

Predicate and Propositional Logic - Tutorial 4

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1. Find tableau proofs of the following tautologies.

(a) $(p \rightarrow (q \rightarrow q))$

(b) $p \leftrightarrow \neg\neg p$

(c) $\neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$

(d) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

(e) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$

Are the constructed tableaux systematic?

2. Applying tableau method prove the following propositions or find counterexamples

(a) $\{\neg q, p \vee q\} \models p$,

(b) $\{q \rightarrow p, r \rightarrow q, (r \rightarrow p) \rightarrow s\} \models s$,

(c) $\{p \rightarrow r, p \vee q, \neg s \rightarrow \neg q\} \models r \rightarrow s$.

3. Applying tableau method determine all models of the following theories.

(a) $\{(\neg p \vee q) \rightarrow (\neg q \wedge r)\}$

(b) $\{\neg q \rightarrow (\neg p \vee q), \neg p \rightarrow q, r \rightarrow q\}$

(c) $\{q \rightarrow p, r \rightarrow q, (r \rightarrow p) \rightarrow s\}$

4. Propose suitable atomic tableaux for Peirce arrow \downarrow (NOR) and for Sheffer stroke \uparrow (NAND).

5. Prove directly (by tableau transformations) the deduction theorem, i.e. for every theory T and propositions φ, ψ ,

$$T \vdash \varphi \rightarrow \psi \text{ if and only if } T, \varphi \vdash \psi.$$

6. Let \mathcal{S} be a countable nonempty family of nonempty finite sets. We say that \mathcal{S} has a *selector* if there exists an injective $f: \mathcal{S} \rightarrow \bigcup \mathcal{S}$ such that $f(S) \in S$ for every $S \in \mathcal{S}$. Prove that \mathcal{S} has a selector if and only if every nonempty finite part of \mathcal{S} has a selector.