Introduction to Artificial Intelligence

English practicals 8: Probabilistic reasoning over time

Marika Ivanová

Department of Theoretical Computer Science and Mathematical Logic (KTIML)
Faculty of Mathematics and Physics

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A small remainder from last practicals

Example: Covid-19 has the following symptoms: caugh (c), high temperature (t), loss of smell (l) and fatigue (f). Knowing the probabilities $P(symptom|Covid = true)$ calculate the probability that a patient exhibiting a combination of symptoms has Covid.

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Chain rule (from lecture):

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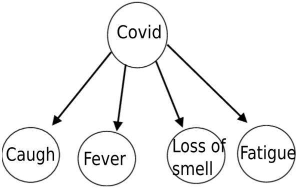

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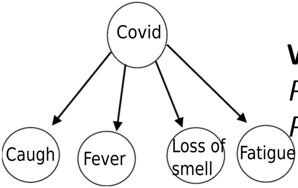

With BN we can use conditional independence:

$$P(c, t, l, f, covid) =$$
$$P(f|covid)P(l|covid)P(t|covid)P(c|covid)P(covid)$$

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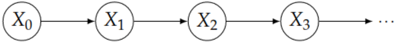
- **Observation model:** $P(E_t | X_{0:t}, E_{1:t-1})$
  - **sensor Markov assumption:** $P(E_t | X_{0:t}, E_{1:t-1}) = P(E_t | X_t)$

Exercise 1: Markov chain

Assume a stationary first order Markov chain:



- States $X = \{(r)ain, (s)un\}$
- Initial state: sun
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X_{t-1}	X_t	$P(X_t \mid X_{t-1})$
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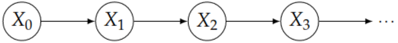

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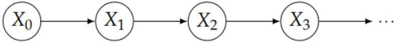

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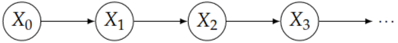

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$$= 0.3 * 0 + 0.9 * 1 = 0.9$$

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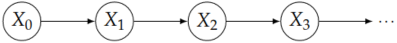

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How about two steps, i.e., $P(X_2)$ given $P(X_1)$ from the previous step?

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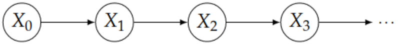

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$$= 0.3 * 0.1 + 0.9 * 0.9 = 0.84$$

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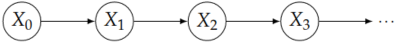

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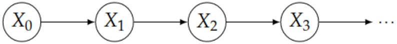

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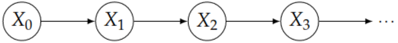

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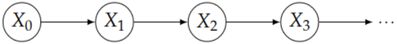

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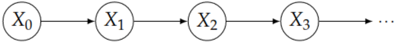

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$$\Rightarrow P(X_\infty) = (0.75, 0.25)$$

Basic inference tasks

- **Filtering** $P(X_t|e_{1:t})$ (Where am I now?)
- **Prediction** $P(X_{t+k}|e_{1:t}), k > 0$ (Where will I be in future?)
- **Smoothing** $P(X_k|e_{1:t}), k < t$ (Where was I in the past?)
- **Most likely explanation** $\arg\max_{x_{1:t}} P(X_{1:t}|e_{1:t})$ (What path did I go through?)

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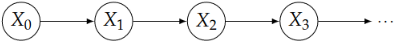
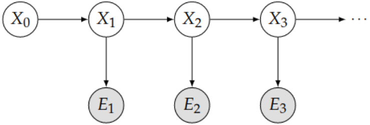
Poll

Markov chain (MC) vs Hidden Markov Model (HMM)

- MC: all variables are observable
- HMM: some variables are observable, some are hidden





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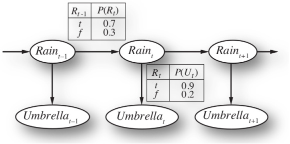

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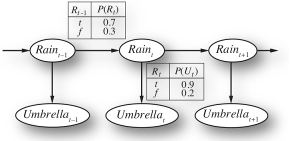

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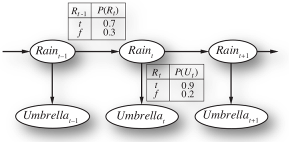

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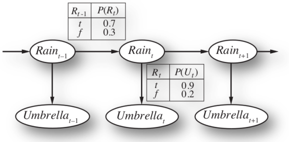

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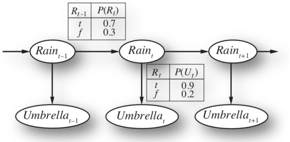

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- Day 1: observation $u_1 = true$
  - Prediction:
    $P(r_1 = true) = \sum_{r_0} P(r_1 = true|r_0) * P(r_0) = 0.7 * 0.5 + 0.3 * 0.5 = 0.5$

