Introduction to Artificial Intelligence

English practicals 7: Probabilistic reasoning

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A small reminder

- **Joint probability:** $P(A, B)$, probability of two events happening at the same time
- **Conditional probability:** $P(B|A)$, probability that event B will occur given that event A has already occurred
  - If evens A and B are **dependent**, then $P(B|A) = P(A, B)/P(A)$
  - If evens A and B are **independent**, then $P(B|A) = P(B)$
- **Full joint probability distribution:** Describes probabilities of all possible worlds (knowledge base)
- **Bayes rule:** $P(A|B) = P(B|A)P(B)/P(A)$
  (because $P(A, B) = P(A|B)/P(B) = P(B|A)/P(A)$)

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Even though the events A and B are not independent, once we know for about the bias, the events are conditionally independent

$P(A = head|B = head, C) = P(A = head|C)$

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$P(A = head|B = head, C) = P(A = head|C)$

Can you think of two events that are statistically independent, but given a certain knowledge they are DEpendent? (conditional dependence)

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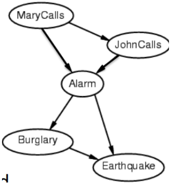
- Why are there arcs MaryCalls$-$ $>$JohnCalls and Burglary$-$ $>$Earthquake?



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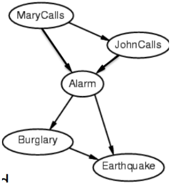

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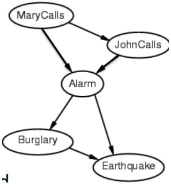
  Because if MaryCalls=True, the probability that also JohnCalls=True increases

  Because if Alarm=Burglary=True, the probability that Earthquake=True decreases, as the alarm was likely triggered by the burglary



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(G)reen party is running for joining the parliament in the next election. It is believed that (M)arijuana is more likely to be legalized if (G) make to the parliament, but it can of course happen even if they are not elected. Let us model the situation as a simple Bayes network:

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$$P(m) = P(m,g) + P(m,\neg g) = P(m|g)P(g) + P(m|\neg g)P(\neg g) = 2/3 * 1/10 + 1/4 * 9/10 = 7/24$$

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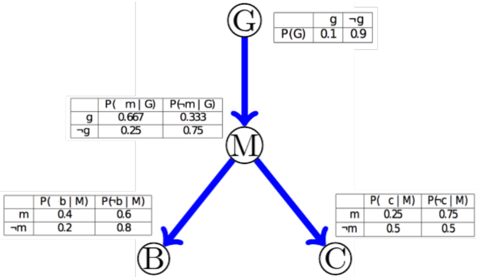

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	P(b | M)	P(¬b | M)
m	0.4	0.6
¬m	0.2	0.8

	P(c | M)	P(¬c | M)
m	0.25	0.75
¬m	0.5	0.5

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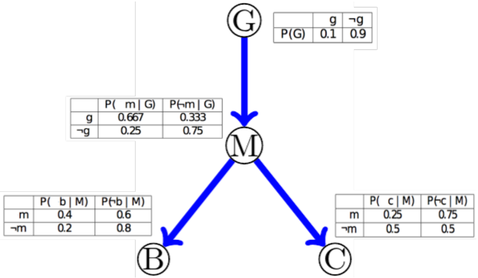

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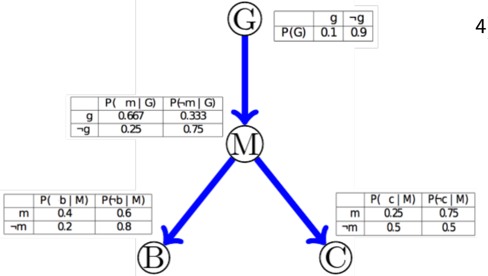

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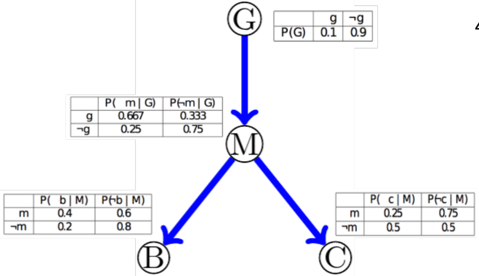

