Introduction to Artificial Intelligence

English practicals 10: Games and multi-agent systems

Marika Ivanová

Department of Theoretical Computer Science and Mathematical Logic (KTIML)
Faculty of Mathematics and Physics

April 26th 2022

A small reminder

- Earlier: uncertain environments
- Today: uncertainty due to other agents' decisions

A small reminder

- Earlier: uncertain environments
- Today: uncertainty due to other agents' decisions
- Turn-taking games with perfect information (fully observable) - minimax, alpha-beta
- Game theory: simultaneous moves and other sources of partial observability

A small reminder: Single-move games

Single-move game

- **Players:** agents who make decisions
- **Actions:** choices available to players
- **Payoff function:** gives utility to each player for each combination of actions by all players.

  Represented by a matrix

A small reminder: Single-move games

Single-move game

- **Players:** agents who make decisions
- **Actions:** choices available to players
- **Payoff function:** gives utility to each player for each combination of actions by all players.

  Represented by a matrix

- Pure strategy: deterministic policy, single action
- Mixed strategy: randomized policy, [p:a1, (1-p):a2]
- Solution: assignment of rational strategy to each player

A small reminder: Single-move games

Single-move game

- **Players:** agents who make decisions
- **Actions:** choices available to players
- **Payoff function:** gives utility to each player for each combination of actions by all players.

  Represented by a matrix

- Pure strategy: deterministic policy, single action
- Mixed strategy: randomized policy, [p:a1, (1-p):a2]
- Solution: assignment of rational strategy to each player
- Example: prisoner's dilemma

	Alice: testify	Alice: refuse
Bob: testify	A=-5, B=-5	A=-10, B=0
Bob: refuse	A=0, B=-10	A=-1, B=-1

A small reminder

- A strategy s of player p **strongly dominates** strategy s', if the outcome of s is better for p than the outcome of s' for every choice of strategies by other(s)
- When each player has a dominant strategy, the combination of those strategies is a **dominant strategy equilibrium** (DSE).
- **(Nash) Equilibrium** (in general): when no player can benefit by switching strategies, given that every other player stick with the same strategy.

Dominant strategy equilibrium

- A strategy s of player p **strongly dominates** strategy s', if the outcome of s is better for p than the outcome of s' for every choice of strategies by other(s)
- When each player has a dominant strategy, the combination of those strategies is a **dominant strategy equilibrium** (DSE).
- **(Nash) Equilibrium** (in general): when no player can benefit by switching strategies, given that every other player stick with the same strategy.

Exercise

Show that every DSE is a Nash equilibrium, but not vice versa

Dominant strategy equilibrium

- A strategy s of player p **strongly dominates** strategy s', if the outcome of s is better for p than the outcome of s' for every choice of strategies by other(s)
- When each player has a dominant strategy, the combination of those strategies is a **dominant strategy equilibrium** (DSE).
- **(Nash) Equilibrium** (in general): when no player can benefit by switching strategies, given that every other player stick with the same strategy.

Exercise

Show that every DSE is a Nash equilibrium, but not vice versa

Proof

$\Rightarrow$: let s^* be DSE. Take any player i. Since s_i^* is a dominant strategy for i, for any given s_i $\forall s_{-i} \in S_{-i} : u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$, particularly $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$,
Since i and s_i are arbitrary, this shows that $s*$ is a Nash equilibrium.
$\Leftarrow$: Counterexample, e.g., Date night dilemma

Date night dilemma (1/2)

- A man and a woman want to spend evening together.
- The man prefers to see MMA fight, while the woman would like to go to ballet.
- We can regard it as a two player single-move game with the following strategic form:

		Woman	
		Ballet	MMA
Man	Ballet	2,3	0,0
	MMA	1,1	3,2

Find all Nash equilibria

Date night dilemma (1/2)

- A man and a woman want to spend evening together.
- The man prefers to see MMA fight, while the woman would like to go to ballet.
- We can regard it as a two player single-move game with the following strategic form:

		Woman	
		Ballet	MMA
Man	Ballet	2,3	0,0
	MMA	1,1	3,2

Find all Nash equilibria

- Two pure strategy Nash equilibria: (ballet, ballet), (MMA, MMA)

Date night dilemma (1/2)

- A man and a woman want to spend evening together.
- The man prefers to see MMA fight, while the woman would like to go to ballet.
- We can regard it as a two player single-move game with the following strategic form:

		Woman	
		Ballet	MMA
Man	Ballet	2,3	0,0
	MMA	1,1	3,2

Find all Nash equilibria

- Two pure strategy Nash equilibria: (ballet, ballet), (MMA, MMA)
- One mixed strategy Nash equilibrium:

Date night dilemma (1/2)

- A man and a woman want to spend evening together.
- The man prefers to see MMA fight, while the woman would like to go to ballet.
- We can regard it as a two player single-move game with the following strategic form:

		Woman	
		Ballet	MMA
Man	Ballet	2,3	0,0
	MMA	1,1	3,2

Find all Nash equilibria

- Two pure strategy Nash equilibria: (ballet, ballet), (MMA, MMA)
- One mixed strategy Nash equilibrium:

$EU_{WB} = 3p_{MB} + (1 - p_{MB})$
$EU_{WM} = 0p_{MB} + 2(1 - p_{MB})$
$EU_{WB} = EU_{WM} \Rightarrow p_{MB} = 1/4$

Date night dilemma (1/2)

- A man and a woman want to spend evening together.
- The man prefers to see MMA fight, while the woman would like to go to ballet.
- We can regard it as a two player single-move game with the following strategic form:

		Woman	
		Ballet	MMA
Man	Ballet	2,3	0,0
	MMA	1,1	3,2

Find all Nash equilibria

- Two pure strategy Nash equilibria: (ballet, ballet), (MMA, MMA)
- One mixed strategy Nash equilibrium:

$$EU_{WB} = 3p_{MB} + (1 - p_{MB})$$
$$EU_{WM} = 0p_{MB} + 2(1 - p_{MB})$$
$$EU_{WB} = EU_{WM} \Rightarrow p_{MB} = 1/4$$

$$EU_{MB} = 2p_{WB} + 0(1 - p_{WB})$$
$$EU_{MM} = p_{WB} + 3(1 - p_{WB})$$
$$EU_{MB} = EU_{MM} \Rightarrow p_{WB} = 3/4$$

Date night dilemma (1/2)

- A man and a woman want to spend evening together.
- The man prefers to see MMA fight, while the woman would like to go to ballet.
- We can regard it as a two player single-move game with the following strategic form:

		Woman	
		Ballet	MMA
Man	Ballet	2,3	0,0
	MMA	1,1	3,2

Find all Nash equilibria

- Two pure strategy Nash equilibria: (ballet, ballet), (MMA, MMA)
- One mixed strategy Nash equilibrium:

$$EU_{WB} = 3p_{MB} + (1 - p_{MB})$$
$$EU_{WM} = 0p_{MB} + 2(1 - p_{MB})$$
$$EU_{WB} = EU_{WM} \Rightarrow p_{MB} = 1/4$$

$$EU_{MB} = 2p_{WB} + 0(1 - p_{WB})$$
$$EU_{MM} = p_{WB} + 3(1 - p_{WB})$$
$$EU_{MB} = EU_{MM} \Rightarrow p_{WB} = 3/4$$

- Both should choose their favourite option with 75% probability.

Date night dilemma (2/2)

		Woman	
		Balley	MMA
Man	Balley	2,3	0,0
	MMA	1,1	3,2

$p_{MB} = 1/4$

$p_{MM} = 3/4$

$p_{WB} = 3/4$

$p_{WM} = 1/4$

Calculate payoffs for this mixed strategy equilibrium.

Date night dilemma (2/2)

		Woman	
		Balley	MMA
Man	Balley	2,3	0,0
	MMA	1,1	3,2

$p_{MB} = 1/4$

$p_{MM} = 3/4$

$p_{WB} = 3/4$

$p_{WM} = 1/4$

Calculate payoffs for this mixed strategy equilibrium.

1. Calculate probabilities of each outcome occurring in equilibrium

Date night dilemma (2/2)

		Woman	
		Balley	MMA
Man	Balley	2,3 3/16	0,0 1/16
	MMA	1,1 9/16	3,2 3/16

$p_{MB} = 1/4$

$p_{MM} = 3/4$

$p_{WB} = 3/4$

$p_{WM} = 1/4$

Calculate payoffs for this mixed strategy equilibrium.

1. Calculate probabilities of each outcome occurring in equilibrium

Date night dilemma (2/2)

		Woman	
		Balley	MMA
Man	Balley	2,3 3/16	0,0 1/16
	MMA	1,1 9/16	3,2 3/16

$p_{MB} = 1/4$

$p_{MM} = 3/4$

$p_{WB} = 3/4$

$p_{WM} = 1/4$

Calculate payoffs for this mixed strategy equilibrium.

1. Calculate probabilities of each outcome occurring in equilibrium
2. For each outcome, multiply that probability by particular player's payoff

Date night dilemma (2/2)

		Woman	
		Balley	MMA
Man	Balley	2,3 3/16	0,0 1/16
	MMA	1,1 9/16	3,2 3/16

$p_{MB} = 1/4$
$p_{MM} = 3/4$
$p_{WB} = 3/4$
$p_{WM} = 1/4$

Calculate payoffs for this mixed strategy equilibrium.

1. Calculate probabilities of each outcome occurring in equilibrium
2. For each outcome, multiply that probability by particular player's payoff
3. Sum all these numbers together

Date night dilemma (2/2)

		Woman	
		Balley	MMA
Man	Balley	2,3 3/16	0,0 1/16
	MMA	1,1 9/16	3,2 3/16

$p_{MB} = 1/4$

$p_{MM} = 3/4$

$p_{WB} = 3/4$

$p_{WM} = 1/4$

Calculate payoffs for this mixed strategy equilibrium.

1. Calculate probabilities of each outcome occurring in equilibrium
2. For each outcome, multiply that probability by particular player's payoff
3. Sum all these numbers together

$$U_M = 2 * 3/16 + 0 * 1/6 + 1 * 9/16 + 3 * 3/16 = 1.5$$

$$U_W = 3 * 3/16 + 0 * 1/6 + 1 * 9/16 + 2 * 3/16 = 1.5$$

Exercise: Dominant strategy and equilibria

What are the dominant strategies and pure strategy equilibria in the following three games?

	L	R
U	1	2
D	3	4

	L	R
U	1	4
D	2	3

	L	R
U	1	3
D	4	2

Exercise: Dominant strategy and equilibria

What are the dominant strategies and pure strategy equilibria in the following three games?

	L	R
U	1	2
D	3	4

	L	R
U	1	4
D	2	3

	L	R
U	1	3
D	4	2

1. Dominant strategy: first player - D, second player - L, pure strategy equilibrium: (D,L)

Exercise: Dominant strategy and equilibria

What are the dominant strategies and pure strategy equilibria in the following three games?

	L	R
U	1	2
D	3	4

	L	R
U	1	4
D	2	3

	L	R
U	1	3
D	4	2

1. Dominant strategy: first player - D, second player - L, pure strategy equilibrium: (D,L)
2. Dominant strategy: second player - L, pure strategy equilibrium: (D,L)

Exercise: Dominant strategy and equilibria

What are the dominant strategies and pure strategy equilibria in the following three games?

