Introduction to Artificial Intelligence

English practicals 3: Constraint satisfaction

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Approaches for solving algorithmic problems

Specialized algorithms

- Dijkstra's shortest path algorithm
- Hungarian algorithm for the assignment problem
- Prim's algorithm for minimum spanning tree

Is there an algorithm for every problem?

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- Infinitely many but countable algorithms
- Uncountable decision problems
- An algorithm does not exist for most of the problems
- There is no algorithm deciding whether a given program stops (The Halting problem)

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General approaches for solving problems

- What class of problems to solve?
- How to encode problems of this class?
- Find a general algorithm for solving this class of problems

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Examples of problem classes

- (Integer) linear programming (ILP)
- Constraint satisfaction programming (CSP)
- SAT (logical formulae in CNF)

A small reminder

- Problem solving is realized via **search**
- **Tree** search vs **graph** search
- **Uninformed** search - DFS, BFS, Uniform cost search vs **Informed** search - Best first search, A*
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  - A **feasible solution** to a CSP is a complete consistent assignment of values to variables

  - The arc (V_i, V_j) is arc consistent iff for each value x from the domain D_i there exists a value y in the domain D_j such that the assignment $V_i = x$ and $V_j = y$ satisfies all the binary constraints on V_i, V_j.

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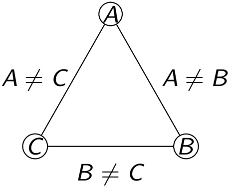

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- Variables: $\{A, B, C\}$
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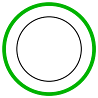

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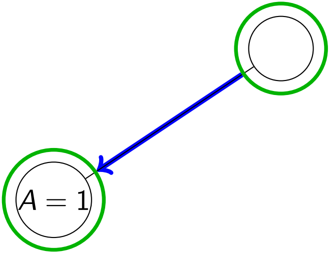

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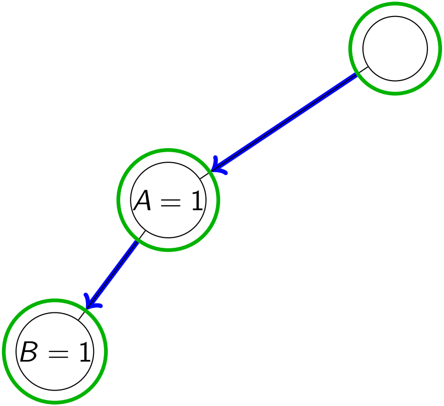

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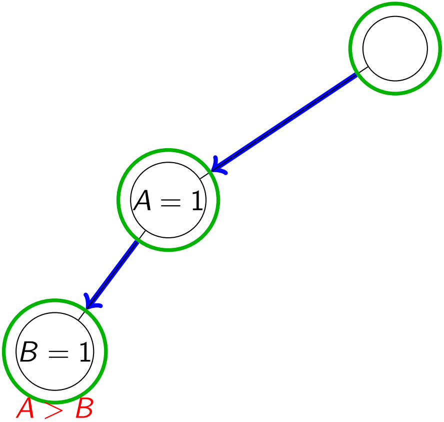

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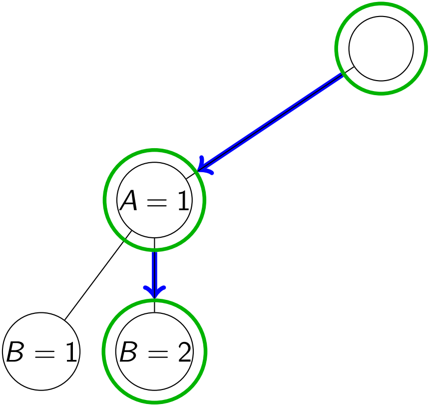

