Introduction to Artificial Intelligence

English practicals 4: (Propositional) logical reasoning

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Solution to assignment # 1

Grid 2D (without diagonals)

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Grid 3D (without diagonals)

- Exactly the same holds in 3D without diagonals. Just add one dimension

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Grid 3D all diagonal

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Grid 3D all diagonal

- Again, the same arguments apply for 3D

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- Better would be $(|x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2|)/2 = 2.5$
- But for the point (0,0,2), we get a better estimate by the maximum heuristic
- Therefore, always use the tighter one: $\max\{(|x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2|)/2, |x_1 - x_2|, |y_1 - y_2|, |z_1 - z_2|\}$

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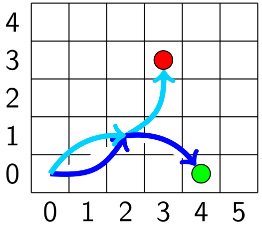
- Consider the two extremes: both positions are in distance 2



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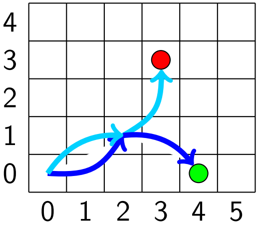

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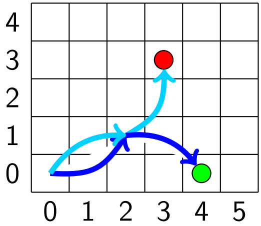

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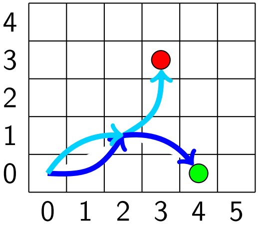
- Again, let's pick the tighter one:

$$\max\{(|x_1 - x_2| + |y_1 - y_2|)/3, |x_1 - x_2|/2, |y_1 - y_2|/2\}$$



A small reminder

Modelling a problem as a boolean formula and finding a satisfying evaluation of variables is another general way of solving combinatorial problems

- Boolean variables attain values 0 or 1
- A formula φ is satisfiable iff there exists a value assignment for each variable so that φ becomes true
- Literal is a single variable or its negation (x, $\neg y$)
- Clause is a disjunction of literals ($x \vee y \vee \neg z$)
- Typically we aim for a CNF formula (conjunction of clauses)
  $(x \vee y \vee \neg z) \wedge (\neg x \vee \neg y) \wedge (\neg z)$
- A DNF formula is a disjunction of conjunctions
  $(x \wedge y \wedge \neg z) \vee (\neg x \wedge \neg y) \vee (\neg z \wedge x)$

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$$p \Leftrightarrow (q \wedge r)$$

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$$p \Leftrightarrow (q \land r)$$

- $(\neg p \lor (q \land r)) \land (\neg(q \land r) \lor p)$
- $(\neg p \lor q) \land (\neg p \lor r) \land (\neg q \lor \neg r \lor p)$

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4. $$(\neg p \vee r) \wedge (q \vee r) \wedge (\neg p \vee \neg s) \wedge (q \vee \neg s)$$

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Prove using resolution that

$$\{(p \Rightarrow q), (q \Rightarrow r)\} \models (p \Rightarrow r)$$

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  7. $\perp$ (4,6, variable r)

SAT modeling

Assume a graph-coloring problem. How do you encode it as a satisfiability problem?

Graph coloring

1. Two adjacent nodes cannot have the same color
2. Each node is assigned at least one of the available colors
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How to find a chromatic number of a graph?

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   $\bigwedge_{k=1}^{9} \bigwedge_{i'=0}^{2} \bigwedge_{j'=0}^{2} \bigwedge_{i=1}^{3} \bigwedge_{j=1}^{3} \bigwedge_{j''=j+1}^{3}$
   $(\neg x_{(3i'+i)(3j'+j)k} \vee \neg x_{(3i'+i)(3j'+j'')k})$
   $\bigwedge_{k=1}^{9} \bigwedge_{i'=0}^{2} \bigwedge_{j'=0}^{2} \bigwedge_{i=1}^{3} \bigwedge_{j=1}^{3} \bigwedge_{i''=x+1}^{3} \bigwedge_{j''=y+1}^{3}$
   $(\neg x_{(3i'+i)(3j'+j)k} \vee \neg x_{(3i'+i'')(3j'+j'')k})$