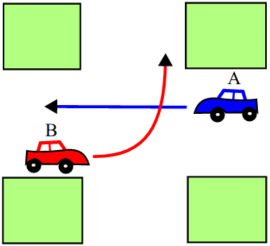
	L	R
U	1	2
D	3	4

	L	R
U	1	4
D	2	3

	L	R
U	1	3
D	4	2

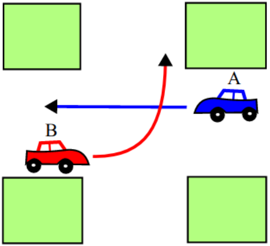
1. Dominant strategy: first player - D, second player - L, pure strategy equilibrium: (D,L)
2. Dominant strategy: second player - L, pure strategy equilibrium: (D,L)
3. Dominant strategy: none, pure strategy equilibrium: none (only mixed strategy equilibrium)

Exercise: Cars meeting at an intersection



- Two cars, both can either wait or go
- If both go and crash, it results in payoff -100 for A and -1000 for B
- If one waits and one goes, the waiting one gets payoff -5, the other one +5
- If both wait, both get payoff -10

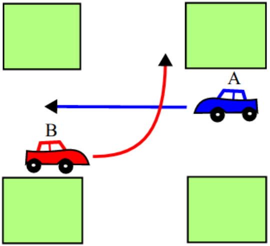
Exercise: Cars meeting at an intersection



- Two cars, both can either wait or go
- If both go and crash, it results in payoff -100 for A and -1000 for B
- If one waits and one goes, the waiting one gets payoff -5, the other one +5
- If both wait, both get payoff -10

Draw the payoff matrix and find pure strategy Nash equilibria.

Exercise: Cars meeting at an intersection



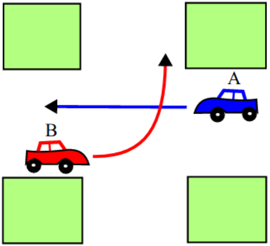
- Two cars, both can either wait or go
- If both go and crash, it results in payoff -100 for A and -1000 for B
- If one waits and one goes, the waiting one gets payoff -5, the other one +5
- If both wait, both get payoff -10

Draw the payoff matrix and find pure strategy Nash equilibria.

B

A		wait	go
	wait	-10, -10	-5,5
	go	5,-5	-100,-1000

Exercise: Cars meeting at an intersection



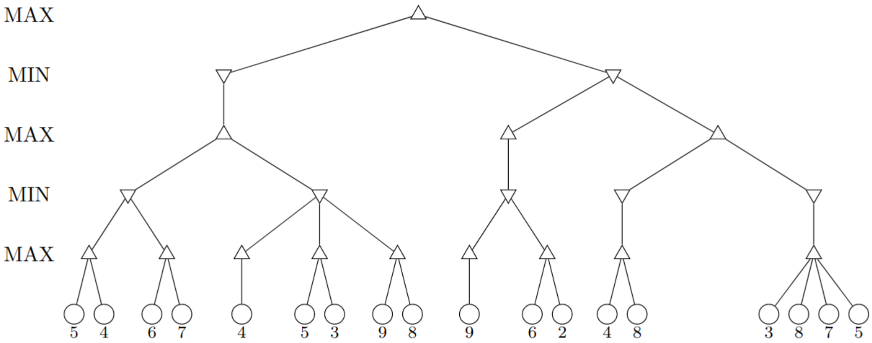
- Two cars, both can either wait or go
- If both go and crash, it results in payoff -100 for A and -1000 for B
- If one waits and one goes, the waiting one gets payoff -5, the other one +5
- If both wait, both get payoff -10

Draw the payoff matrix and find pure strategy Nash equilibria.

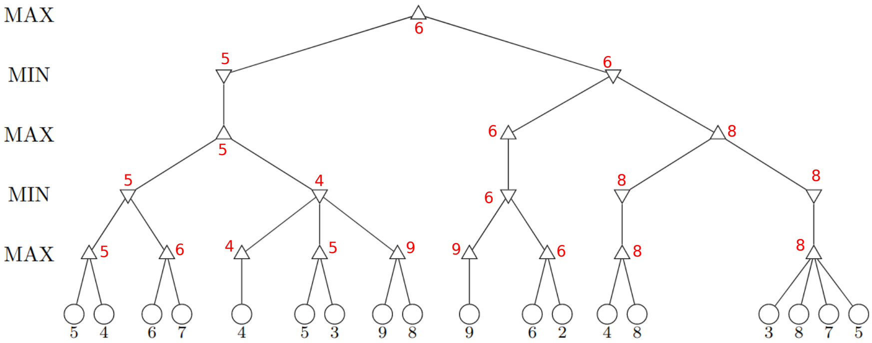
		B: wait	B: go
A	wait	-10, -10	-5,5
A	go	5,-5	-100,-1000

Pure strategy Nash equilibria: (wait, go) and (go, wait)

Exercise: minimax



Exercise: minimax



Exercise: alpha-beta pruning

Selected quiz questions

- Which nodes in the game tree are evaluated by the utility function?

Selected quiz questions

- Which nodes in the game tree are evaluated by the utility function?
  Terminal nodes.

Selected quiz questions

- Which nodes in the game tree are evaluated by the utility function?
  Terminal nodes.
- Do minimax and alpha-beta algorithms always return the same solution?

Selected quiz questions

- Which nodes in the game tree are evaluated by the utility function?

  Terminal nodes.

- Do minimax and alpha-beta algorithms always return the same solution?

  Yes

Selected quiz questions

- Which nodes in the game tree are evaluated by the utility function?

  Terminal nodes.

- Do minimax and alpha-beta algorithms always return the same solution?

  Yes

- Describe the evaluation function for the game Tic-Tac-Toe (5 needed).

Selected quiz questions

- Which nodes in the game tree are evaluated by the utility function?

  Terminal nodes.

- Do minimax and alpha-beta algorithms always return the same solution?

  Yes

- Describe the evaluation function for the game Tic-Tac-Toe (5 needed).

  Scan the state and assign ∞ if it is a winning position for the evaluated player

  Look for 4 consecutive symbols, or 2x3 consecutive symbols with intersection, if found, add high positive number

  Evaluate similarly for the opponent, but add negative values

Selected quiz questions

- What are the major problems and advantages of English auction? Why English auction discourages competition?

Selected quiz questions

- What are the major problems and advantages of English auction? Why English auction discourages competition?

  Simple dominant strategy: bid if you would not pay more than your private value v_i

  Competition discouraged: in case there is a known rich bidder, others do not bid more and the rich one can get it for a lower price

Selected quiz questions

- What are the major problems and advantages of English auction? Why English auction discourages competition?

  Simple dominant strategy: bid if you would not pay more than your private value v_i

  Competition discouraged: in case there is a known rich bidder, others do not bid more and the rich one can get it for a lower price

- Explain sealed-bid and Vickrey auctions. What is the major difference between them? What are the rational strategies for these auctions?

Selected quiz questions

- What are the major problems and advantages of English auction? Why English auction discourages competition?

  Simple dominant strategy: bid if you would not pay more than your private value v_i

  Competition discouraged: in case there is a known rich bidder, others do not bid more and the rich one can get it for a lower price

- Explain sealed-bid and Vickrey auctions. What is the major difference between them? What are the rational strategies for these auctions?

  Sealed bid: everyone makes a single bid, not knowing bids of others. No simple dominant strategy, one should bid $b_0 + \epsilon$, if $b_0 \leq v_i$, where b_0 is believed to be the maximum bid of the others

Selected quiz questions

- What are the major problems and advantages of English auction? Why English auction discourages competition?

  Simple dominant strategy: bid if you would not pay more than your private value v_i

  Competition discouraged: in case there is a known rich bidder, others do not bid more and the rich one can get it for a lower price

- Explain sealed-bid and Vickrey auctions. What is the major difference between them? What are the rational strategies for these auctions?

  Sealed bid: everyone makes a single bid, not knowing bids of others. No simple dominant strategy, one should bid $b_0 + \epsilon$, if $b_0 \leq v_i$, where b_0 is believed to be the maximum bid of the others

  Vickrey: same as before, but the winner pays the price of the second highest bid

  Simple dominant strategy is to bid v_i

Selected quiz questions

- What are the major problems and advantages of English auction? Why English auction discourages competition?

  Simple dominant strategy: bid if you would not pay more than your private value v_i

  Competition discouraged: in case there is a known rich bidder, others do not bid more and the rich one can get it for a lower price

- Explain sealed-bid and Vickrey auctions. What is the major difference between them? What are the rational strategies for these auctions?

  Sealed bid: everyone makes a single bid, not knowing bids of others. No simple dominant strategy, one should bid $b_0 + \epsilon$, if $b_0 \leq v_i$, where b_0 is believed to be the maximum bid of the others

  Vickrey: same as before, but the winner pays the price of the second highest bid

  Simple dominant strategy is to bid v_i

- What is tragedy of common? Give some examples. How is this problem solved?

Selected quiz questions

- What are the major problems and advantages of English auction? Why English auction discourages competition?

  Simple dominant strategy: bid if you would not pay more than your private value v_i

  Competition discouraged: in case there is a known rich bidder, others do not bid more and the rich one can get it for a lower price

- Explain sealed-bid and Vickrey auctions. What is the major difference between them? What are the rational strategies for these auctions?

  Sealed bid: everyone makes a single bid, not knowing bids of others. No simple dominant strategy, one should bid $b_0 + \epsilon$, if $b_0 \leq v_i$, where b_0 is believed to be the maximum bid of the others

  Vickrey: same as before, but the winner pays the price of the second highest bid

  Simple dominant strategy is to bid v_i

- What is tragedy of common? Give some examples. How is this problem solved?

  Situation in which individuals with access to a shared resource (also called a common) act in their own interest and, in doing so, ultimately deplete the resource.

Selected quiz questions

- What are the major problems and advantages of English auction? Why English auction discourages competition?

  Simple dominant strategy: bid if you would not pay more than your private value v_i

  Competition discouraged: in case there is a known rich bidder, others do not bid more and the rich one can get it for a lower price

- Explain sealed-bid and Vickrey auctions. What is the major difference between them? What are the rational strategies for these auctions?

  Sealed bid: everyone makes a single bid, not knowing bids of others. No simple dominant strategy, one should bid $b_0 + \epsilon$, if $b_0 \leq v_i$, where b_0 is believed to be the maximum bid of the others

  Vickrey: same as before, but the winner pays the price of the second highest bid

  Simple dominant strategy is to bid v_i

- What is tragedy of common? Give some examples. How is this problem solved?

  Situation in which individuals with access to a shared resource (also called a common) act in their own interest and, in doing so, ultimately deplete the resource.

  Examples: overfishing/overhunting, traffic congestion (air pollution, slower traffic

  Solution: paying tax for the resources (e.g., Vickrey-Clarks-Groves mechanism)

Exercise: Dominant strategy for Vickrey auction

The winner pays the price of the second highest bid

Proposition

In a Vickrey auction, it is a dominant strategy to bid one's value, $b_i(s_i) = s_i$.

Proof

□

Exercise: Dominant strategy for Vickrey auction

The winner pays the price of the second highest bid

Proposition

In a Vickrey auction, it is a dominant strategy to bid one's value, $b_i(s_i) = s_i$.

Proof

Suppose i's value is s_i and bids $b_i > s_i$. Let $\hat{b}$ denote the highest bid of the other bidders $j \neq i$
There are three possible outcomes from i's perspective:

□

Exercise: Dominant strategy for Vickrey auction

The winner pays the price of the second highest bid

Proposition

In a Vickrey auction, it is a dominant strategy to bid one's value, $b_i(s_i) = s_i$.

