists x)\psi(x)$,
   (b) $(\forall x)(\varphi(x) \wedge \psi(x)) \leftrightarrow (\forall x)\varphi(x) \wedge (\forall x)\psi(x)$,
   (c) $(\varphi \vee (\forall x)\psi(x)) \rightarrow (\forall x)(\varphi \vee \psi(x))$ where x is not free in φ,
   (d) $(\varphi \wedge (\exists x)\psi(x)) \rightarrow (\exists x)(\varphi \wedge \psi(x))$ where x is not free in φ.
   (e) $(\exists x)(\varphi \rightarrow \psi(x)) \rightarrow (\varphi \rightarrow (\exists x)\psi(x))$ where x is not free in φ,
   (f) $(\exists x)(\varphi \wedge \psi(x)) \rightarrow (\varphi \wedge (\exists x)\psi(x))$ where x is not free in φ,
   (g) $(\exists x)(\varphi(x) \rightarrow \psi) \rightarrow ((\forall x)\varphi(x) \rightarrow \psi)$ where x is not free in ψ,
   (h) $((\exists x)\varphi(x) \rightarrow \psi) \rightarrow (\forall x)(\varphi(x) \rightarrow \psi)$ where x is not free in ψ.

4. Let T^* be a theory with axioms of equality. Prove by tableau method that
   (a) $T^* \models x = y \ \rightarrow \ y = x$ (symmetry of =)
   (b) $T^* \models (x = y \ \wedge \ y = z) \ \rightarrow \ x = z$ (transitivity of =)

   Hint: To show (a) apply the axiom of equality (*iii*) for $x_1 = x$, $x_2 = x$, $y_1 = y$ a $y_2 = x$, to show (b) apply (*iii*) for $x_1 = x$, $x_2 = y$, $y_1 = x$ a $y_2 = z$.

5. Let L be a language with equality containing a binary relation symbol $\leq$ and let T be a theory of L such that T has an infinite model and the axioms of linear ordering are valid in T. Applying the compactness theorem show that T has a model $\mathcal{A}$ with an *infinite decreasing chain*; that is, there are elements c_i for every $i \in \mathbb{N}$ in A such that

   $$\cdots < c_{n+1} < c_n < \cdots < c_0.$$

   (This show that the notion of *well-ordering* is not definable in a first-order language.)