Introduction to Artificial Intelligence

English practicals 2: Problem solving and search

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Selected quiz questions

- Explain the difference between atomic and factored representations of states using N-queens problem.

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  State - representation of physical configuration. Search node - data structure, two different search nodes may correspond to one state

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  basic:

  1. States: coordinates of queens on board
  2. Initial state: empty board
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  4. Search space size: $(n \times n)^n$

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  2. Initial state: empty board
  3. Goal state: test that all queens are on the board and do not attack each other
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  queens initially assigned to columns:

  1. States: row number of each queen
  2. Initial state: empty board
  3. Goal state: test that all queens are on the board and do not attack each other
  4. Search space size: n^n

Tree vs Graph Search

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- Possible duplicates (multiple nodes with identical state)

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- Search tree can be infinite even for finite state space

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- Measures of time and space complexity:
  - b - maximum branching factor
  - d - shallowest depth of a solution
  - m - maximum depth of the search space

BFS vs DFS

BFS

DFS

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BFS

- Expand shallowest node in the frontier

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- Expand deepest node in the frontier
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- Complete for finite b (search space can be infinite)

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- Time: $\mathcal{O}(b^d)$
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- Time: $\mathcal{O}(b^d)$
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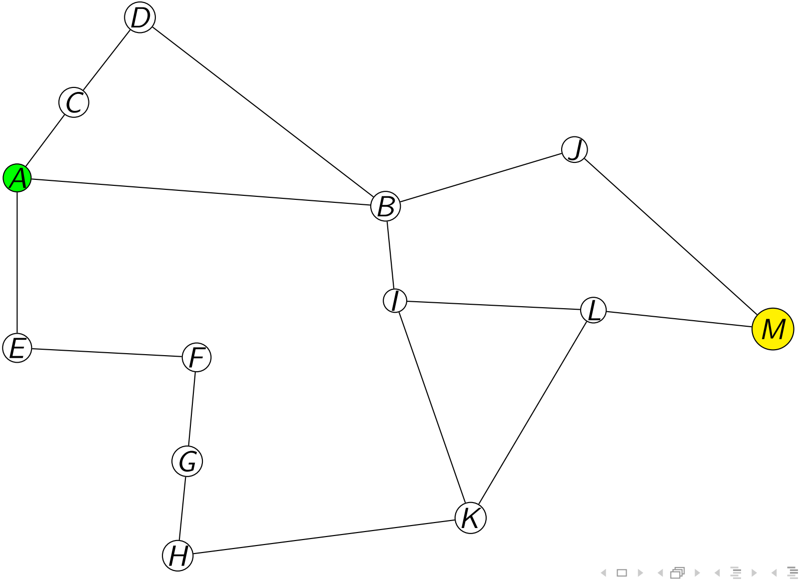

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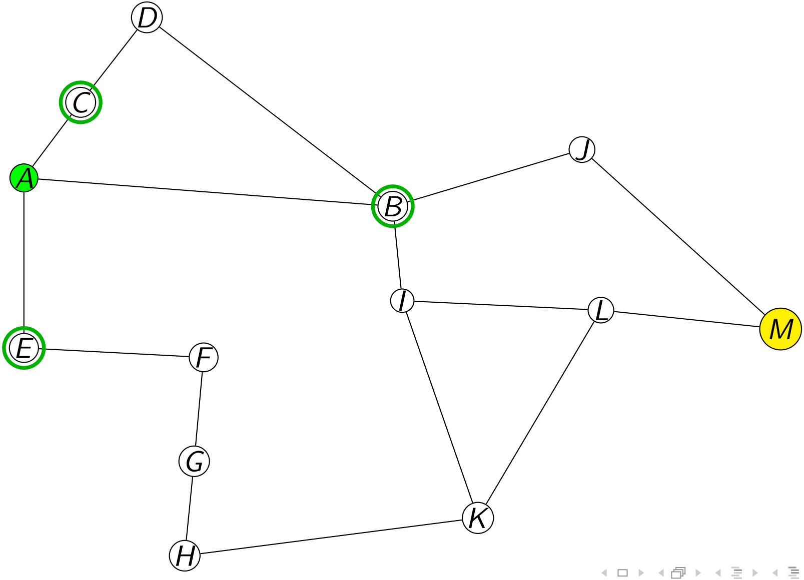

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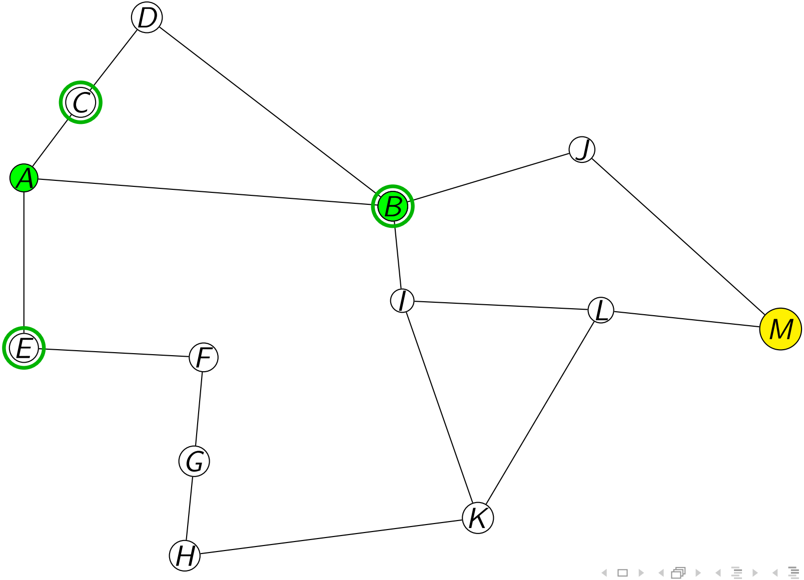

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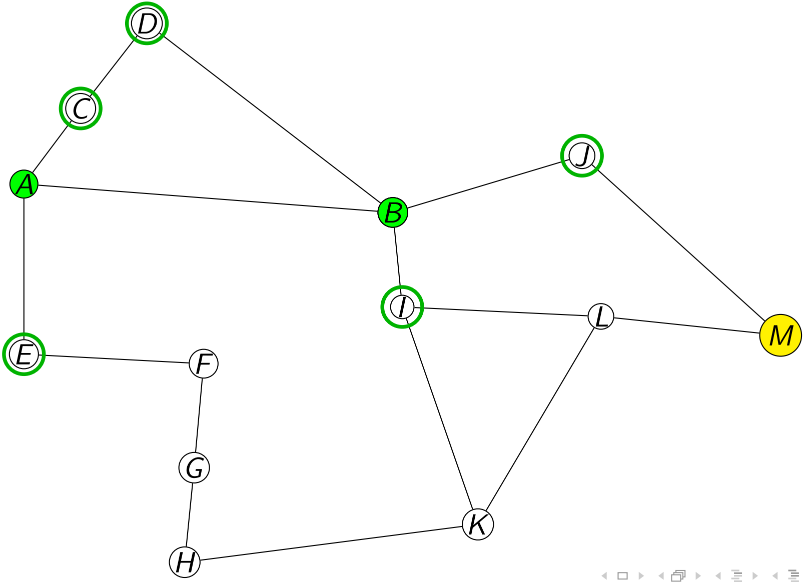

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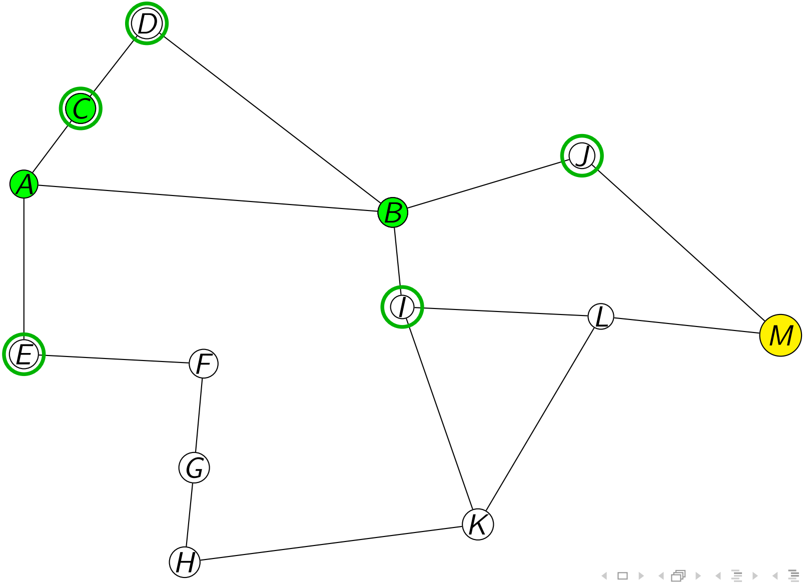