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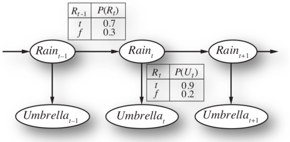

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  - Update by the new evidence: $P(r_1 = true|u_1 = true) = \alpha P(u_1 = true|r_1 = true)P(r_1 = true) = \alpha * 0.9 * 0.5 = \alpha * 0.45 \approx 0.818$

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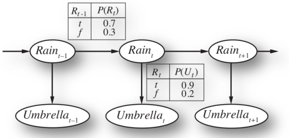

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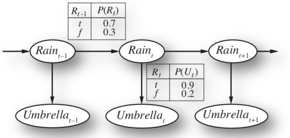

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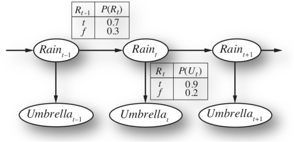
  - Update by the new evidence: $P(r_1 = true|u_1 = true) = \alpha P(u_1 = true|r_1 = true)P(r_1 = true) = \alpha * 0.9 * 0.5 = \alpha * 0.45 \approx 0.818$
- Day 2: observation $u_2 = true$
  - Prediction: $P(r_2 = true|u_1 = true) = \sum_{r_1} P(r_2 = true|r_1) * P(r_1|u_1 = true) = 0.7 * 0.818 + 0.3 * 0.182 \approx 0.627$

Exercise 2



Assume that the probability of rain at day 0 is 0.5. What is the probability of rain at day 2, given that we observed an umbrella at day 1 and 2?

Filtering: Where am I now? $P(X_{t+1}|E_{1:t+1}) = \alpha P(e_{t+1}|X_{t+1}) \sum_{x_t} P(X_{t+1}|x_t)P(x_t|e_{1:t})$

- Day 0: no observation, only the prior belief $P(R_0) = (0.5, 0.5)$
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Exercise 3: Hidden Markov model

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	R	**G**	**B**
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Chain rule:

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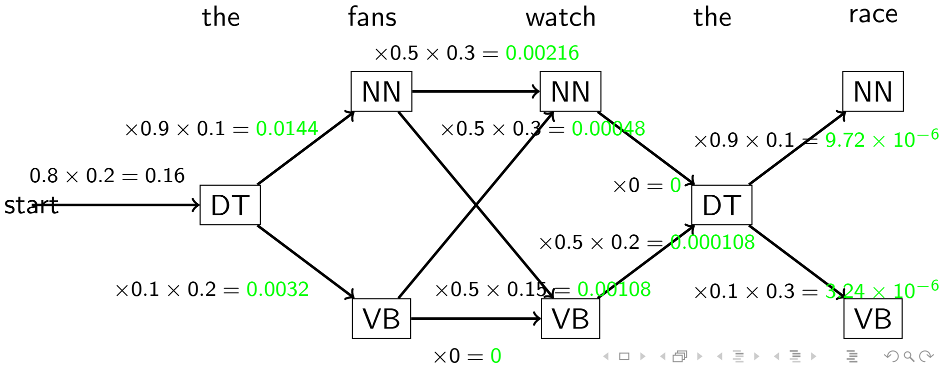
- HHH: $0.1 * 0.1 * 0.8 * 0.7^3 = 0.0027$
- SSH: $0.3 * 0.5 * 0.8 * 0.5 * 0.5 * 0.6 = 0.018$
- ...

Viterbi algorithm: NLP example

	DT	NN	VB
(Start)	0.8	0.2	0
DT	0	0.9	0.1
NN	0	0.5	0.5
VB	0.5	0.5	0

	THE	FANS	WATCH	RACE
DT	0.2	0	0	0
NN	0	0.1	0.3	0.1
VB	0	0.2	0.15	0.3



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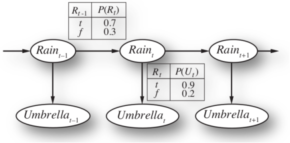

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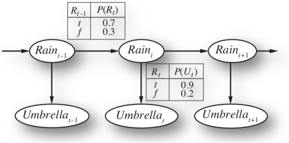

0.3

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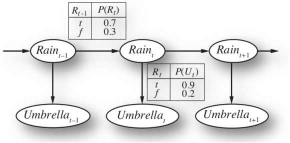

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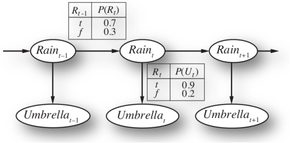

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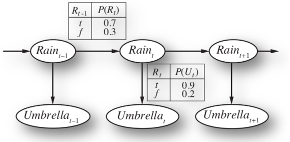

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  Then, construct a sequence of most likely states

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  However, the most likely sequence is different:

$$\arg\max_{r_{1:t}} P(r_{1:t}|u_{1:t})$$