Proof

Suppose i's value is s_i and bids $b_i > s_i$. Let $\hat{b}$ denote the highest bid of the other bidders $j \neq i$
There are three possible outcomes from i's perspective:

1. $\hat{b} > b_i > s_i$
2. $b_i > \hat{b} > s_i$
3. $b_i > s_i > \hat{b}$

□

Exercise: Dominant strategy for Vickrey auction

The winner pays the price of the second highest bid

Proposition

In a Vickrey auction, it is a dominant strategy to bid one's value, $b_i(s_i) = s_i$.

Proof

Suppose i's value is s_i and bids $b_i > s_i$. Let $\hat{b}$ denote the highest bid of the other bidders $j \neq i$
There are three possible outcomes from i's perspective:

1. $\hat{b} > b_i > s_i$
2. $b_i > \hat{b} > s_i$
3. $b_i > s_i > \hat{b}$

In (1) or (3), the bidder i would have done equally well to bid s_i rather than $b_i > s_i$.

□

Exercise: Dominant strategy for Vickrey auction

The winner pays the price of the second highest bid

Proposition

In a Vickrey auction, it is a dominant strategy to bid one's value, $b_i(s_i) = s_i$.

Proof

Suppose i's value is s_i and bids $b_i > s_i$. Let $\hat{b}$ denote the highest bid of the other bidders $j \neq i$
There are three possible outcomes from i's perspective:

1. $\hat{b} > b_i > s_i$
2. $b_i > \hat{b} > s_i$
3. $b_i > s_i > \hat{b}$

In (1) or (3), the bidder i would have done equally well to bid s_i rather than $b_i > s_i$.

In (1), i doesn't win anyway, in (3) i wins and pays $\hat{b}$ anyway.

□

Exercise: Dominant strategy for Vickrey auction

The winner pays the price of the second highest bid

Proposition

In a Vickrey auction, it is a dominant strategy to bid one's value, $b_i(s_i) = s_i$.

Proof

Suppose i's value is s_i and bids $b_i > s_i$. Let $\hat{b}$ denote the highest bid of the other bidders $j \neq i$
There are three possible outcomes from i's perspective:

1. $\hat{b} > b_i > s_i$
2. $b_i > \hat{b} > s_i$
3. $b_i > s_i > \hat{b}$

In (1) or (3), the bidder i would have done equally well to bid s_i rather than $b_i > s_i$.

In (1), i doesn't win anyway, in (3) i wins and pays $\hat{b}$ anyway.

in (2), i wins and pays more than would like to, which would not happen if the bid was s_i .

□

Exercise: Dominant strategy for Vickrey auction

The winner pays the price of the second highest bid

Proposition

In a Vickrey auction, it is a dominant strategy to bid one's value, $b_i(s_i) = s_i$.

Proof

Suppose i's value is s_i and bids $b_i > s_i$. Let $\hat{b}$ denote the highest bid of the other bidders $j \neq i$
There are three possible outcomes from i's perspective:

1. $\hat{b} > b_i > s_i$
2. $b_i > \hat{b} > s_i$
3. $b_i > s_i > \hat{b}$

In (1) or (3), the bidder i would have done equally well to bid s_i rather than $b_i > s_i$.

In (1), i doesn't win anyway, in (3) i wins and pays $\hat{b}$ anyway.

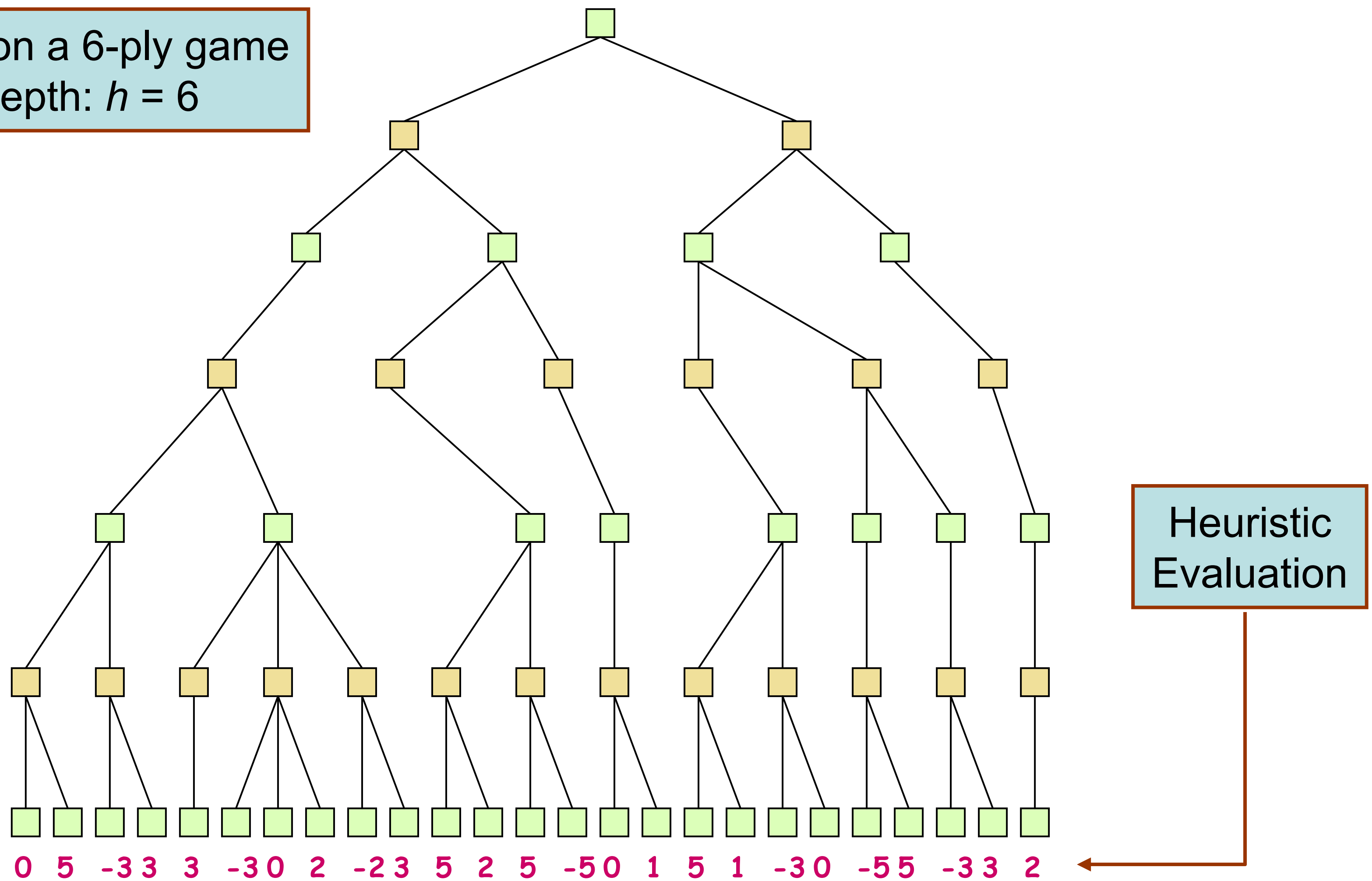
in (2), i wins and pays more than would like to, which would not happen if the bid was s_i .

Similarly for $b_i < s_i$.