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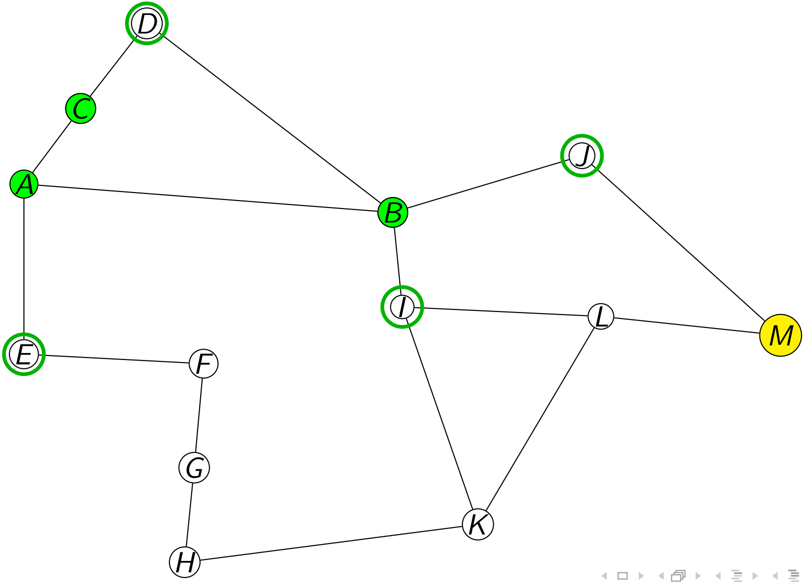

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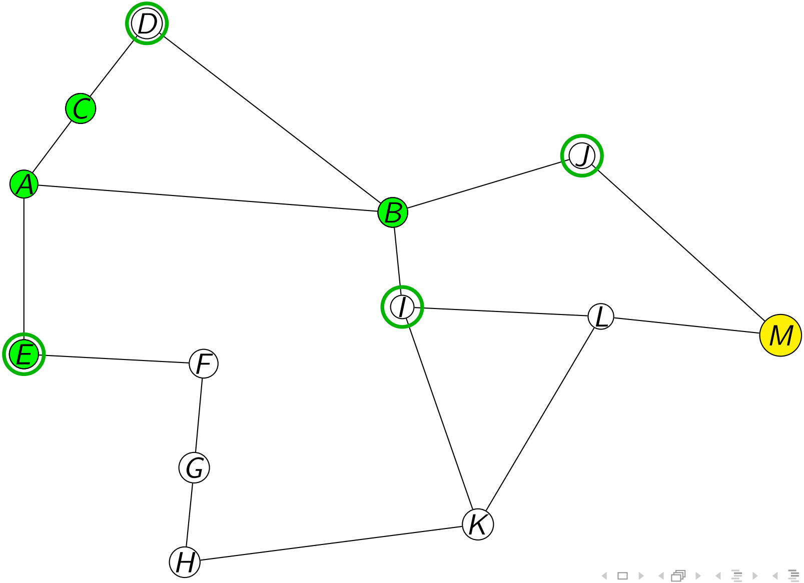

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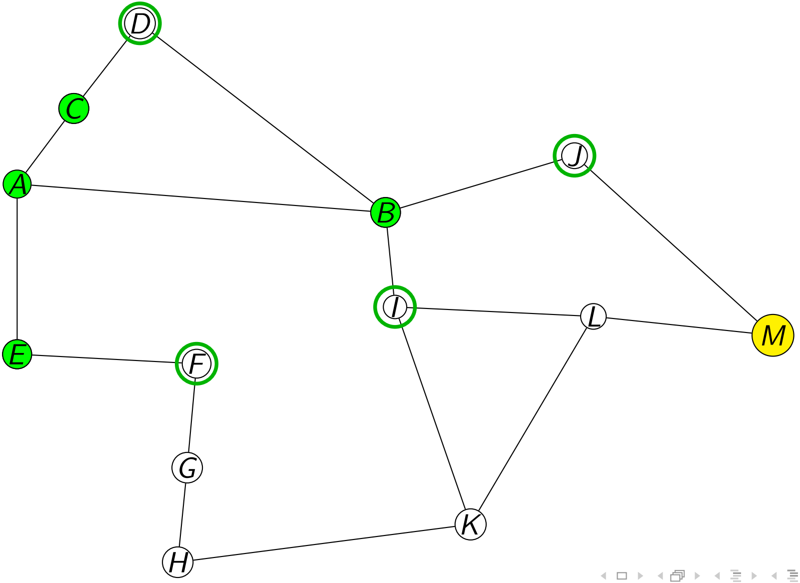

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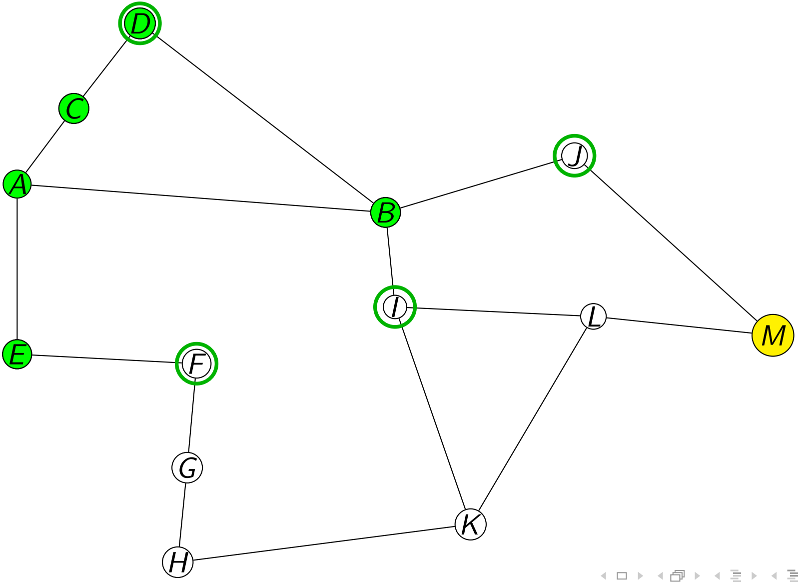

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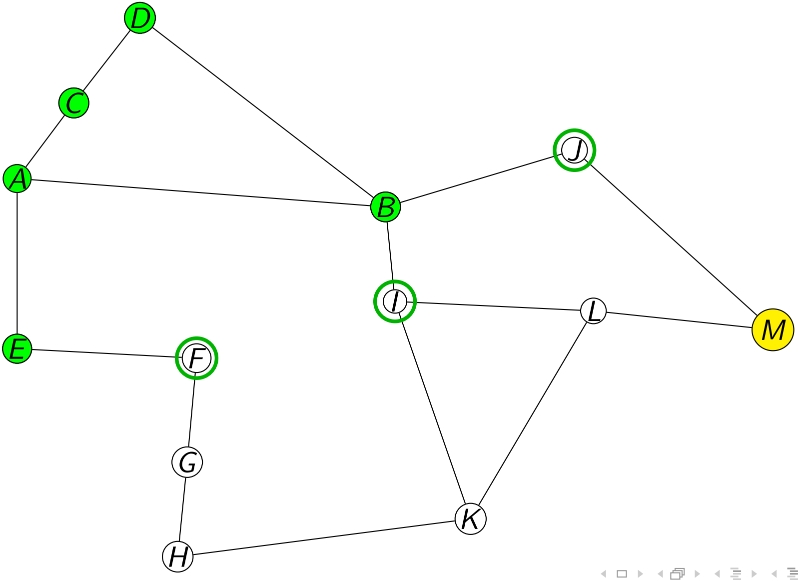

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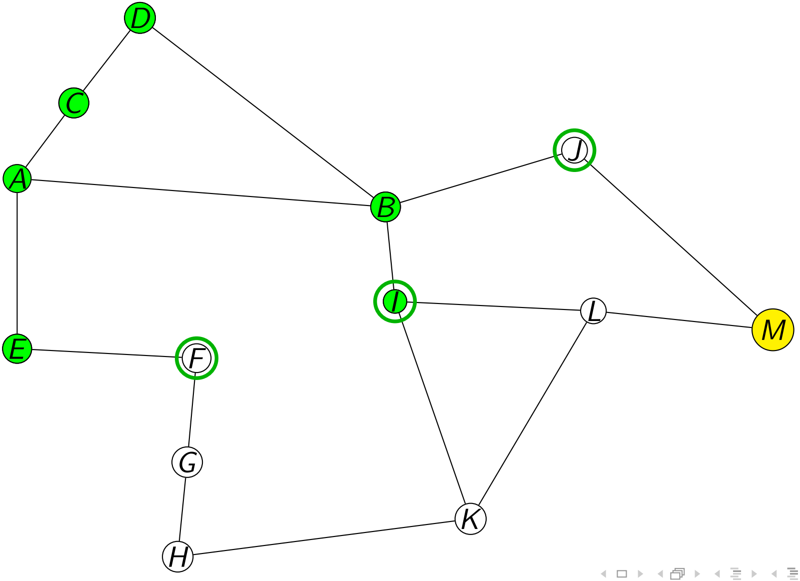