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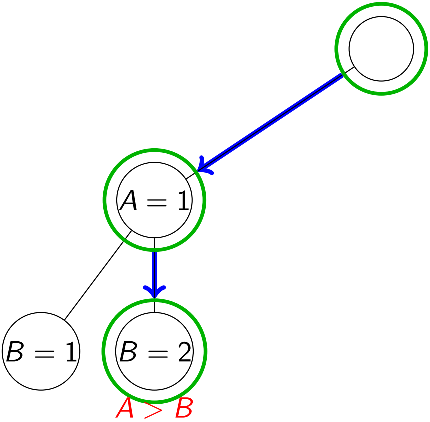

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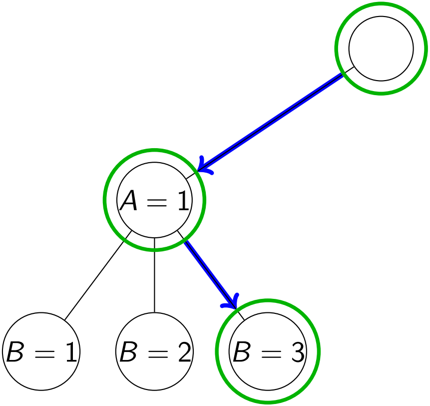

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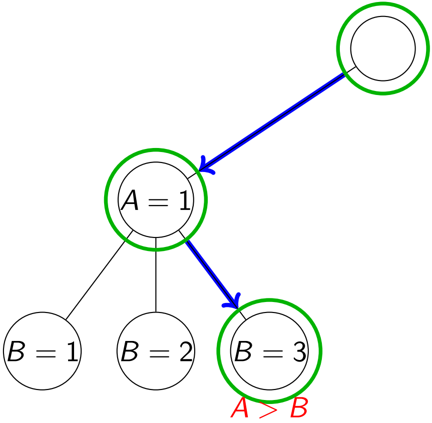

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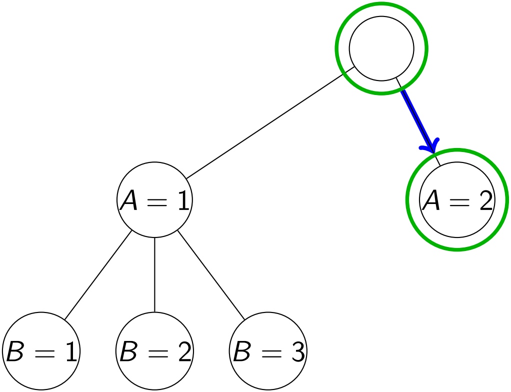

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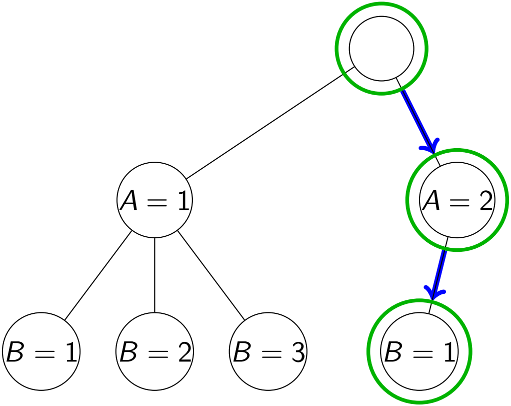

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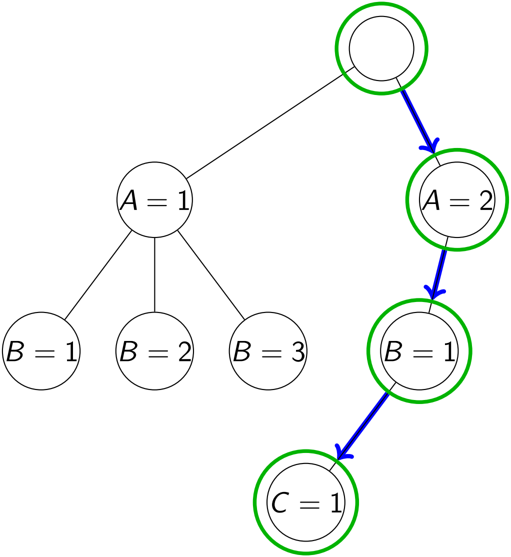