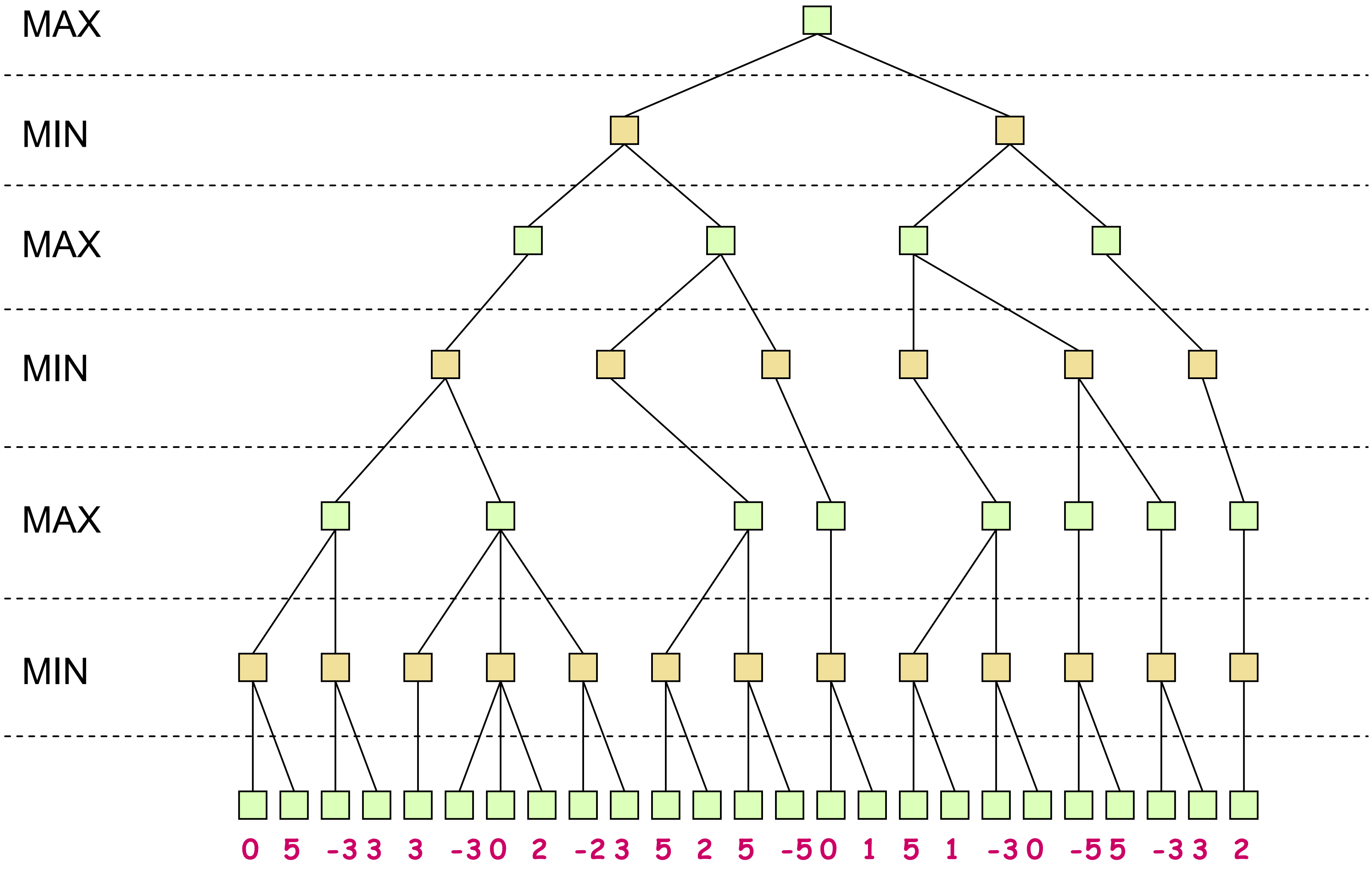
□

Alpha-Beta Pruning – Example

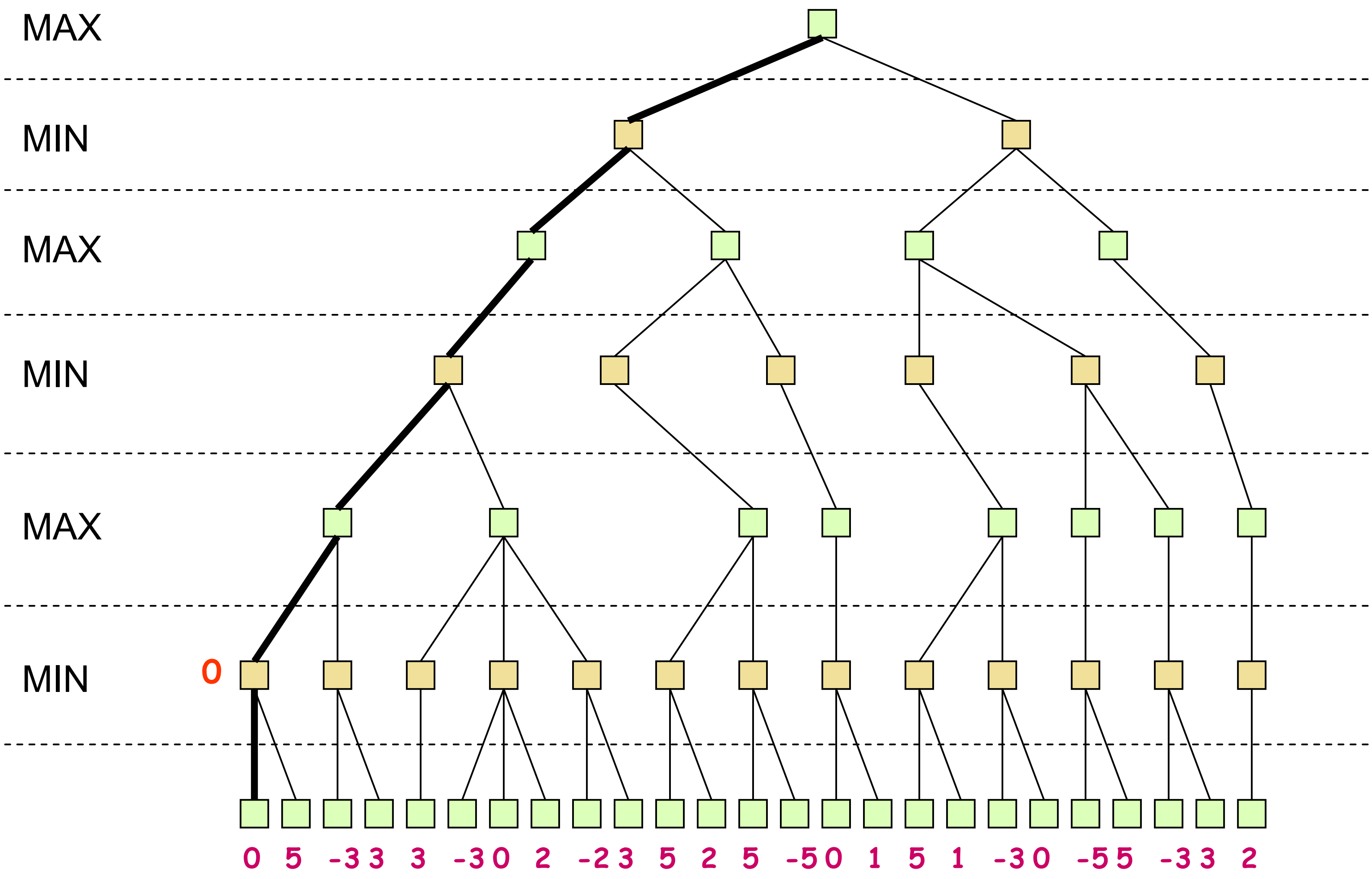
Minimax on a 6-ply game
Horizon depth: h = 6