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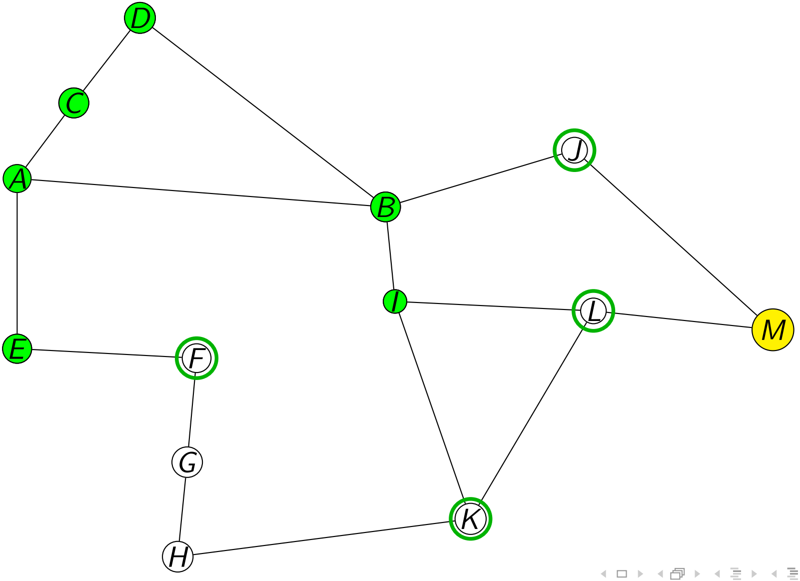

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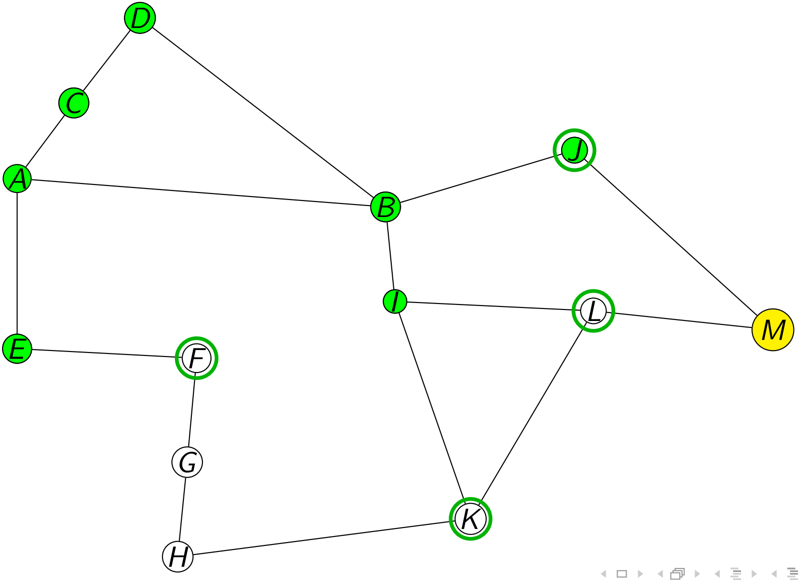

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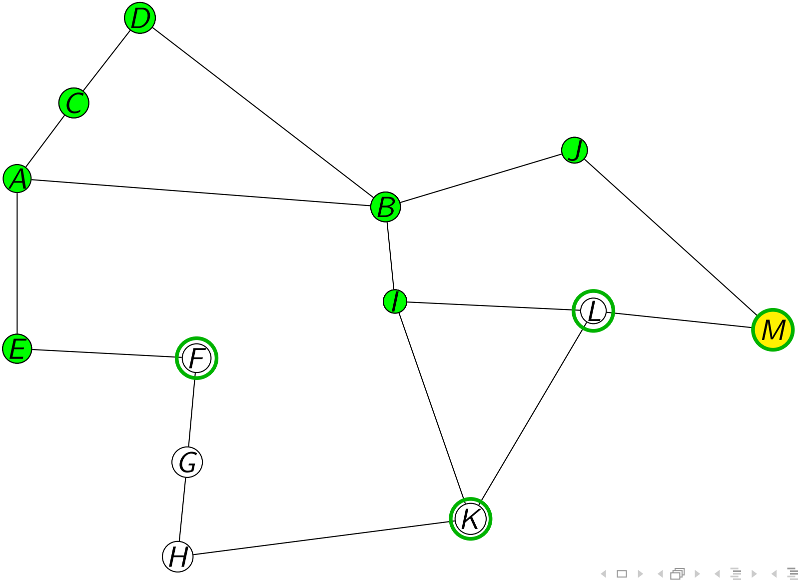

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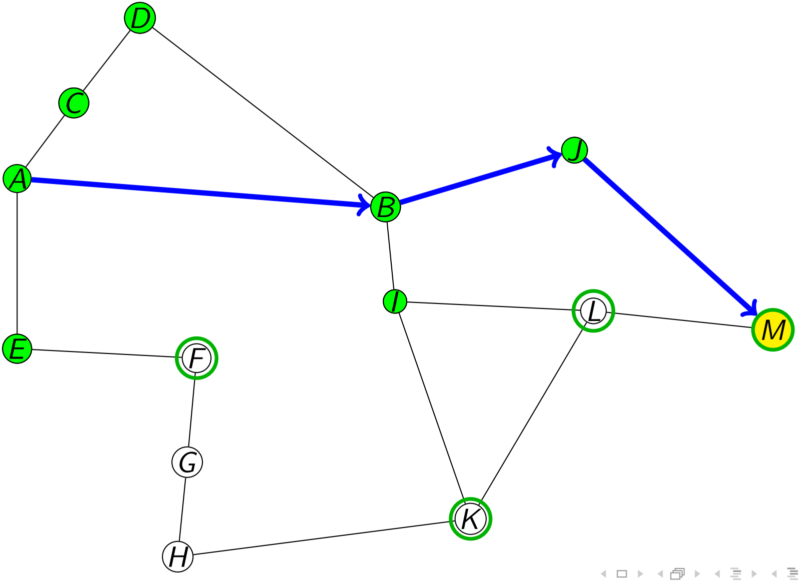

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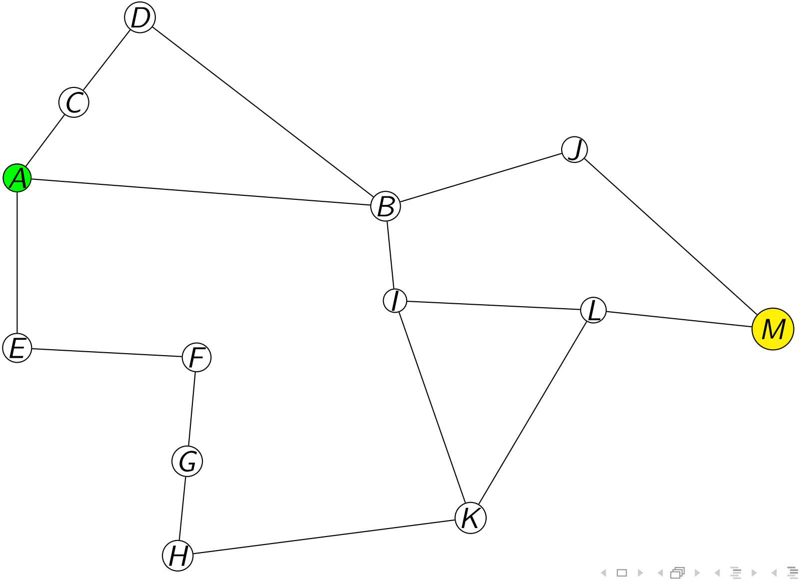

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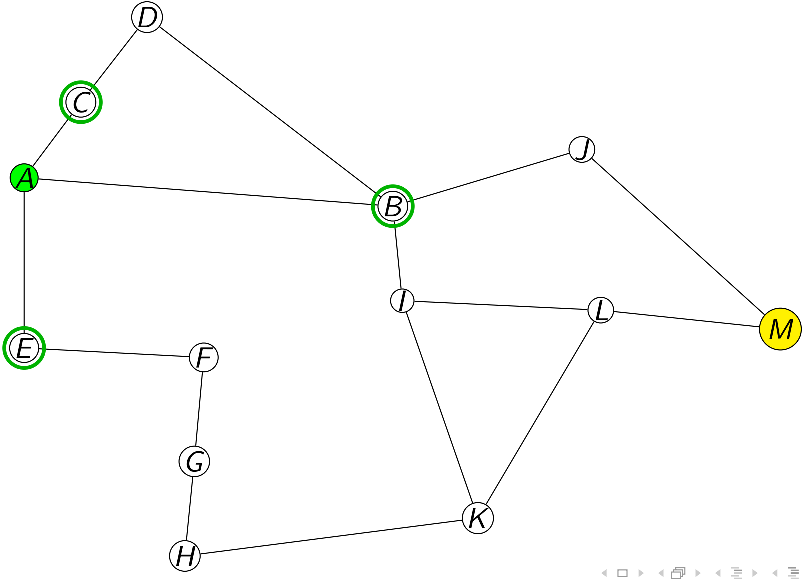