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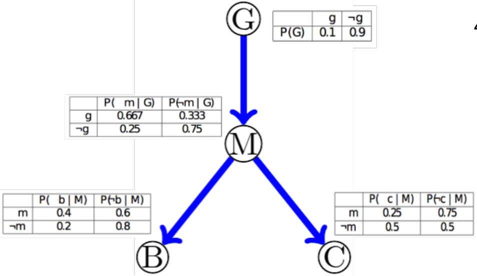

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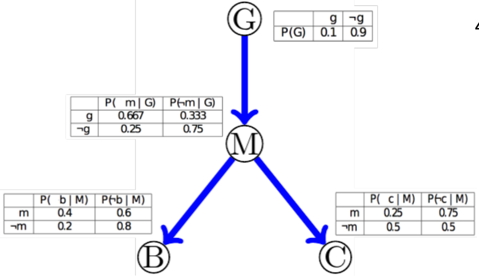

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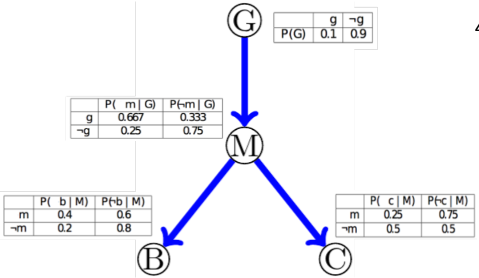

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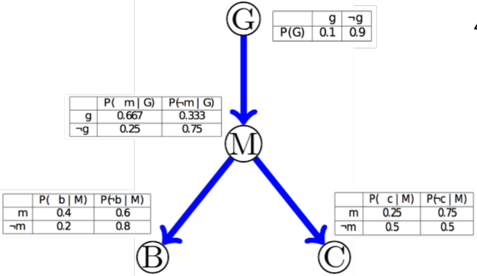

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b) $P(b) = \sum_{G,M,C} P(G,M,b,C) = 31/120$ (Summed from full joint probabilities)

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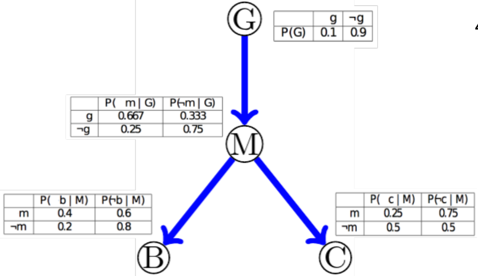

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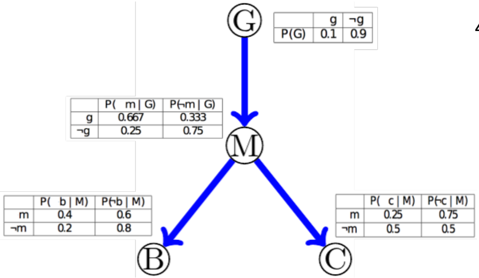
c) $P(c|b) =$

Exercise 1: election (2/2)

We can make better inference using more evidence. Assume the legalization of marijuana influences whether the budget is balanced and also class attendance of students.

G	M	B	C	$P(G,M,B,C)$	G	M	B	C	$P(G,M,B,C)$
T	T	T	T	$1/150$	F	T	T	T	$9/400$
T	T	T	F		F	T	T	F	$27/400$
T	T	F	T	$1/100$	F	T	F	T	$27/800$
T	T	F	F	$3/100$	F	T	F	F	$81/800$
T	F	T	T	$1/300$	F	F	T	T	$27/400$
T	F	T	F	$1/300$	F	F	T	F	$27/400$
T	F	F	T		F	F	F	T	
T	F	F	F	$1/75$	F	F	F	F	$27/100$