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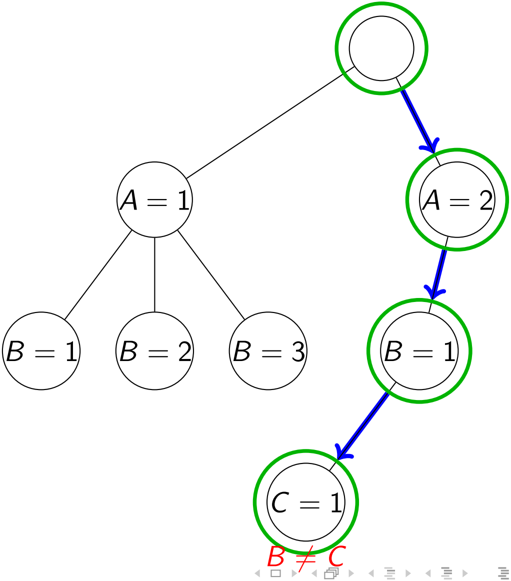

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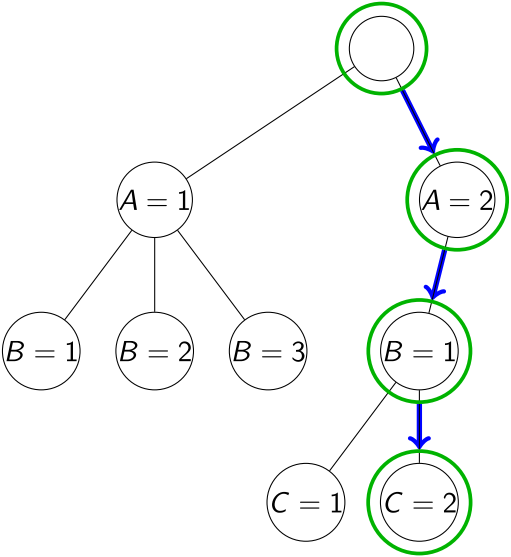

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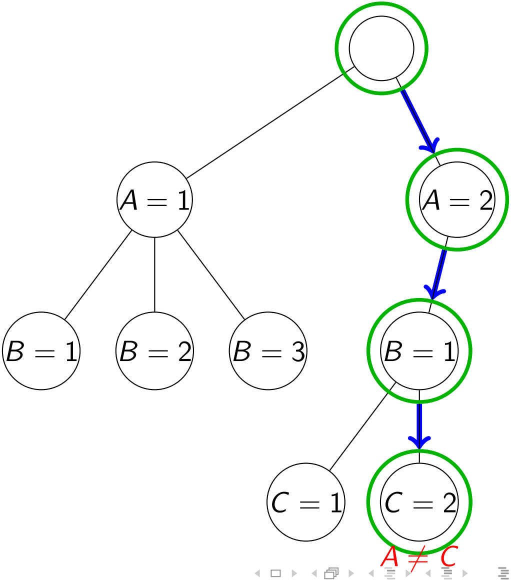

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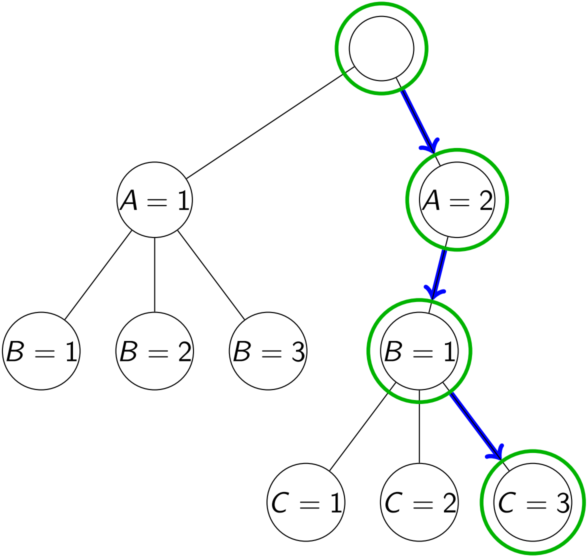