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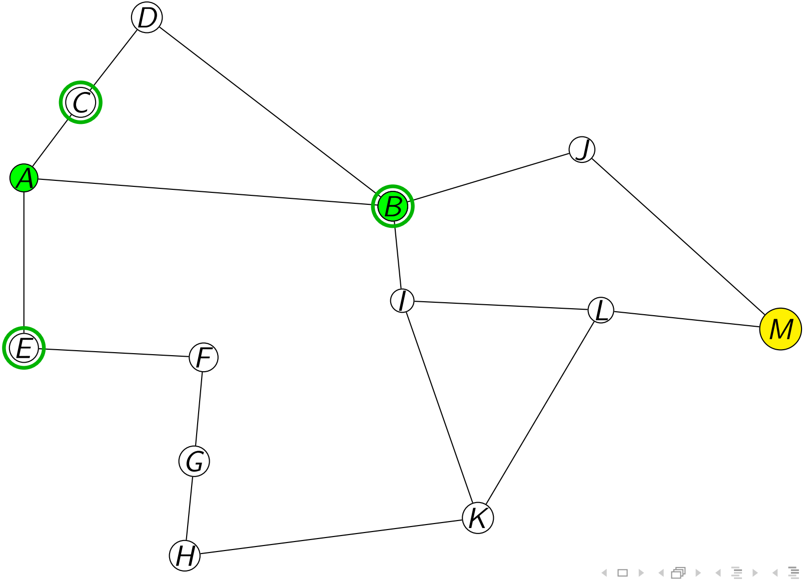

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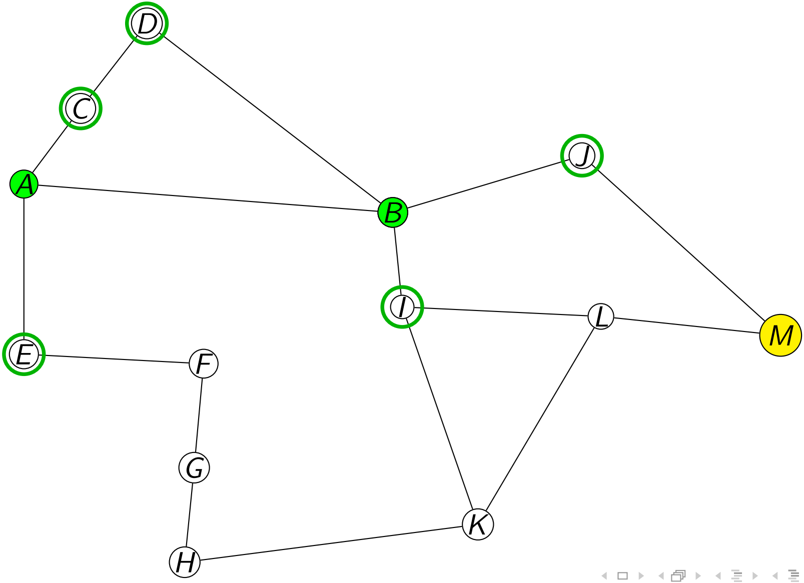

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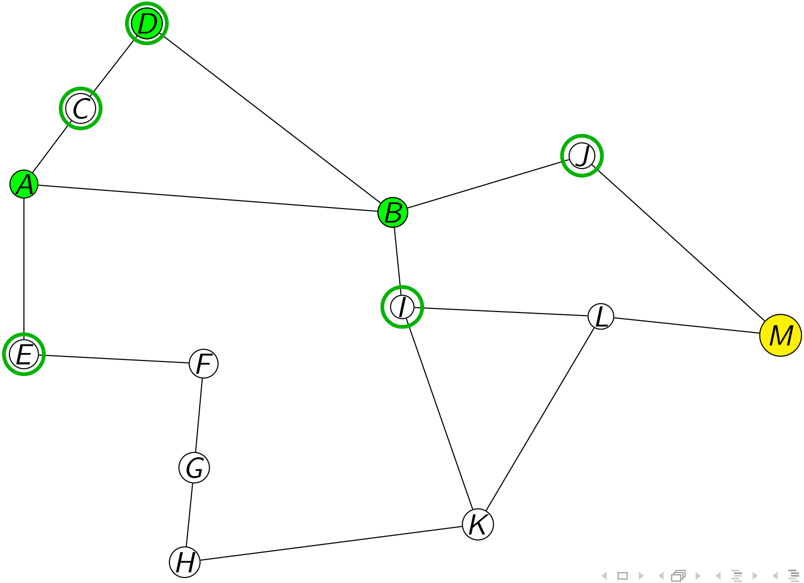

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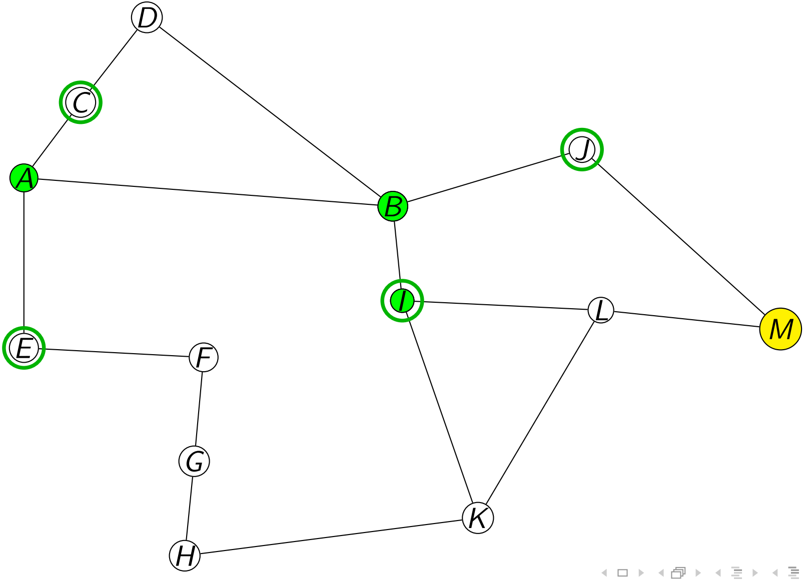

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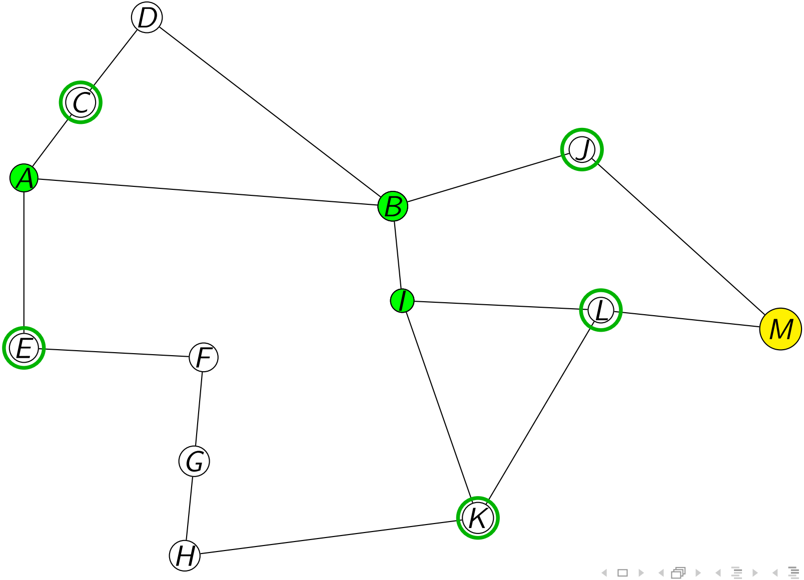

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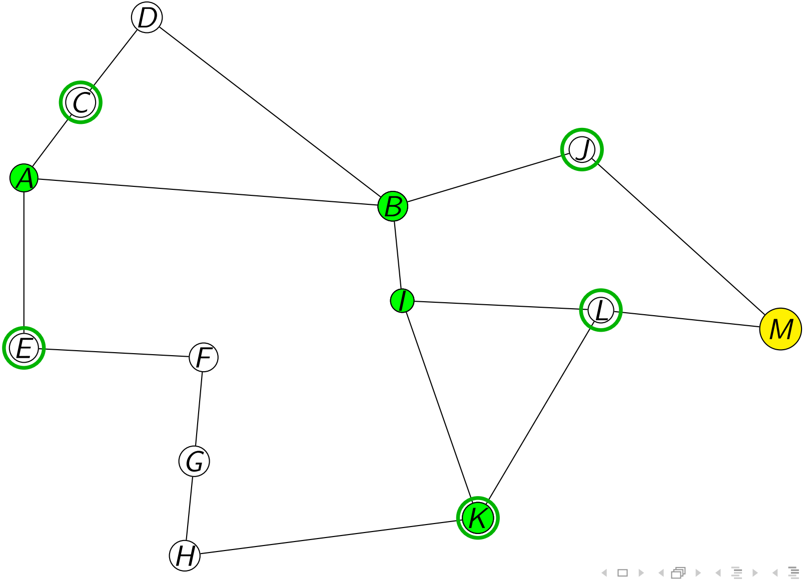

