

Midterm test

Nov 23, 2023

1. Let the propositional variables w, m, r represent (respectively) that “*people wear umbrellas*”, “*meteorologists forecast rain*”, “*it rains*” and denote $\mathbb{P} = \{w, m, r\}$. We know that
 - (i) *If it does not rain, people do not wear umbrellas.*
 - (ii) *If meteorologists forecast rain, people wear umbrellas.*
 - (iii) *If meteorologists do not forecast rain, it rains.*
 - (a) Write propositions $\varphi_1, \varphi_2, \varphi_3$ in \mathbb{P} expressing (respectively) (i), (ii), (iii), transform them into 2-CNF and use the implication graph to show that the theory $T = \{\varphi_1, \varphi_2, \varphi_3\}$ is not contradictory. (30 points)
 - (b) Write T in clausal form and show by resolution that from T it follows *it rains*. Represent the resolution proof by a resolution tree. (30 points)
 - (c) Is the theory T an extension of the theory S over $\{w, m\}$ with the following single axiom (iv)? Is T a conservative extension of S ? Justify your answers. (20 points)
 - (iv) *People wear umbrellas only if meteorologists forecast rain.*
 - (d) How many there exist mutually non-equivalent propositions over \mathbb{P} that are independent in T ? Write at least two of them, if they exist. (20 points)