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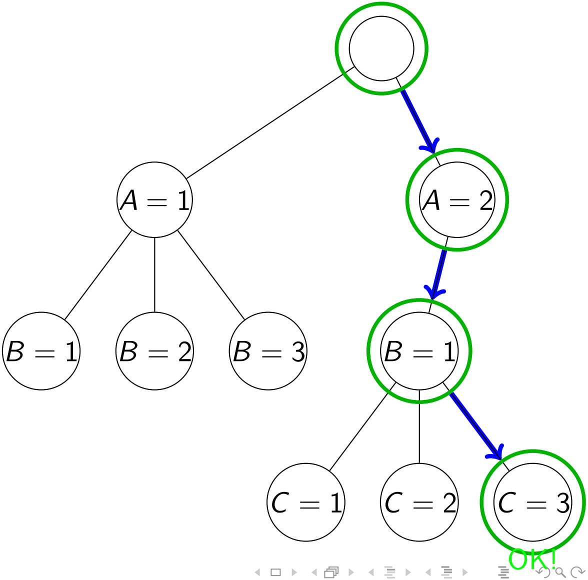

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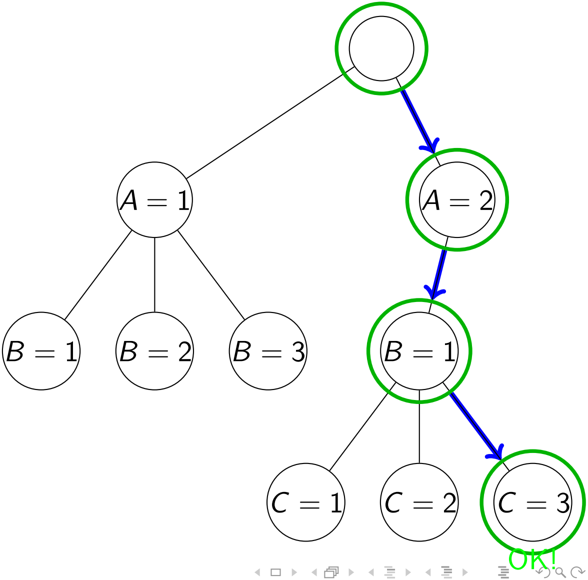
- Feasible solution:
  $A = 2$
  $B = 1$
  $C = 3$



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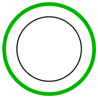

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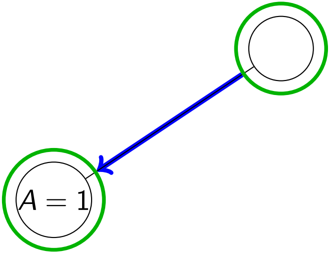

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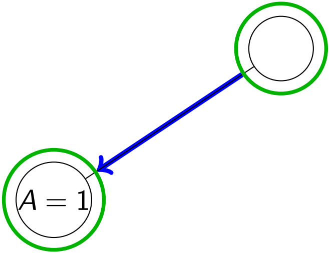

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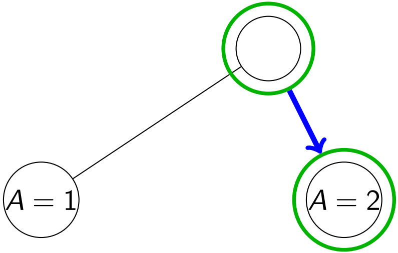