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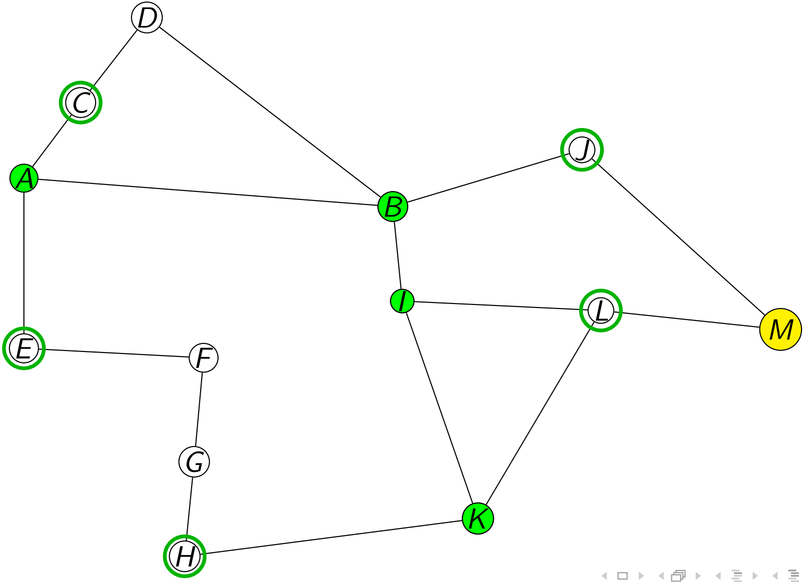

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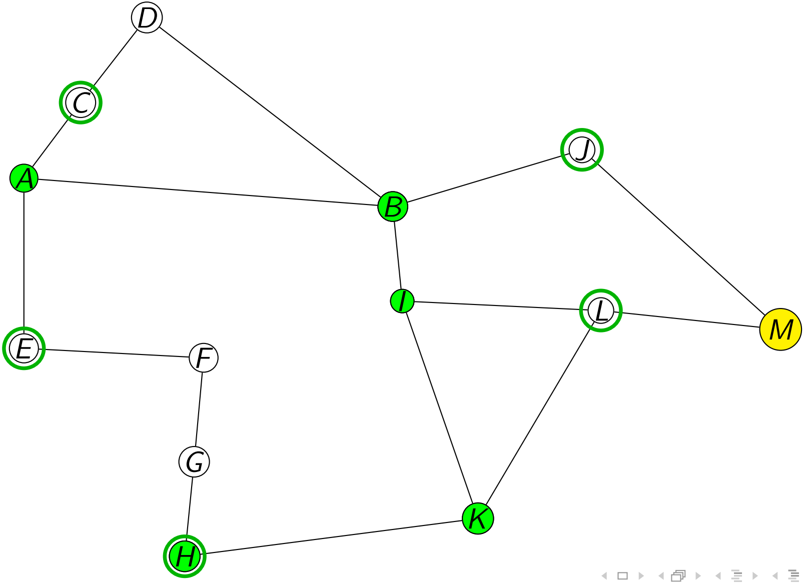

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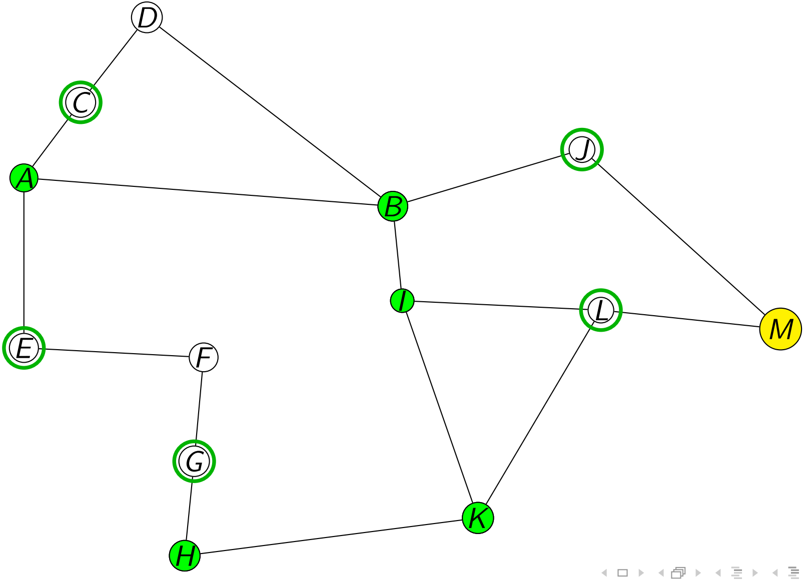

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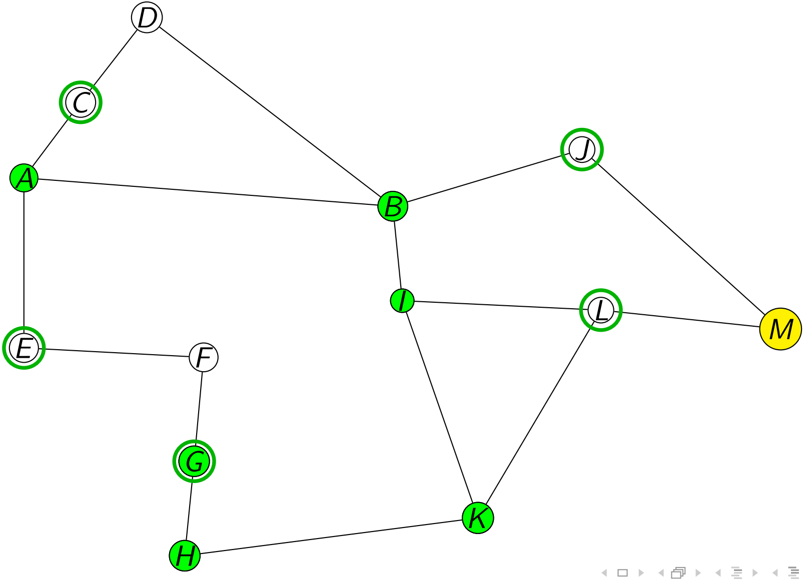

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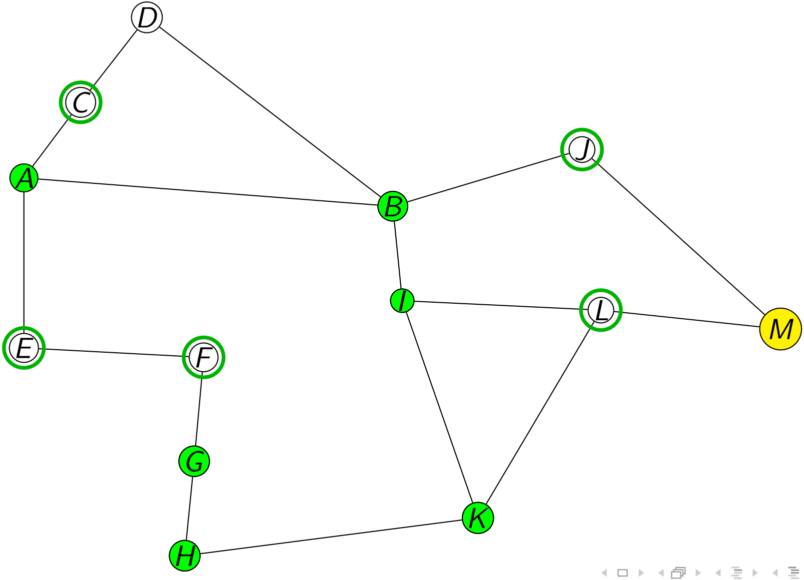

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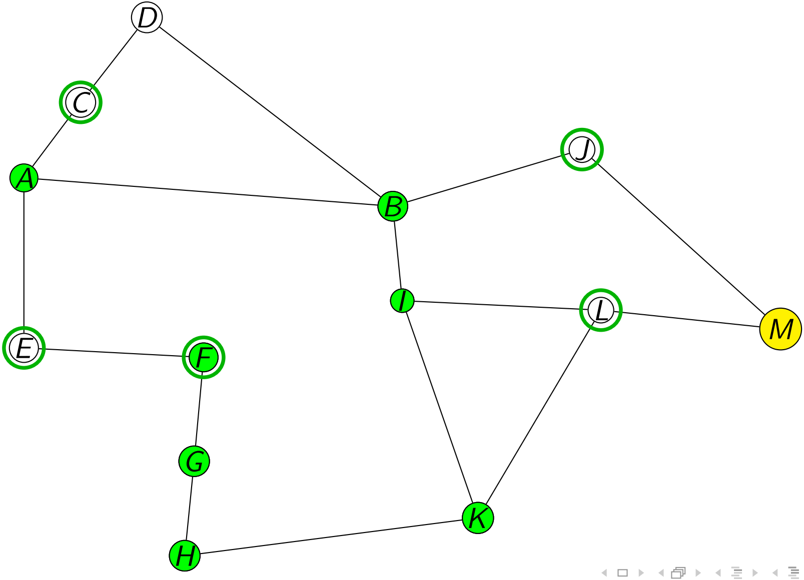

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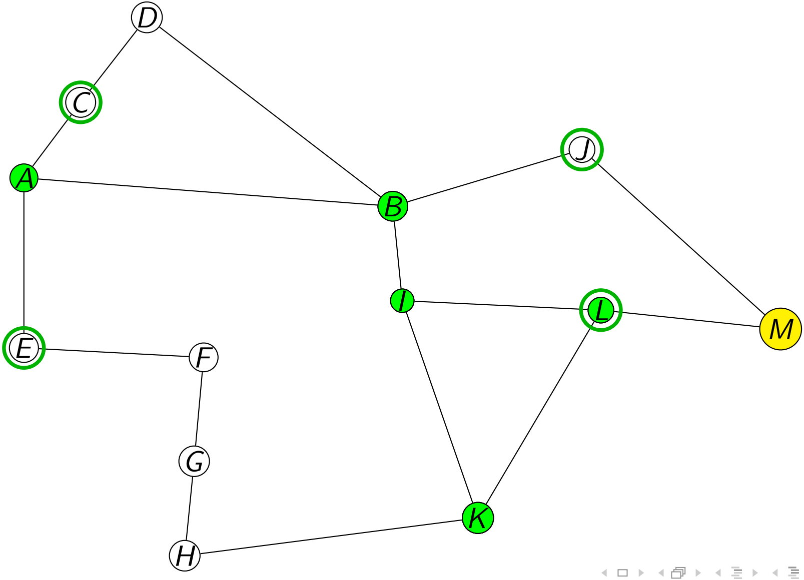

