

Practical sessions for Introduction to Complexity and Computability - 1

October 8th, 2019

Evaluation

Evaluation will be based on points from homeworks. Each practicals, several problems totally for about 3 points will be assigned for homework. The deadline for each homework will be the next practicals. You will need at least $2/3$ points from the first 6 practicals to obtain the credit, that is 12 points. Additional points will be available for homework assigned on the last (7th) practical on Jan 7th.

Exercises

1. Turing machines: motivation, definition, computation, recognized language, examples.
 - (a) Construct TM that reverses input words (in some fixed alphabet).
 - (b) Construct TM recognizing the language $\{a^n b^n c^n \mid n > 0\}$.
 - (c) Construct TM that computes $f(n) = n - 1$ for integer $n \geq 1$ (in standard binary encoding).
 - (d) Construct TM that transforms natural numbers in binary encoding into quaternary encoding.
 - (e) Let $J(n)$ for $n \geq 1$ be the last remaining person when n people gather in a circle and every second person goes out, started by skipping the first person. For example $J(5) = 3$. Construct TM that computes $J(n)$ for $n \geq 1$ (in binary encoding).
2. Modifications of Turing machines.
 - (a) Show that for every TM there is an equivalent TM with only a right-sided tape. Assume that the first cell contains a special symbol $\#$ which cannot be rewritten and the head cannot move left from it.
 - (b) Show that for every TM there is an equivalent TM that performs at most two operations in each step (out of three operations: change state, move head, write on tape); that is, it has no instructions of type $(p, a) \rightarrow (q, b, M)$ where $p \neq q$, $a \neq b$, and $M \in \{L, R\}$.
 - (c) Show that for every TM there is an equivalent TM that performs at most one operation in each step (see above).
 - (d) Show that for every TM there is an equivalent TM (up to encoding of the input) with only a single symbol alphabet (aside from λ).
 - (e) Show that for every multi-tape TM there is an equivalent (single-tape) TM.
 - (f) Show that every TM is equivalent to some pushdown automata with two stacks (and without the input tape).
 - (g) Show that for every TM there is an equivalent TM with only two active (i.e. nonaccepting) states. [*]

Homework

Problems 1(e) and 2(a). Hint for 1(e): It is important that the input is in the binary encoding. Both problems are for 2 points, but a trivial solution (that is also much longer and harder to verify) will be for 1 point.