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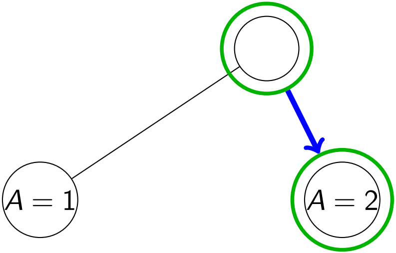

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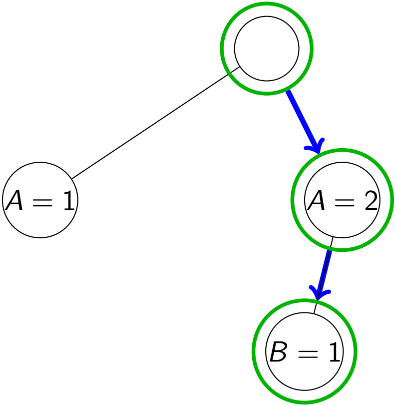

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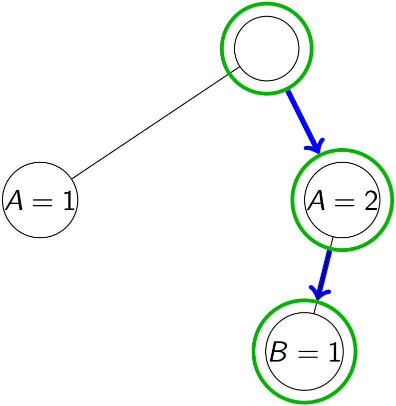

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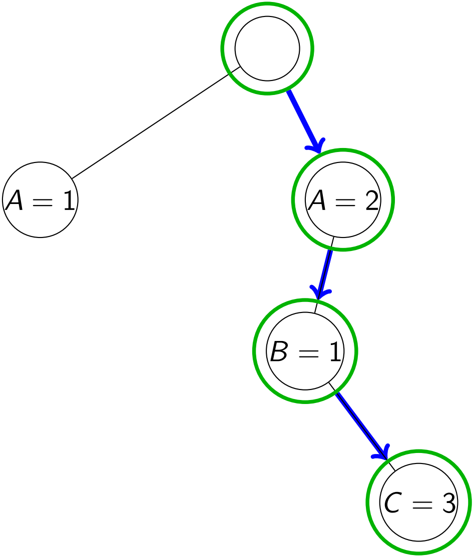

AC-3

- Turn each binary constraint into two arcs
  ($A < B$ becomes $A < B$ and $B > A$)
- Add all arcs to queue Q
- Repeat until Q is empty:
  - remove an arc (X_i, X_j) from Q and check if for every value of X_i there is a value in X_j
  - remove any inconsistent values from X_i
  - if the domain of X_i changed, add all arcs (X_k, X_i) to Q

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Arcs:

$A > B$

$B < A$

$B = C$

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Example: AC-3

- Variables:
  $\{A, B, C\}$
- Domains:
  $D_A = \{1, 2, 3\}$
  $D_B = \{1, 2, 3\}$
  $D_C = \{1, 2, 3\}$
- Constraints:
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Empty Q - the problem is arc consistent.

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Will the algorithm always terminate?

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  $\forall x_{ij} \in X : x_{ij} \in C_j, x_{ij} \in R_i, x_{ij} \in B_{(i-1)/3+1,(j-1)/3+1}$

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  - $\forall j \in \{1, \ldots, 9\}$: all_different(C_j)
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- Each variable evaluation satisfying all constraints corresponds to a solution to a given sudoku instance and vice versa

Modelling example: Perfect matching

Perfect matching

A <u>perfect matching</u> in a graph $G = (V, E)$ is a subset $M \subseteq E$ in which every vertex in V is incident to exactly one edge in M.

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- Bipartite graph with node sets A and B with $|A| = |B|$

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- Constraints: $\forall a, a' \in A$ such that $a \neq a'$: $x_a \neq x'_a$