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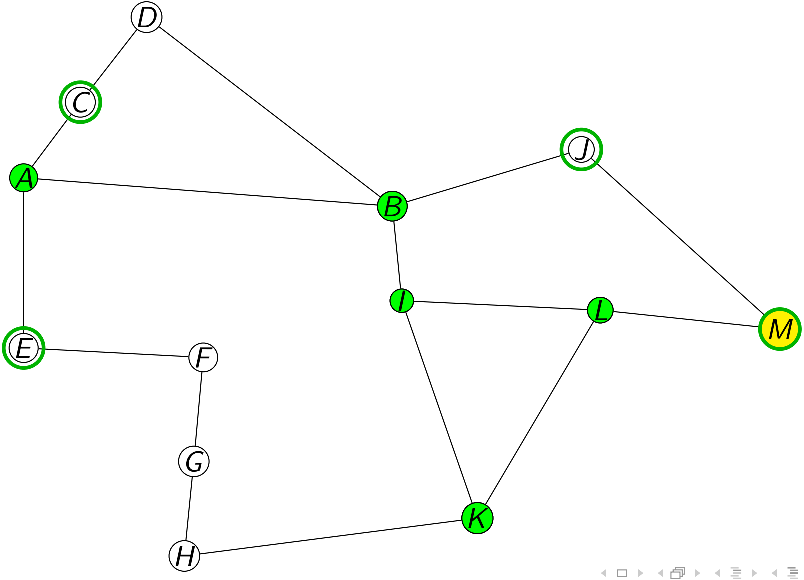

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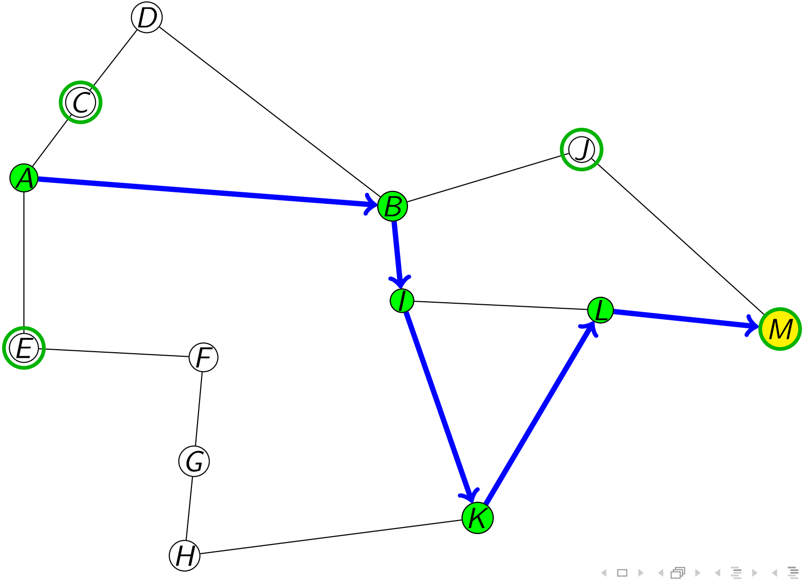

Uniform-cost search (Dijkstra)

Like BFS, but:

- $g(n)$ - lowest path cost from root to n
- Frontier implemented as priority queue g
- Goal test performed when a node is **selected**
- Test if a better path is find to a node in the frontier

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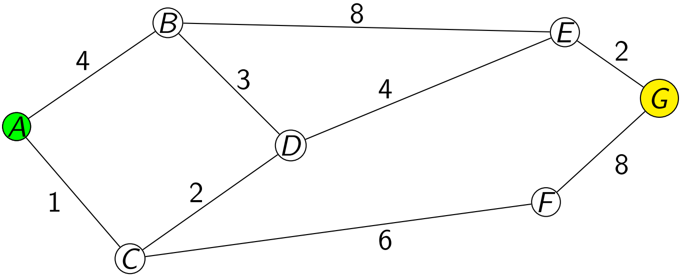

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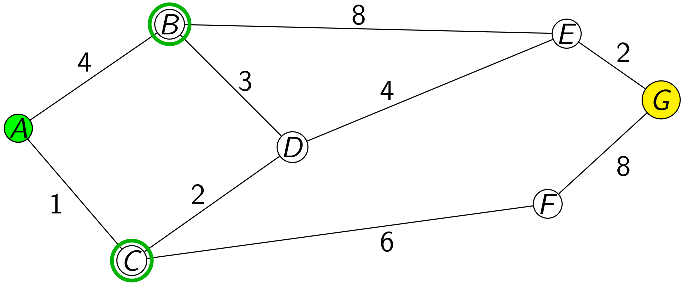

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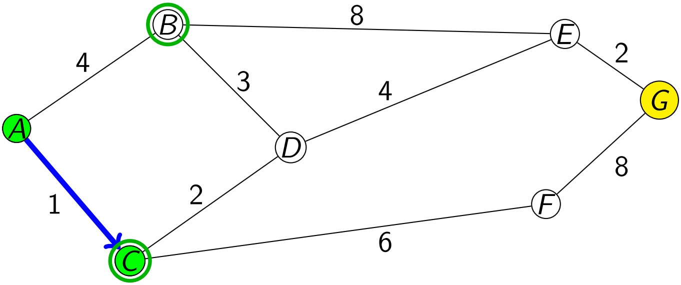

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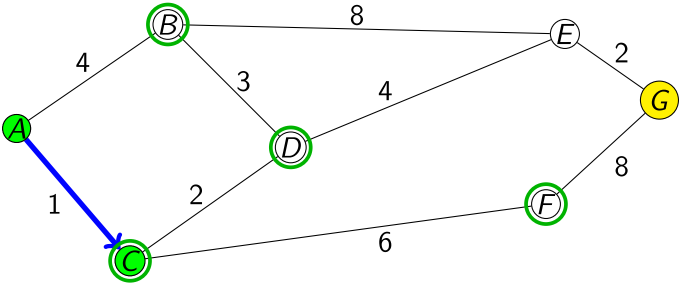

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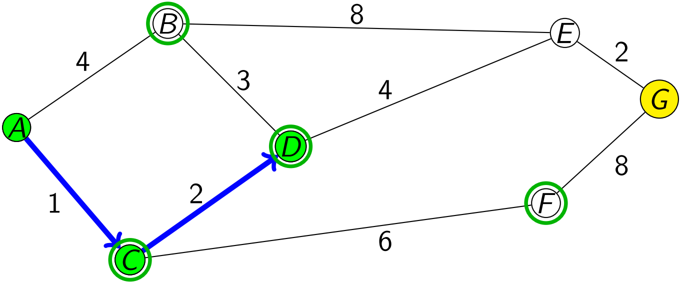

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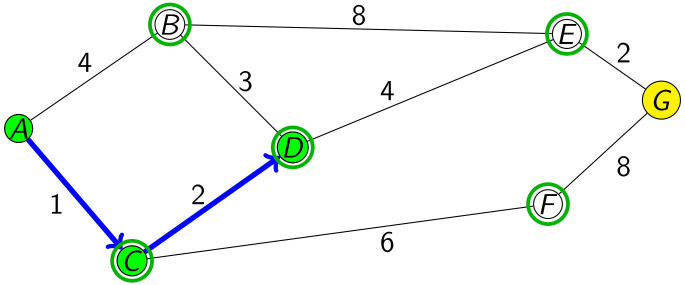

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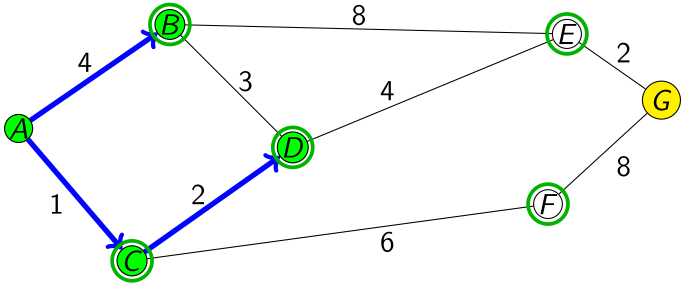

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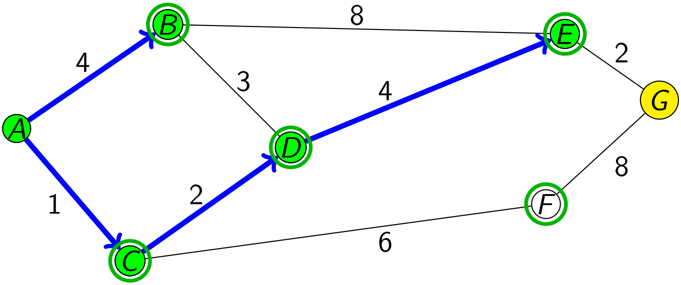

