Practical sessions for Introduction to Complexity and Computability - 1

October 4th, 2018

Evaluation

Evaluation will be based on points from homeworks. Each practicals, several problems totally for about 3 points will be assigned for homework. The deadline for each homework will be the next practicals. You will need at least 2/3 points from the first 6 practicals to obtain the credit, that is 12 points. Additional points will be available for homework assigned on the last (7th) practicals.

Exercises

1. Turing machines: motivation, definition, computation, recognized language, examples.
   (a) Construct TM that reverses input words (in some fixed alphabet).
   (b) Construct TM recognizing the language $\{a^n b^n c^n \mid n > 0\}$.
   (c) Construct TM that computes $f(n) = n - 1$ for integer $n \geq 1$ (in standard binary encoding).
   (d) Construct TM that transforms natural numbers in binary encoding into quaternary encoding.
   (e) Let $J(n)$ for $n \geq 1$ be the last remaining person when n people gather in a circle and every second person goes out, started by skipping the first person. For example $J(5) = 3$. Construct TM that computes $J(n)$ for $n \geq 1$ (in binary encoding).
2. Modifications of Turing machines.
   (a) Show that for every TM there is an equivalent TM with only a right-sided tape. Assume that the first cell contains a special symbol # which cannot be rewritten and the head cannot move left from it.
   (b) Show that for every TM there is an equivalent TM that performs at most two operations in each step (out of three operations: change state, move head, write on tape); that is, it has no instructions of type $(p, a) \to (q, b, M)$ where $p \neq q$, $a \neq b$, and $M \in \{L, R\}$).
   (c) Show that for every TM there is an equivalent TM that performs at most one operation in each step (see above).
   (d) Show that for every TM there is an equivalent TM (up to encoding of the input) with only a single symbol alphabet (aside from λ).
   (e) Show that for every multi-tape TM there is an equivalent (single-tape) TM.
   (f) Show that every TM is equivalent to some pushdown automata with two stacks (and without the input tape).
   (g) Show that for every TM there is an equivalent TM with only two active (i.e. nonaccepting) states. [*]

Homework

Problems 1(e) and 2(a). Hint for 1(e): consider $J(2n)$ and $J(2n+1)$, then derive an explicit formula for $J(n)$, which will provide TM with less than 10 instructions. Both problems are for 2 points, but a trivial solution (that is also much longer and harder to verify) will be for 1 point.