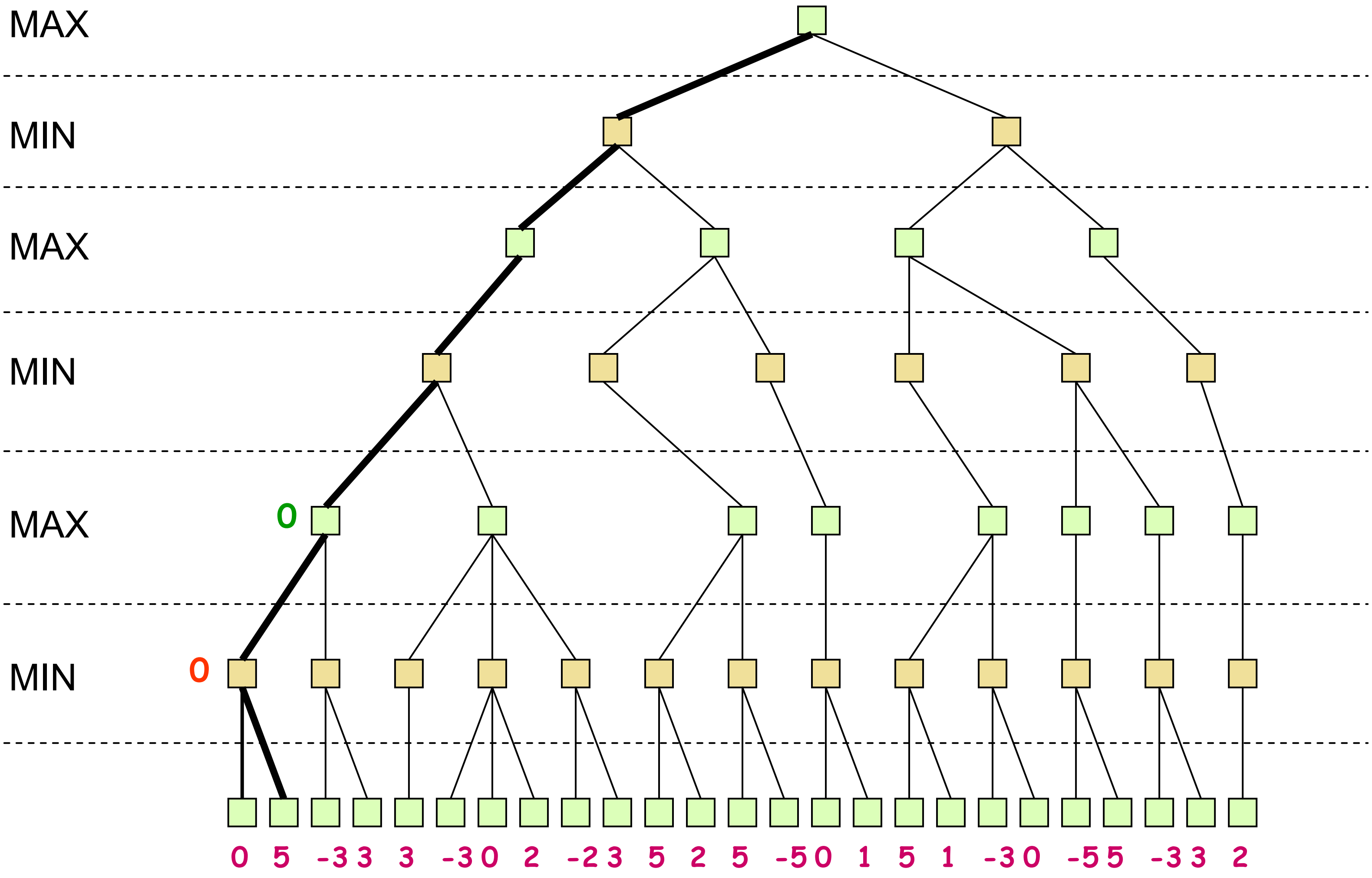
Alpha-Beta Pruning – Example



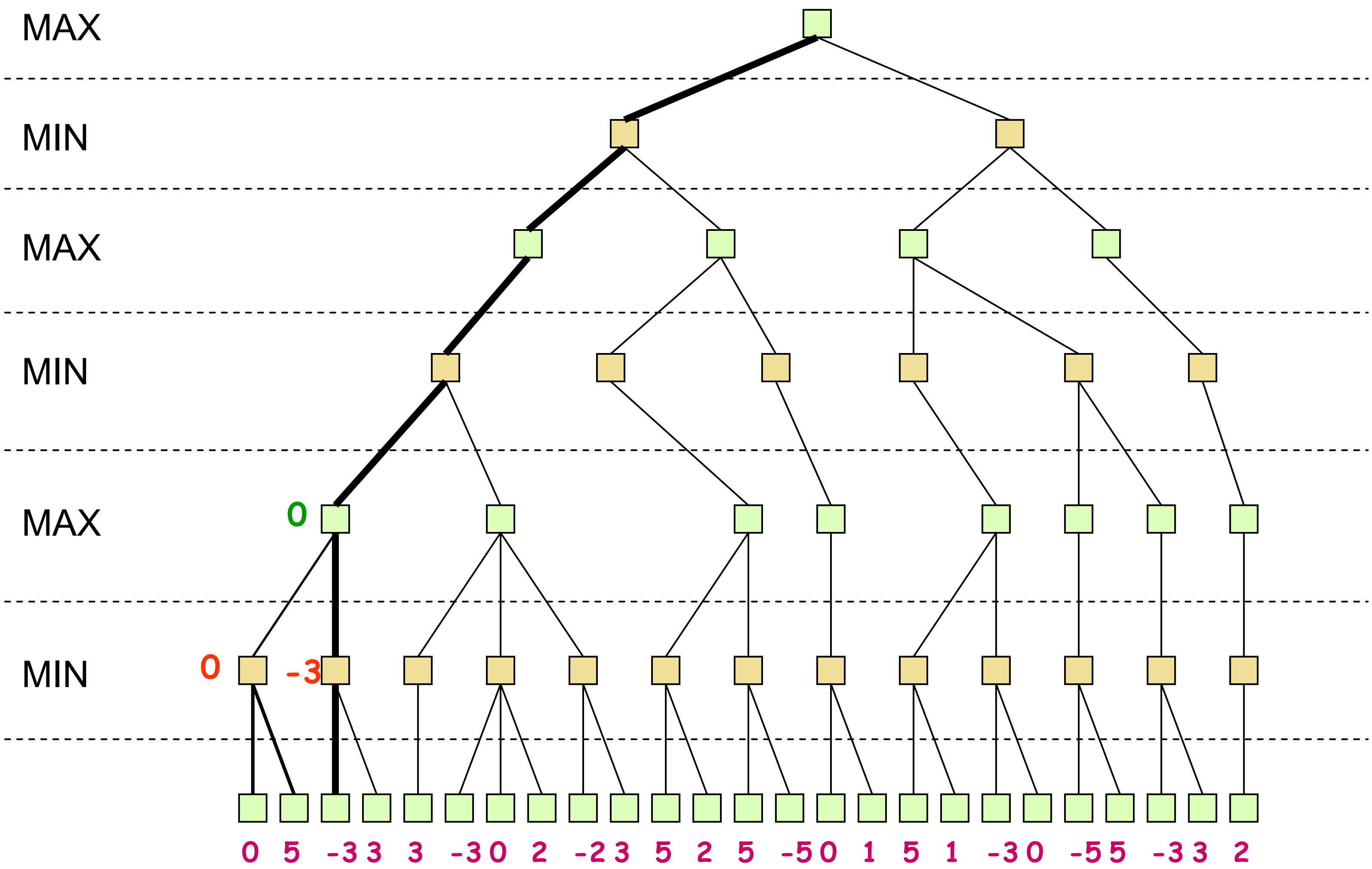
Alpha-Beta Pruning – Example



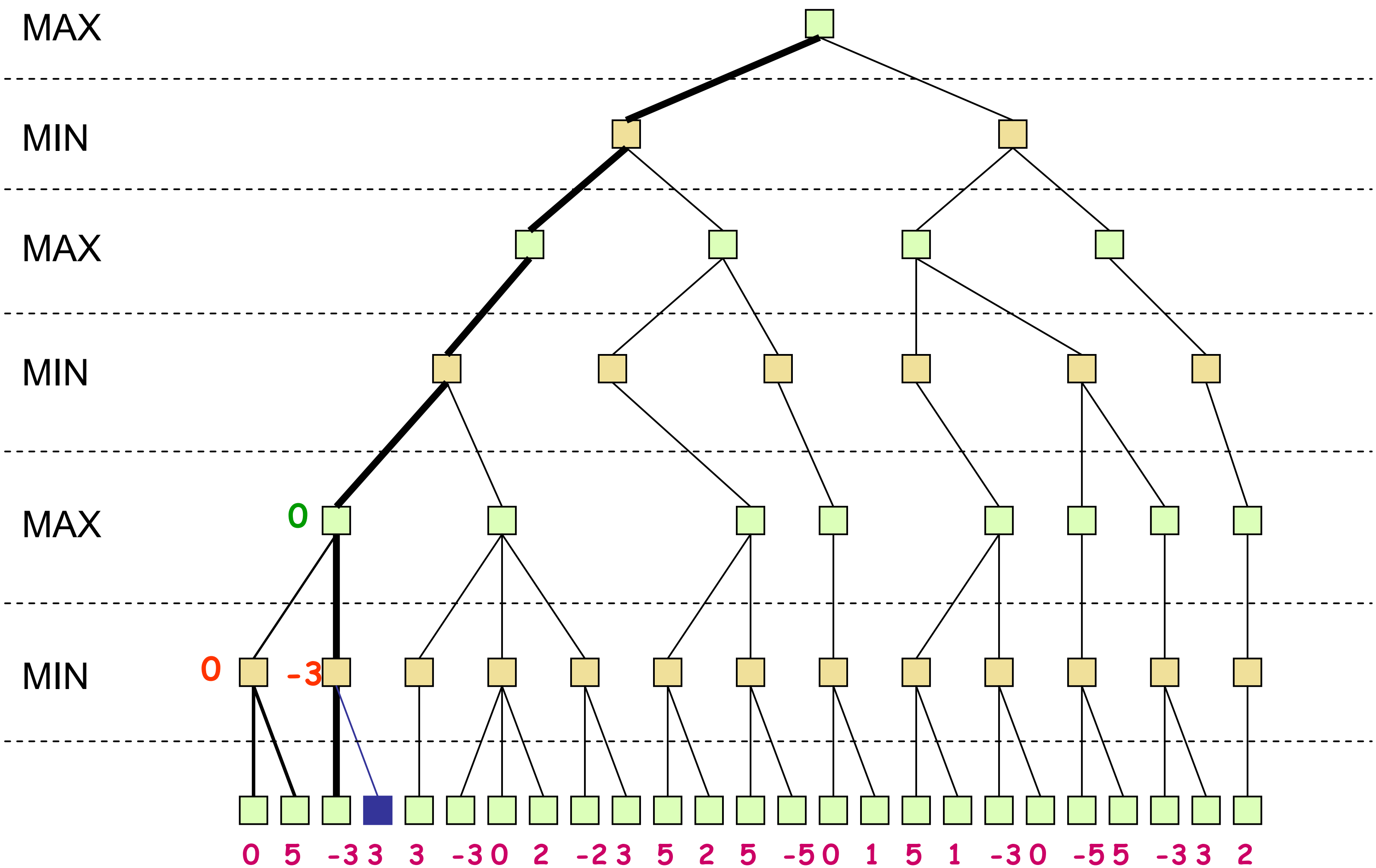
Alpha-Beta Pruning – Example



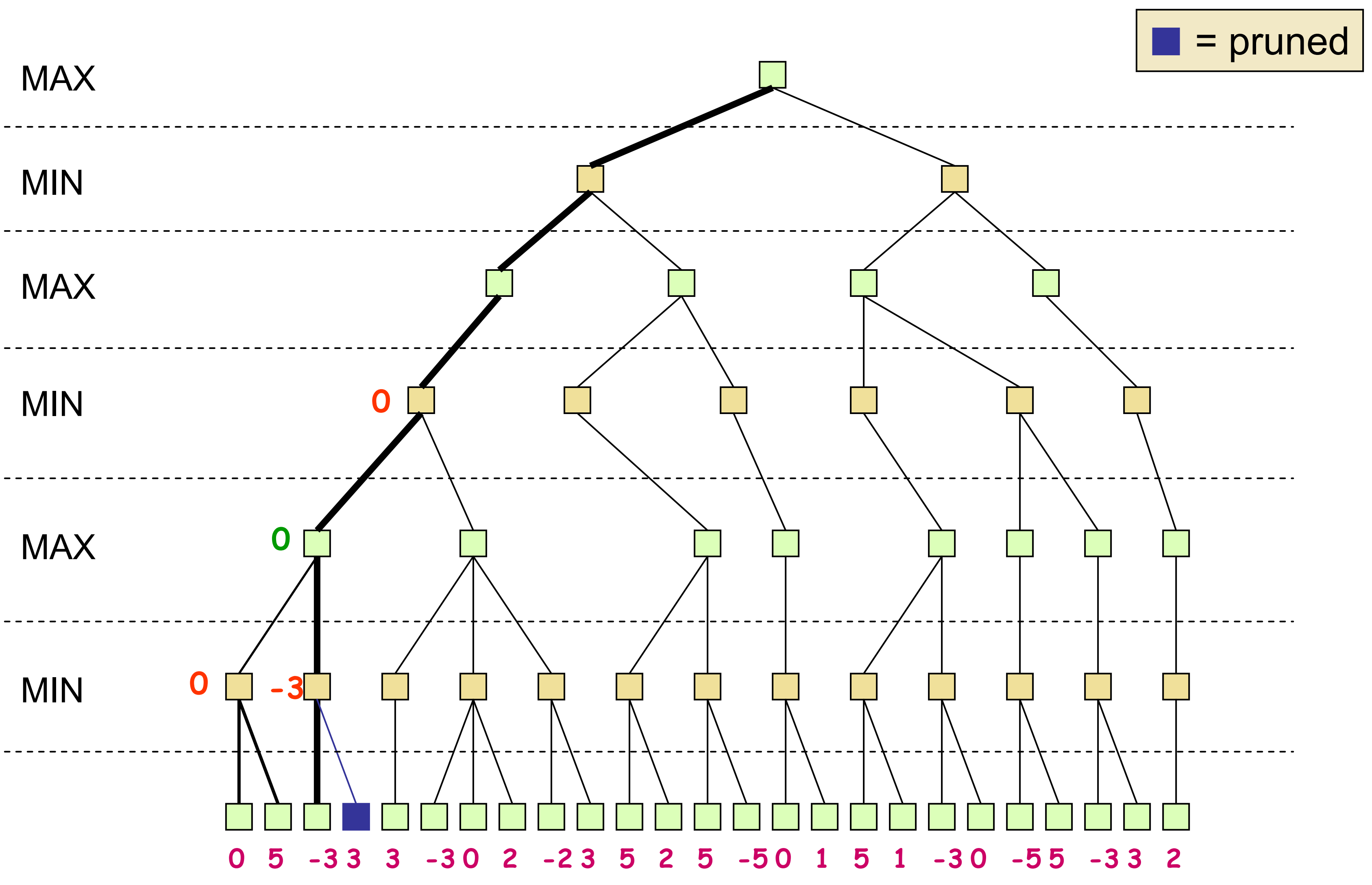