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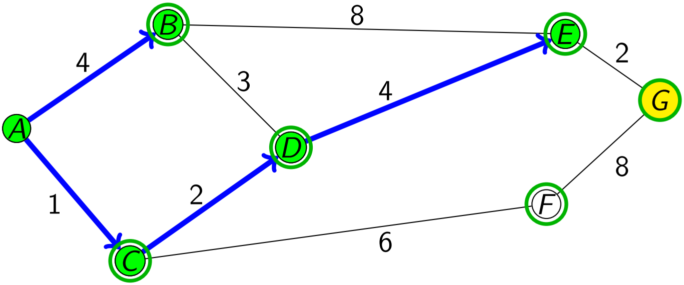

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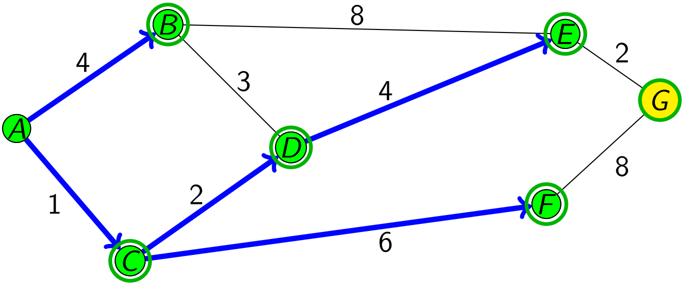

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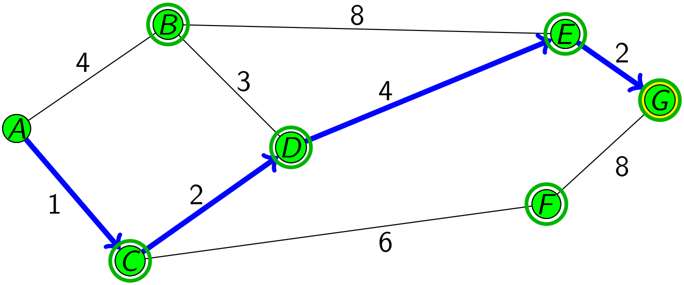
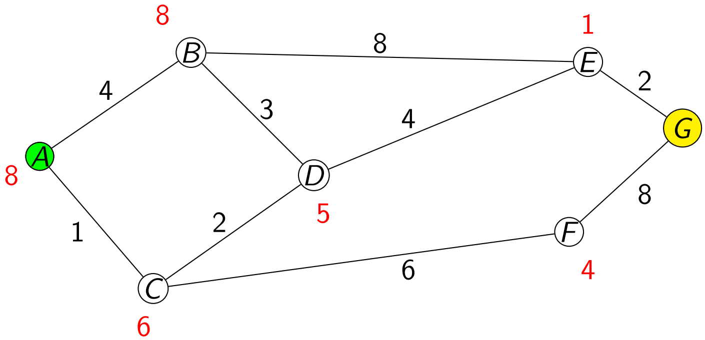


Greedy best first search



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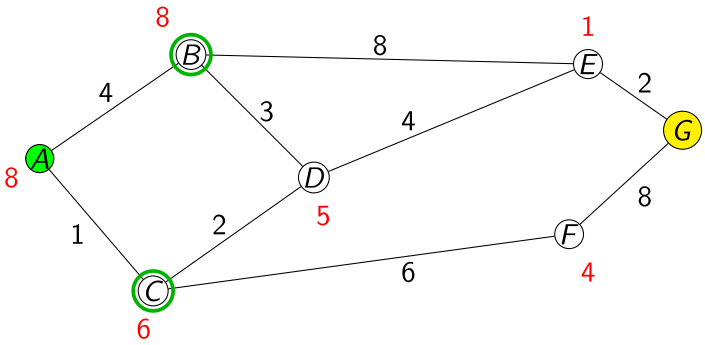

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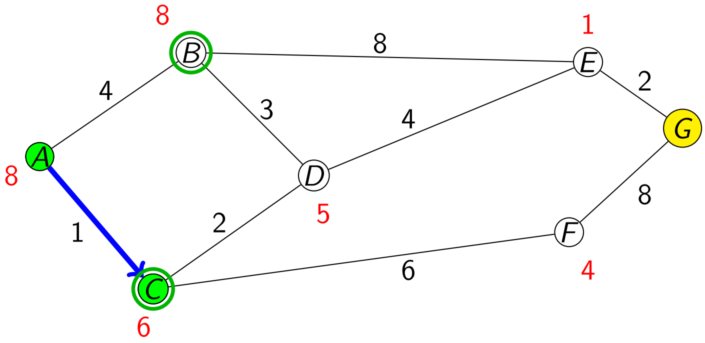

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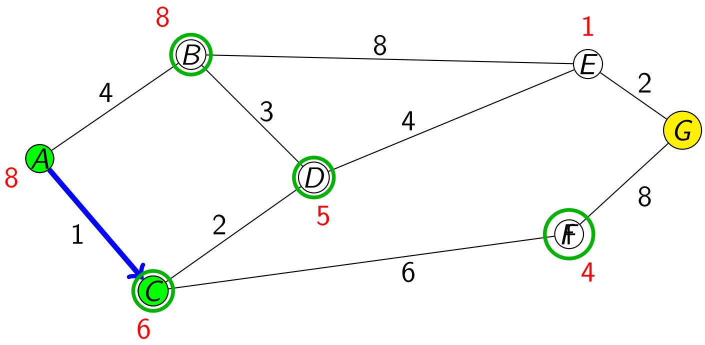

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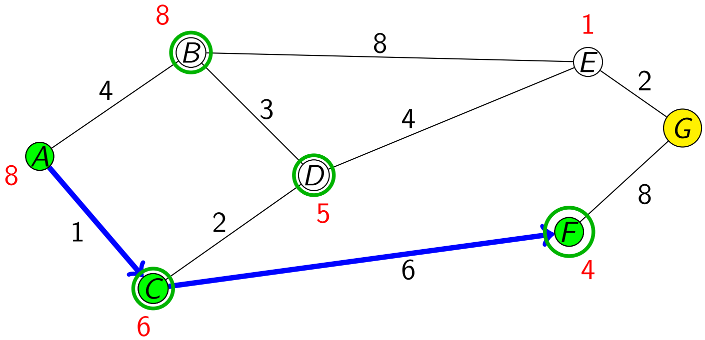

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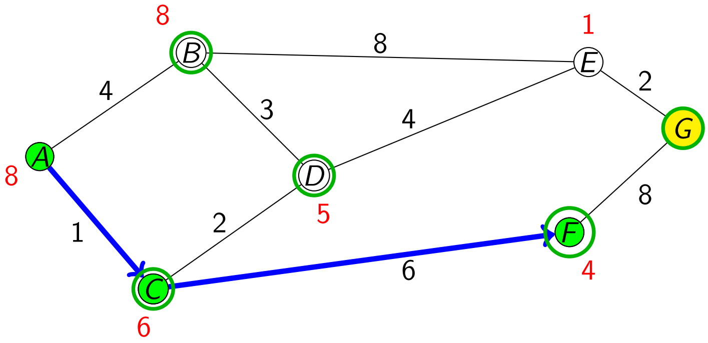

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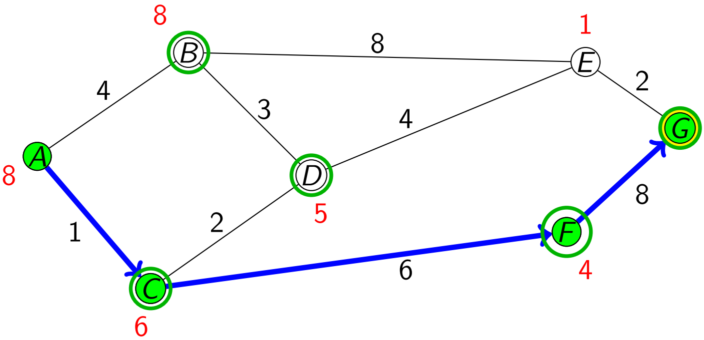

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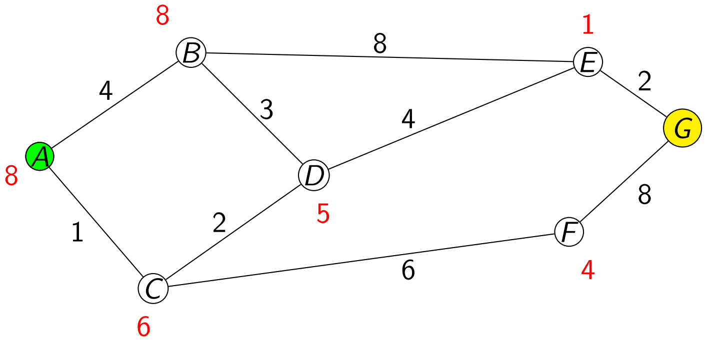

