## Test 2

1. People of Springfield and Greenville organize a fair. We know that:

   ($i$) Everyone knows everyone who lives in the same town.

   ($ii$) From each of the both towns, someone comes to the fair.

   ($iii$) If someone comes to the fair, everyone who knows them also comes.

   ($iv$) John lives in Springfield and Mary lives in Greenville.

   We want to show that:

   ($v$) Both John and Mary come to the fair.

   (a) Express the statements ($i$) to ($v$) by <u>sentences</u> $\varphi_1$ to $\varphi_5$ of the language $L = \langle L, K, C, m, j, s, g\rangle$ without equality, where $L$, $K$ are binary relation symbols with $L(x, y)$, $K(x, y)$ expressing that, "$x$ lives in town $y$" and "$x$ knows $y$", respectively, $C$ is a unary relation symbol with $C(x)$ expressing that "$x$ comes to the fair", and $m$, $j$, $s$, $g$ are constant symbols denoting Mary, John, Springfield, and Greenville, respectively. (20 pts)

   (b) By skolemization of the previous formulas find an open theory $T$ (possibly in an extended language), that is unsatisfiable, if and only if $\{\varphi_1, \varphi_2, \varphi_3, \varphi_4\} \models \varphi_5$. Transform $T$ into the clausal form (set representation). (20 pts)

   (c) Use resolution to show that $T$ is not satisfiable. Express the refutation as a resolution tree. Show the unifications used in each step. Alternatively, use the tableau method to show that $T$ is unsatisfiable. (40 pts)

   (d) Find a conjunction of ground instances of the axioms of $T$ that is unsatisfiable. (10 pts)

   (e) Is $T \vdash_{LI} \square$ ? Explain. (10 pts)