Alpha-Beta Pruning – Example



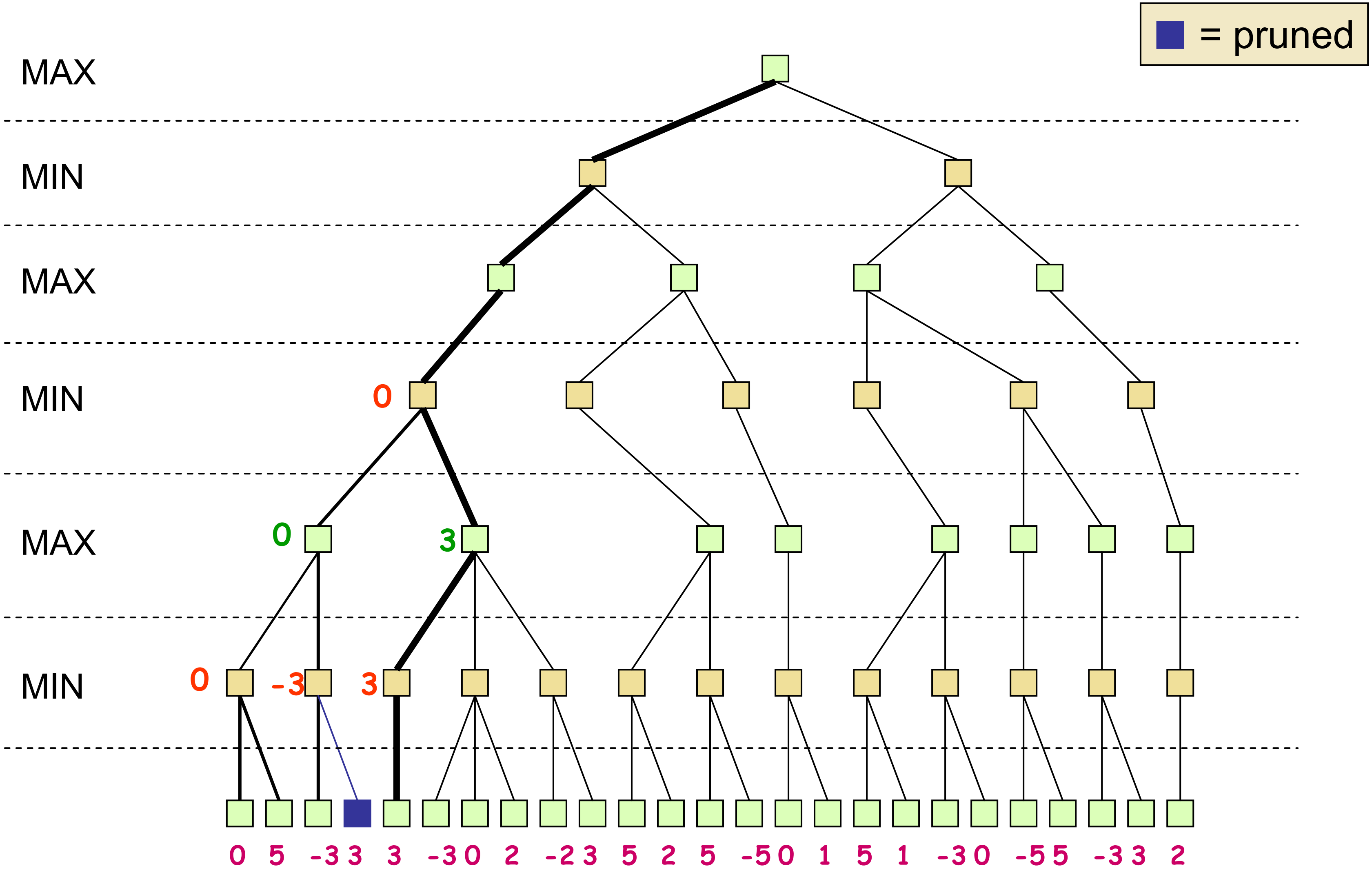
Alpha-Beta Pruning – Example



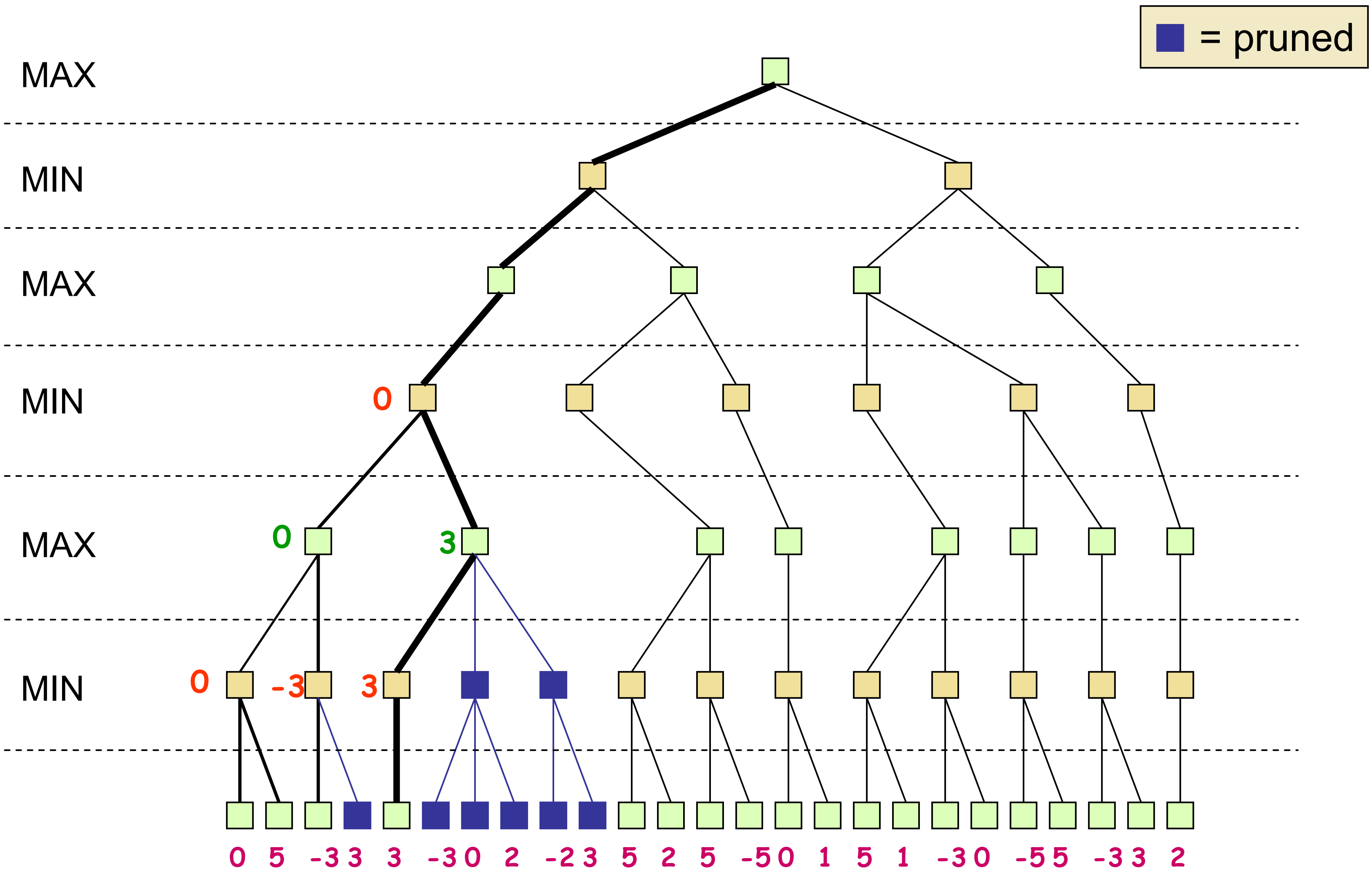
Alpha-Beta Pruning – Example



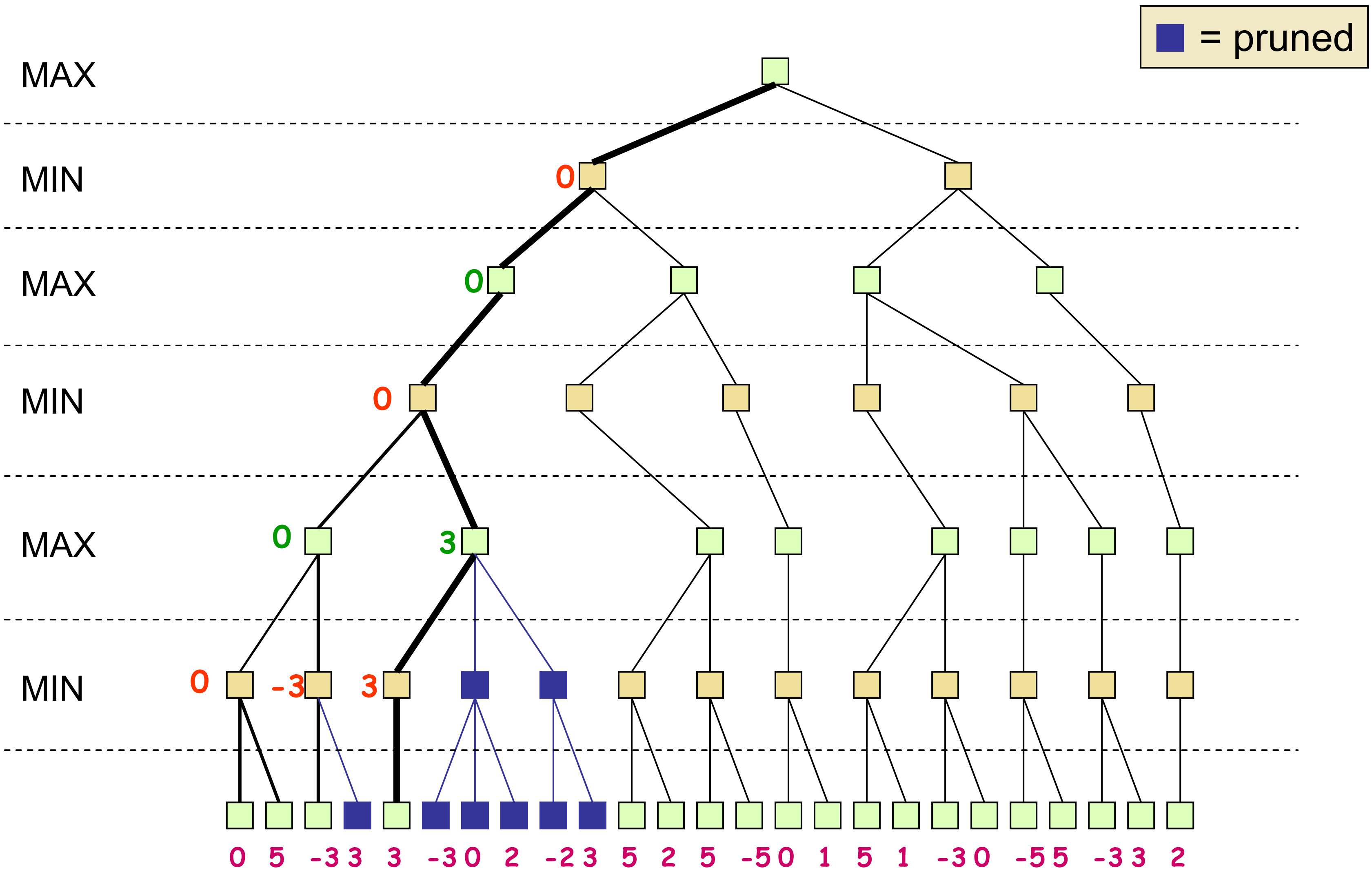
Alpha-Beta Pruning – Example



