Practical sessions for Introduction to Complexity and Computability - 5

November 29th, 2018

Exercises

1. (previous HW) Let A be a finite problem (i.e. the language of its positive instances is finite). Determine for which functions f the class TIME($f(n)$) contains A. Is A in the class $\mathbf{P}$?

2. (previous HW) For a language A and $k \in \mathbb{N}$ let $A_k = \{x \in A;\ |x| = k\}$ be the restriction of A to inputs of size k. Prove or disprove that for any language A, if $A_k \in \mathbf{P}$ for every k, then $A \in \mathbf{P}$.

3. (previous HW) Consider a Turing machine M running in the following time $f(n)$ where $k \in \mathbb{N}$. In which cases the language $L(M)$ belongs to $\mathbf{P}$? In which cases it belongs to $\mathbf{EXP}$? Explain why.

   (a) $f(n) = n^k$
   (b) $f(n) = k^n$
   (c) $f(n) = n^n$
   (d) $f(n) = k^{k \log n}$
   (e) $f(n) = n^{\log \log n}$
   (f) $f(n) = k^{n^k}$
   (g) $f(n) = k^{n^n}$
   (h) $f(n) = n^{n^k}$
   (i) $f(n) = n^{n^n}$

4. Recall that a *literal* is a propositional variable or its negation, a *clause* is a disjunction of (any number of) literals, a formula in CNF (conjunctive normal form) is a conjunction of (any number of) clauses. Decide which of the following problems are in $\mathbf{P}$ and which are in $\mathbf{NP}$. Note that some answers may not be known. Given a formula φ in CNF,

   (a) Is φ satisfiable? (SAT problem)
   (b) Is $\neg\varphi$ satisfiable?
   (c) Is φ valid (i.e. a tautology)?
   (d) Is φ contradictory (i.e. $\neg\varphi$ is valid)?

5. Consider the above problem for formulas in DNF (disjunctive normal form). A propositional formula in DNF is a disjunction of (any number of) elementary conjunctions. An *elementary conjunction* (also called a *term*) is a conjunction of (any number of) literals.

6. Let A be the language over the alphabet $\{(,)\}$ of all words that have properly nested parentheses. For example, $(())$ is in A, but $)($ is not. Let B be the language over the alphabet $\{(,),[,]\}$ of all words that have properly nested parentheses and brackets. For example, $([()]()[])$ is in B but $([)]$ is not. Show that both A and B are in the class $\mathbf{L}$.

7. For each pair of complexity classes below decide if there is any inclusion between them (by using theorems and propositions from the lecture). Note that some relations may not be known (by our knowledge from the lecture).

   (a) SPACE(n) and SPACE(n^3)
   (b) SPACE(n^3) and TIME(2^{n^4})
   (c) TIME(2^{n^3}) and NSPACE($n \log n$)

(d) NSPACE($n \log n$) and NTIME($n \log n$)

(e) NTIME($n \log n$) and SPACE(n)

(f) SPACE(n) and TIME(2^{n^3})

(g) SPACE(n^3) and NSPACE($n \log n$)

(h) TIME(2^{n^3}) and NTIME($n \log n$)

(i) NSPACE($n \log n$) and SPACE(n)

(j) NTIME($n \log n$) and SPACE(n^3)

8. In the cases above where there is some inclusion, decide if it is a proper inclusion (i.e. $\subsetneq$) (by using theorems and propositions from the lecture). Note that it may not be known (by our knowledge from the lecture).

Homework

Problems 6 (1 point), 7 (chose any 5 subproblems from (b)-(j), 2 points), and 8 (on the same 5 subproblems, 1 point). For 8 you may need also time hierarchy theorem which will be on the next lecture.