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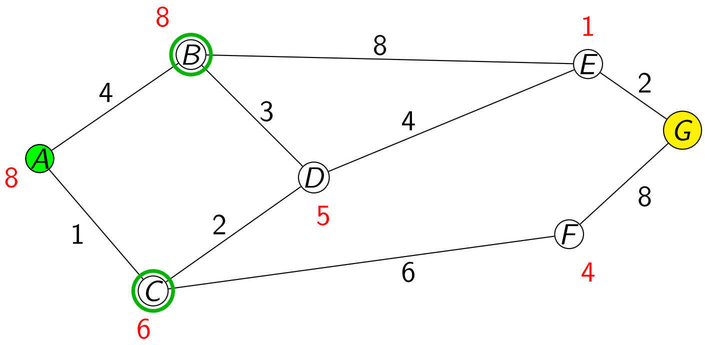

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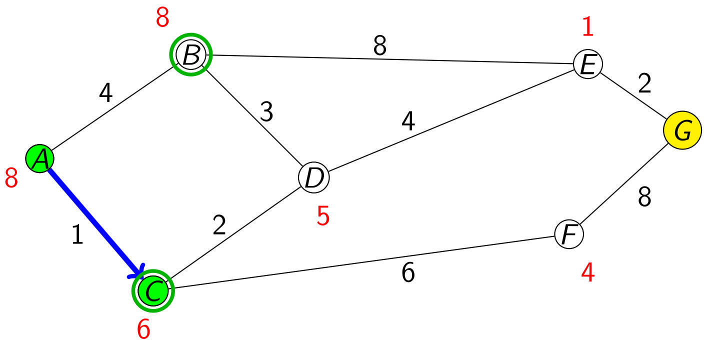

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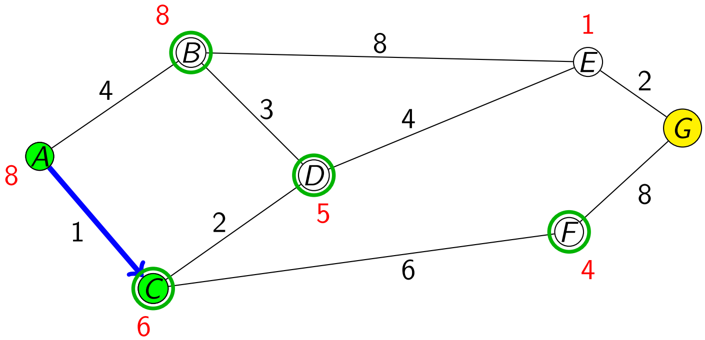

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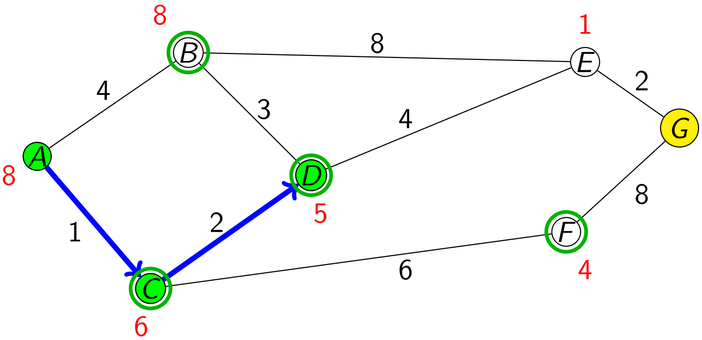

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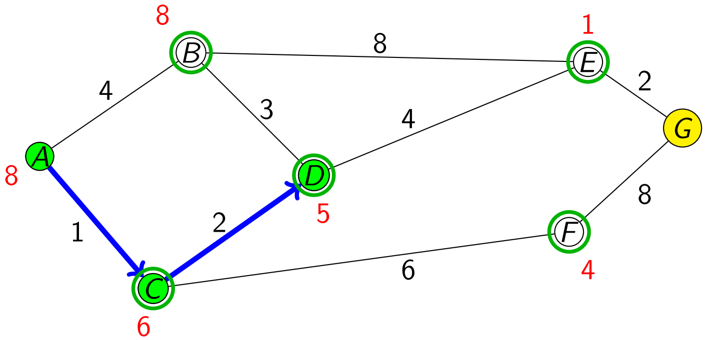

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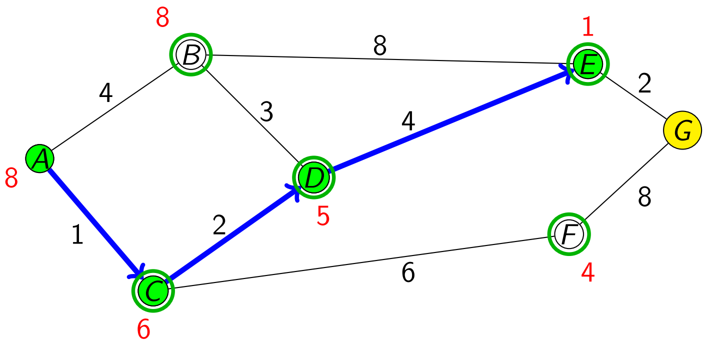

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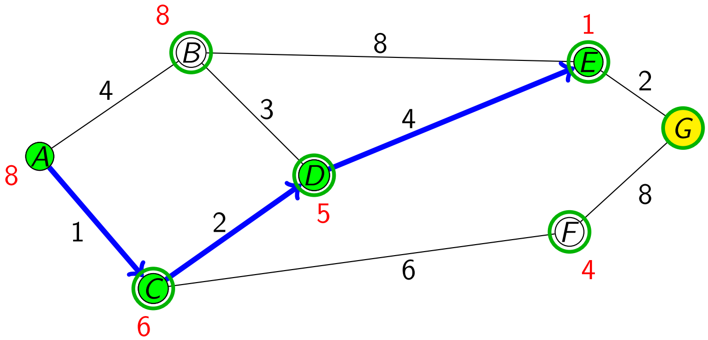

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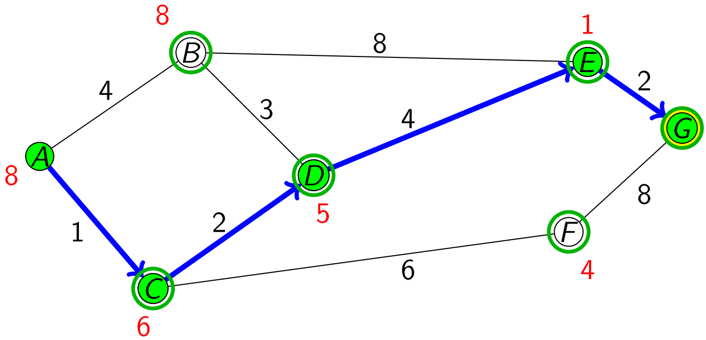


Heuristic functions

- Heuristic function $h(n)$ is an estimate of the distance from n to goal g
- if $h(n) = 0$ for all n, we get Dijkstra
- $h(n)$ should be calculated quickly, ideally in $\mathcal{O}(1)$

Heuristic functions

Definition

$h(n)$ is admissible if for all nodes n, $0 \leq h(n) \leq p^*(n)$, where p^* is the length of a shortest path from n to g.

Definition

$h(n)$ is monotonous if $h(n) \leq h(n') + c(n, n')$, for all nodes n and edges (n, n')

Exercises

Decide whether the following heuristics are admissible and monotonous for a traffic network (cities and roads).

- Euclidean distance: $h(n) = \sqrt{(g_1 - n_1)^2 + (g_2 - n_2)^2}$
- Manhattan metric: $h(n) = |g_1 - n_1| + |g_2 - n_2|$
- Maximum metric: $h(n) = \max\{|g_1 - n_1|, |g_2 - n_2|\}$

Heuristic functions

Proposition

All monotonous heuristics are also admissible.

Exercise

Find a heuristic function that is admissible, but not monotonous.

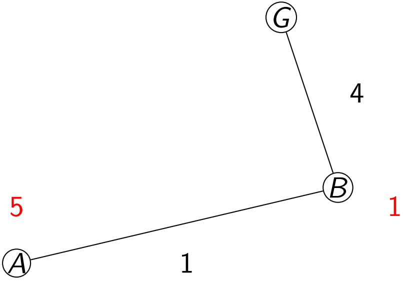
Heuristic functions

Proposition

All monotonous heuristics are also admissible.

Exercise

Find a heuristic function that is admissible, but not monotonous.



Summary

	Compl.*	Opt.	Time	Space	Node selection
BFS	Yes	Yes	$\mathcal{O}(b^d)$	$\mathcal{O}(b^d)$	FIFO
DFS	No	No	$\mathcal{O}(b^m)$	$\mathcal{O}(bm)$	LIFO
Dijkstra	Yes	Yes	$\mathcal{O}(b^{1+C^*/\epsilon})$	$\mathcal{O}(b^{1+C^*/\epsilon})$	$f(n) = g(n)$
Best FS	No	No	$\mathcal{O}(b^m)$	$\mathcal{O}(b^m)$	$f(n) = h(n)$
A*	Yes	Yes**	depends	depends	$f(n) = g(n) + h(n)$

C^* is the cost of an optimal solution

ϵ smallest action (edge) cost

* depends on assumptions about the state space

** depends on heuristic