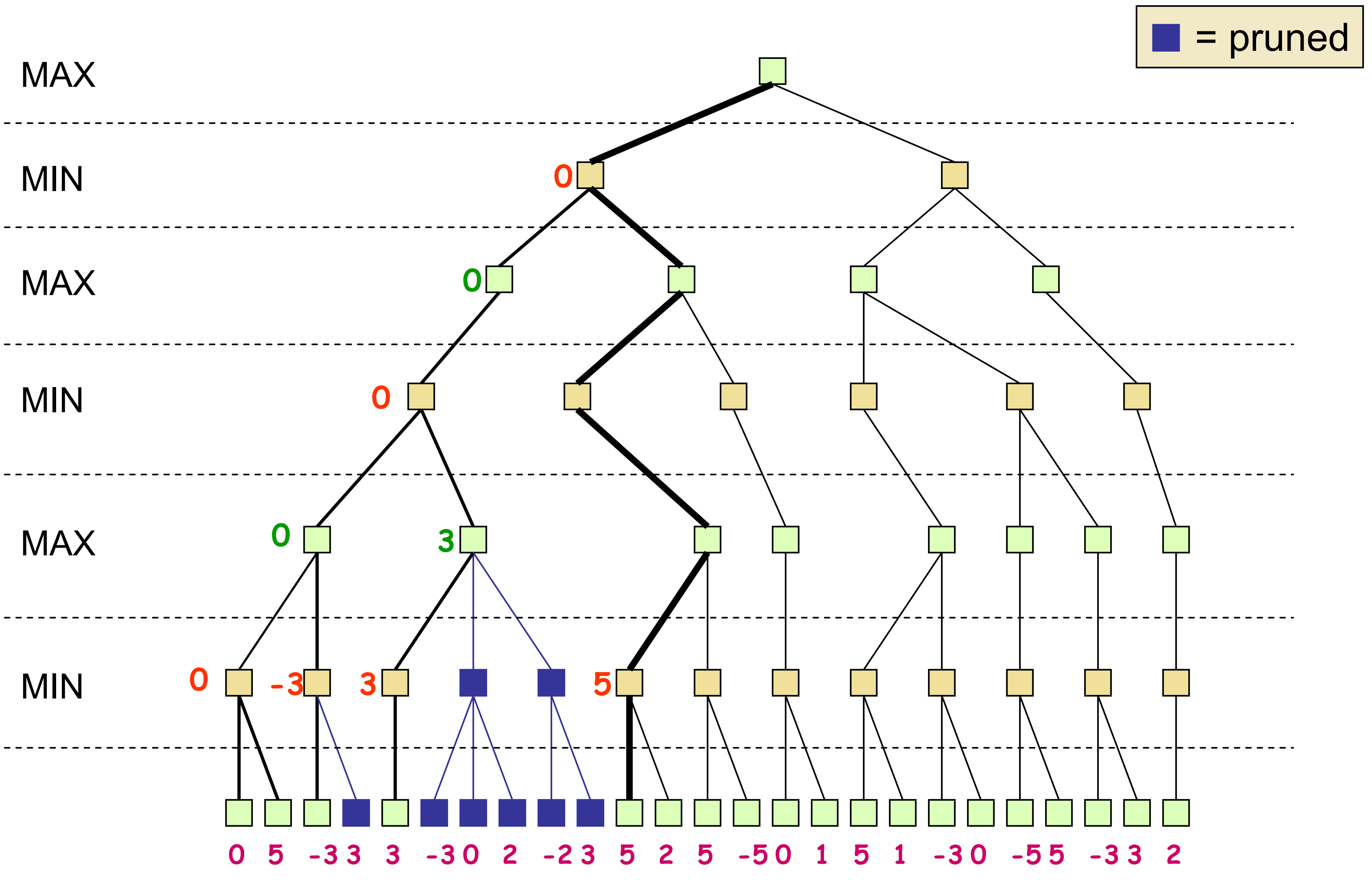
Alpha-Beta Pruning – Example



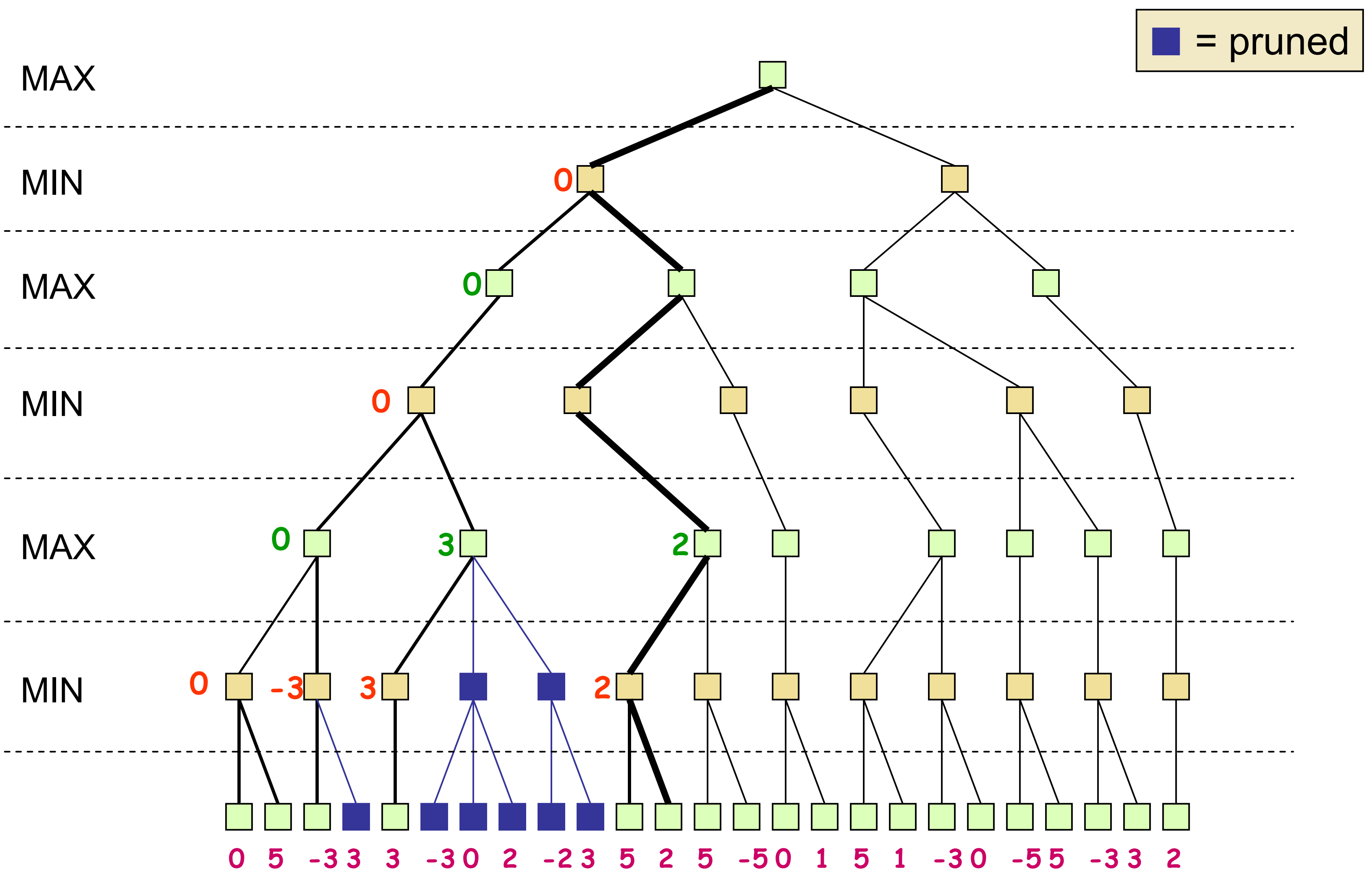
Alpha-Beta Pruning – Example



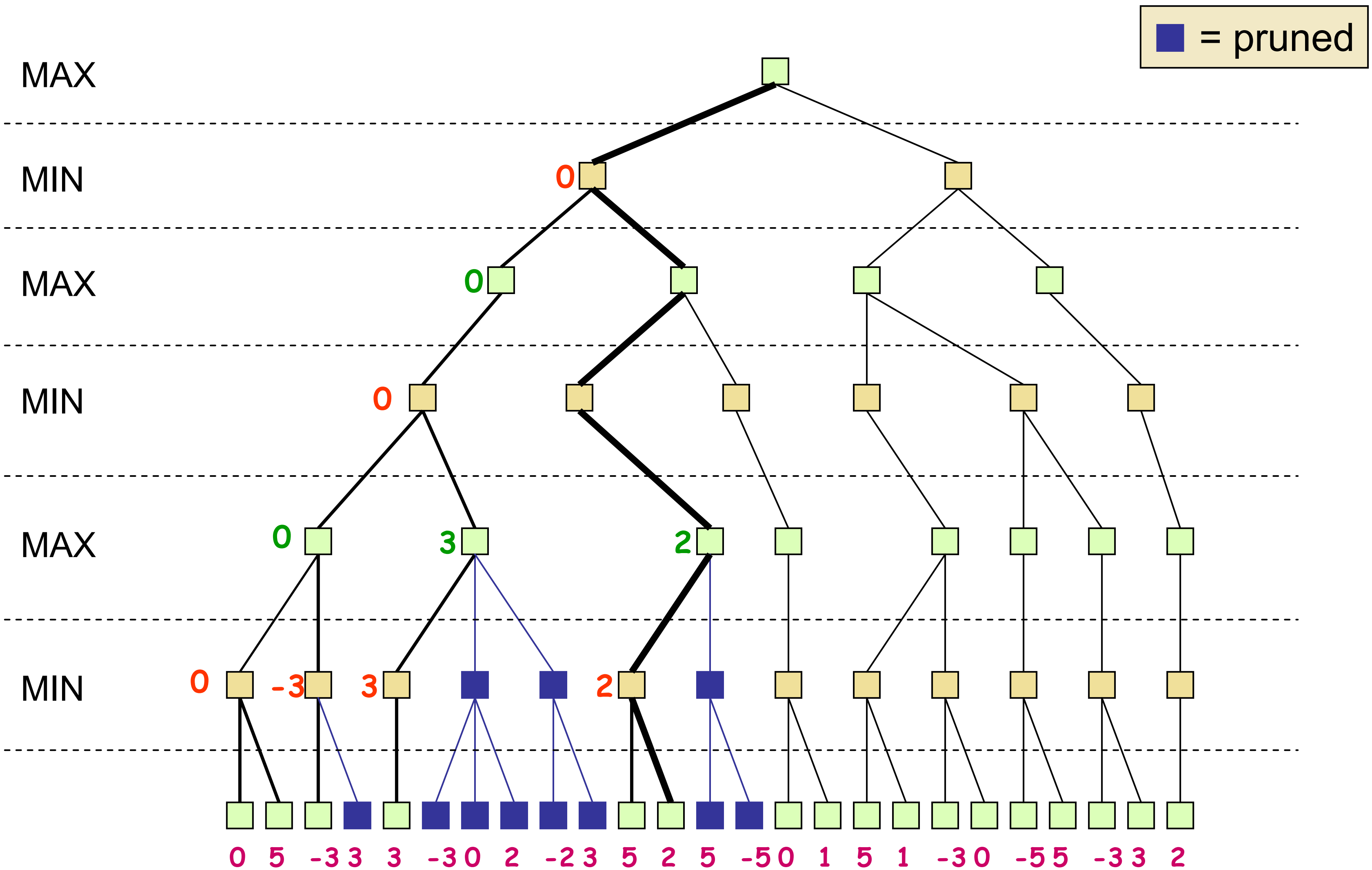
Alpha-Beta Pruning – Example



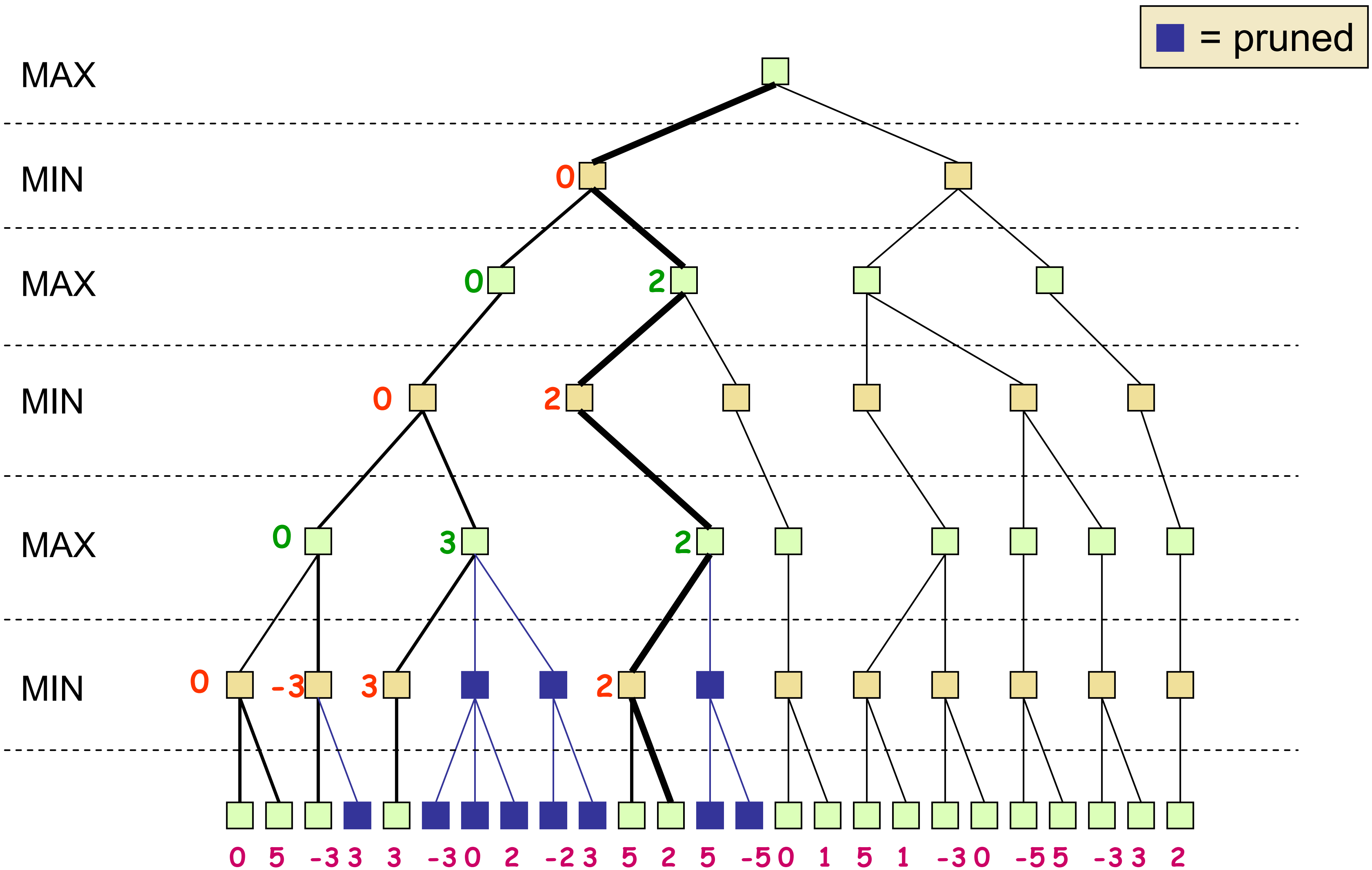