4) Fill in the missing values

5) Determine the following probabilities:

a) $P(b|m,g) =$ 4/10, from BN since B and G are conditionally independent given M

b) $P(b) = \sum_{G,M,C} P(G,M,b,C) = 31/120$ (Summed from full joint probabilities)

c) $P(c|b) = \frac{P(b,c)}{P(b)} = \frac{\sum_{G,M} P(G,M,b,c)}{31/120} = \frac{12}{31}$

Exercise 2: Monty Hall (1/2)

Monty Hall's problem

- Three doors, one host and one contestant
- There is prize money behind one of the doors and a goat behind each of the other two
- The host knows the location of the money, the contestant has no prior information
- The contestant points to a certain door and the host opens another door with a goat
- The contestant has a chance to change their mind.

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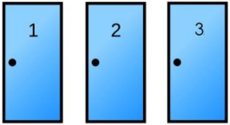

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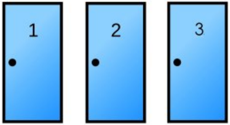

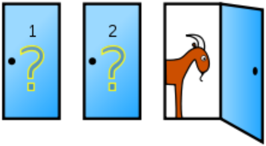

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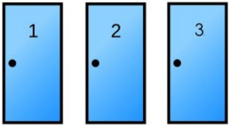

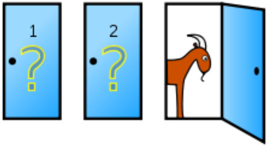

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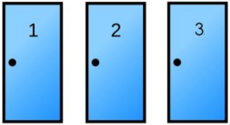

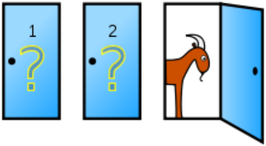

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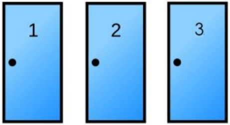

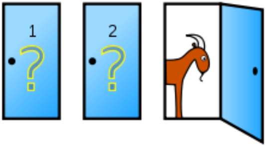

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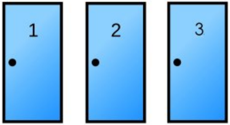

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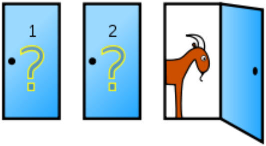

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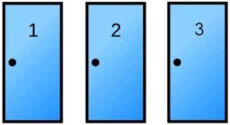

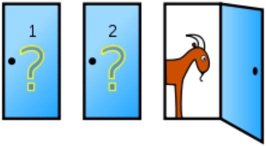

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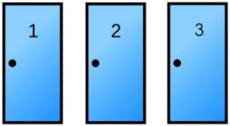

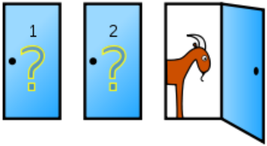

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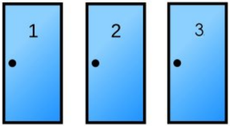

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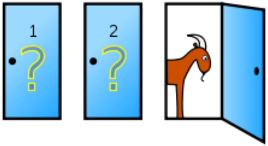

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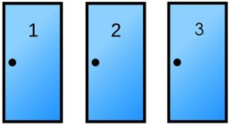

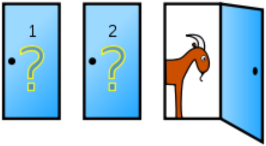

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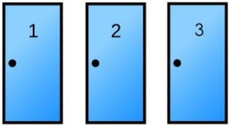

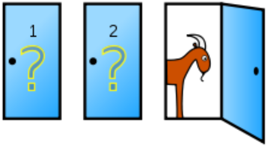

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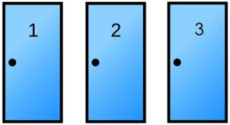

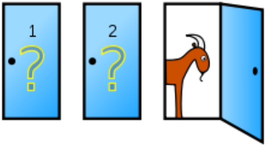

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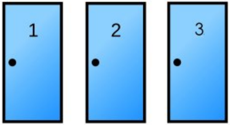

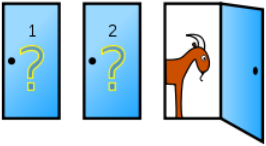

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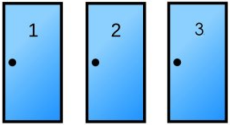

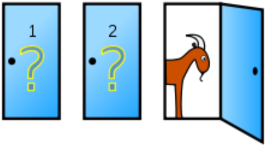

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Exercise 3: Bayes network

Vehicle diagnostics

Consider the following Boolean random variables describing the state of a car:

- (B)attery: is the battery charged?
- (F)uel: is the fuel tank empty?
- (I)gnition: does the ignition system work?
- (M)oves: does the car move?
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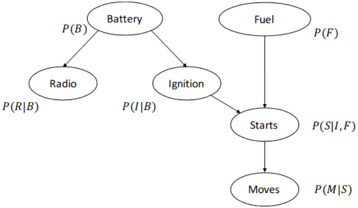

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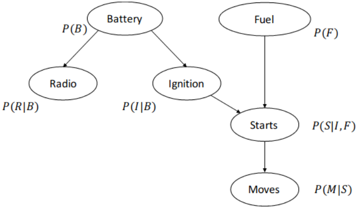

$$P(B, F, I, M, R, S) =$$
$$P(B)P(F)P(R|B)P(I|B)P(S|I, F)P(M, S)$$

Application: spam filtering

Naive Bayes spam filtering:

- Training: Learning probabilities. User manually indicates spam/no spam. Adjust probabilities of each word (or a subset) that it appears in a spam ("refinance", "prize money",...)

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Consider the word "Vicodin" - event v

$$P(s|v) = \frac{P(v|s)P(s)}{P(v|s)P(s) + P(v|\neg s)P(\neg s)}$$

- $P(s|v)$ - probability that a message in which the word "Vicodin" appears is a spam
- $P(s), P(\neg s)$ - probability that a message is a spam and genuine, respectively
- $P(v|s)$ - probability that the word "Vicodin" appears in a spam message
- $P(v|\neg s)$ - probability that the word "Vicodin" appears in a genuine message

Exercise 4: Naïve Bayes classifier

	Spam		**No spam**	
Total	25		75	
"buy"	20		5	
"cheap"	15		10	
"buy" & "cheap"				

Exercise 4: Naïve Bayes classifier

	Spam		**No spam**	
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	Spam		No spam	
Total	25		75	
"buy"	20	4/5	5	1/15
"cheap"	15	3/5	10	2/15
"buy" & "cheap"	12	12/25		

What is the probability that a message containing both "buy" and "cheap" is a spam?

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$$= \frac{P(b|s)P(c|s)P(s)}{P(b|s)P(c|s)P(s) + P(b|\neg s)P(c|\neg s)P(\neg s)} =$$

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$$= \frac{P(b|s)P(c|s)P(s)}{P(b|s)P(c|s)P(s) + P(b|\neg s)P(c|\neg s)P(\neg s)} =$$

$$= \frac{4/5 * 3/5 * 1/4}{4/5 * 3/5 * 1/4 + 1/15 * 2/15 * 3/4} \approx 0.947$$

Exercise 4: Naïve Bayes classifier

	Spam		**No spam**	
Total	25		75	
"buy"	20	4/5	5	1/15
"cheap"	15	3/5	10	2/15
"buy" & "cheap"	12	12/25	2/3	2/225

What is the probability that a message containing both "buy" and "cheap" is a spam?

$$P(s|b,c) = \frac{P(b,c|s)P(s)}{P(b,c|s)P(s) + P(b,c|\neg s)P(\neg s)} =$$

$$= \frac{P(b|s)P(c|s)P(s)}{P(b|s)P(c|s)P(s) + P(b|\neg s)P(c|\neg s)P(\neg s)} =$$

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Intuitively: $\frac{12}{12+2/3} \approx 0.947$

Assignment #5 - minesweeper