Alpha-Beta Pruning – Example



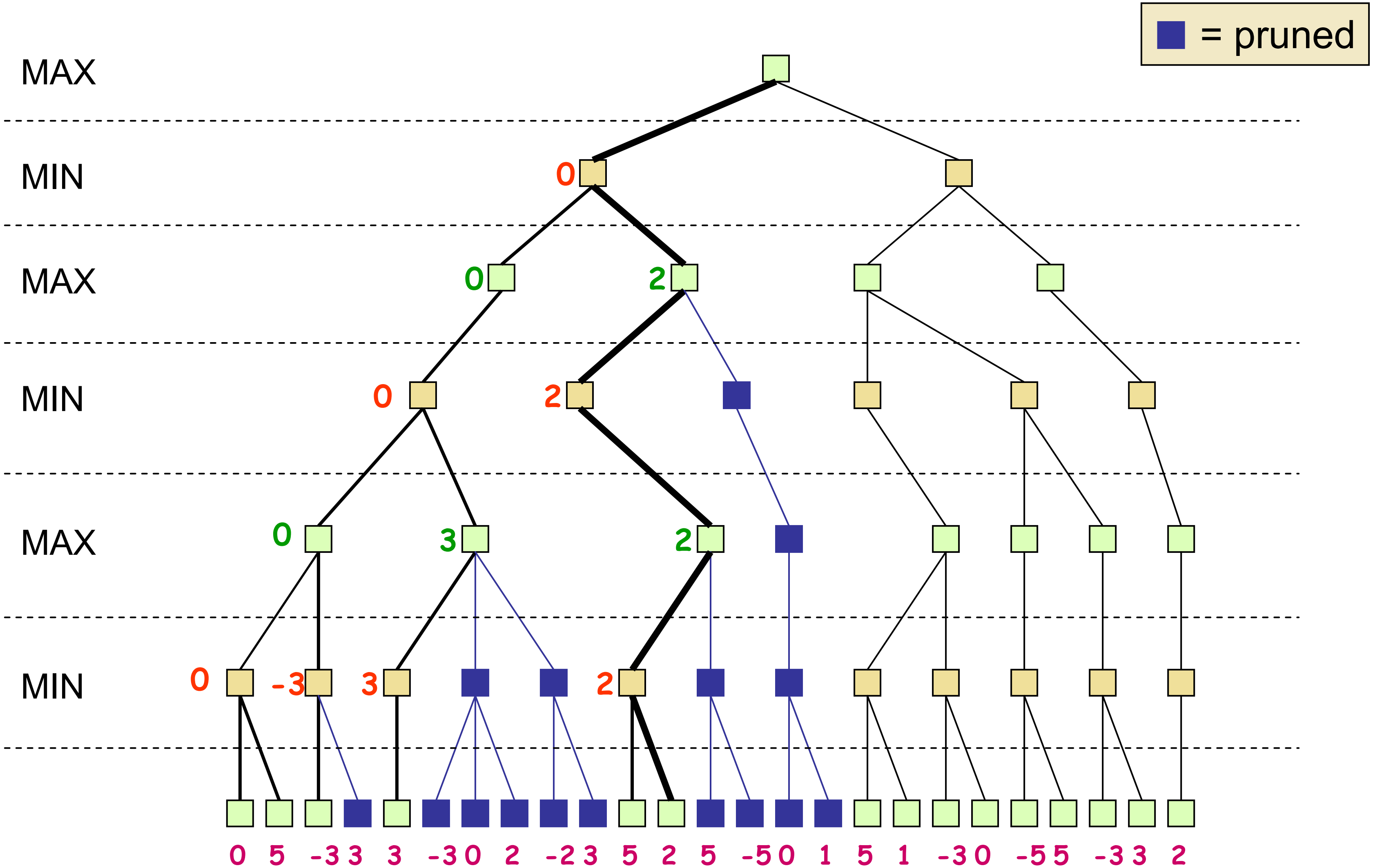
Alpha-Beta Pruning – Example



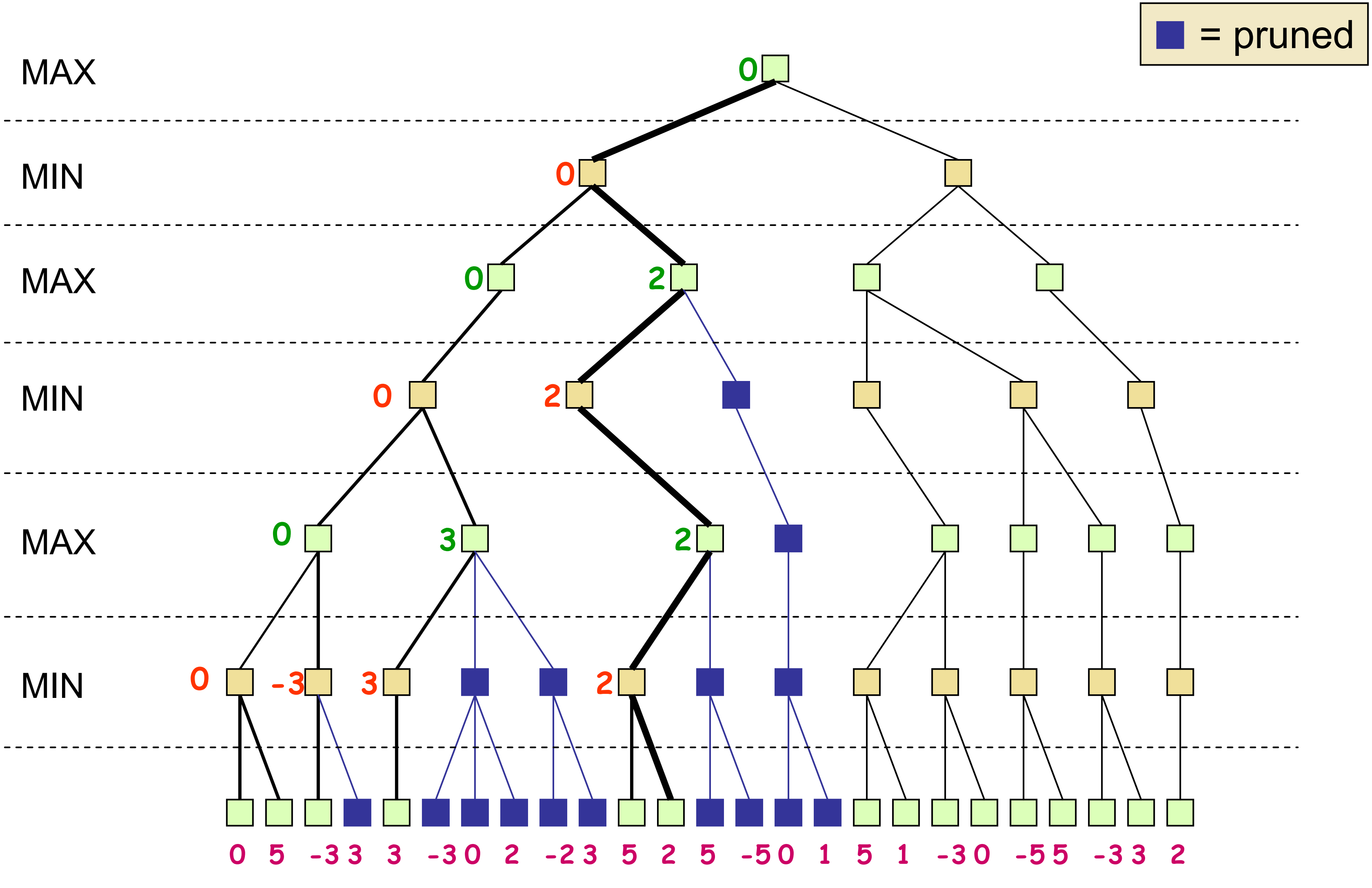
Alpha-Beta Pruning – Example



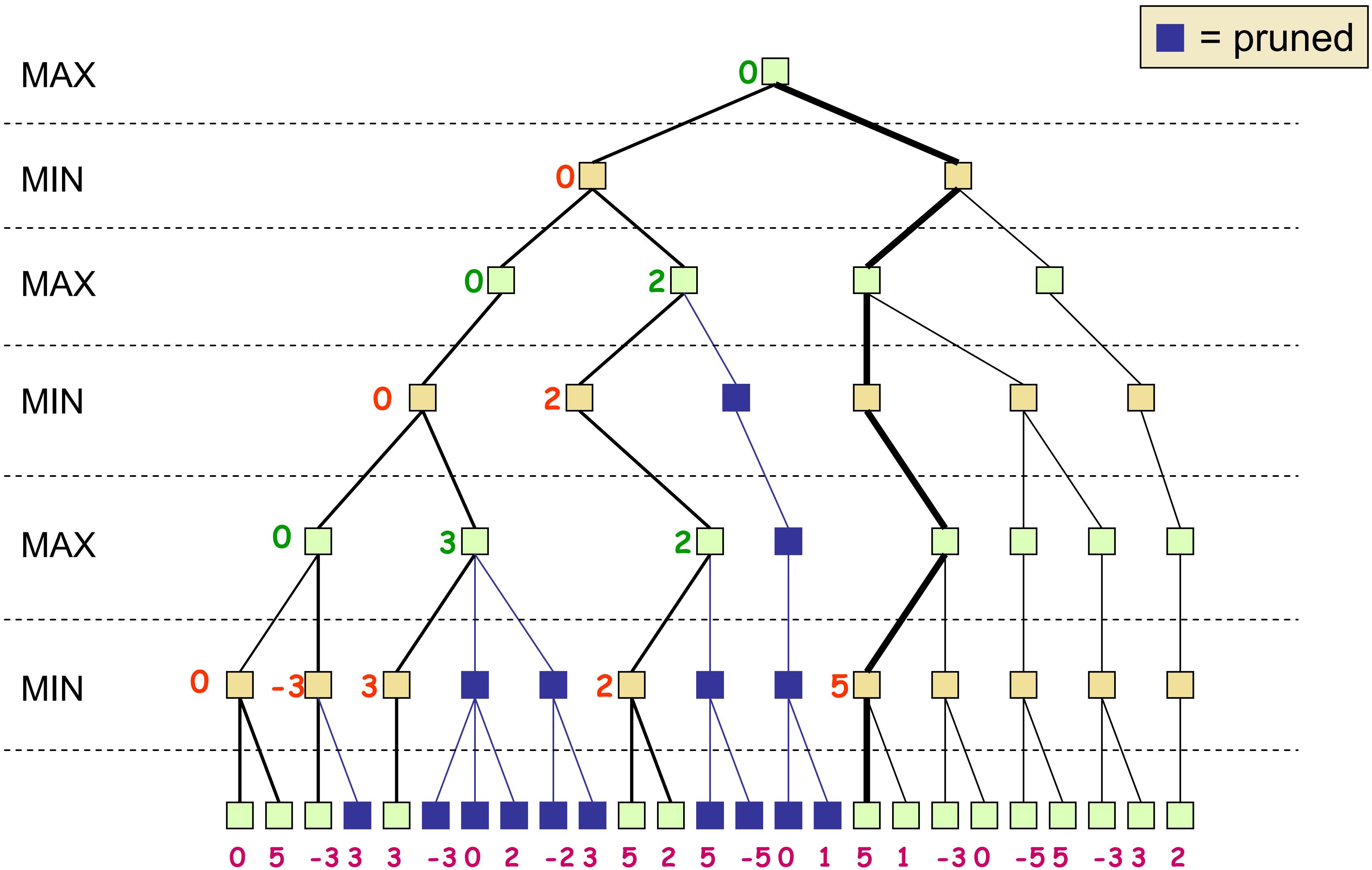
Alpha-Beta Pruning – Example



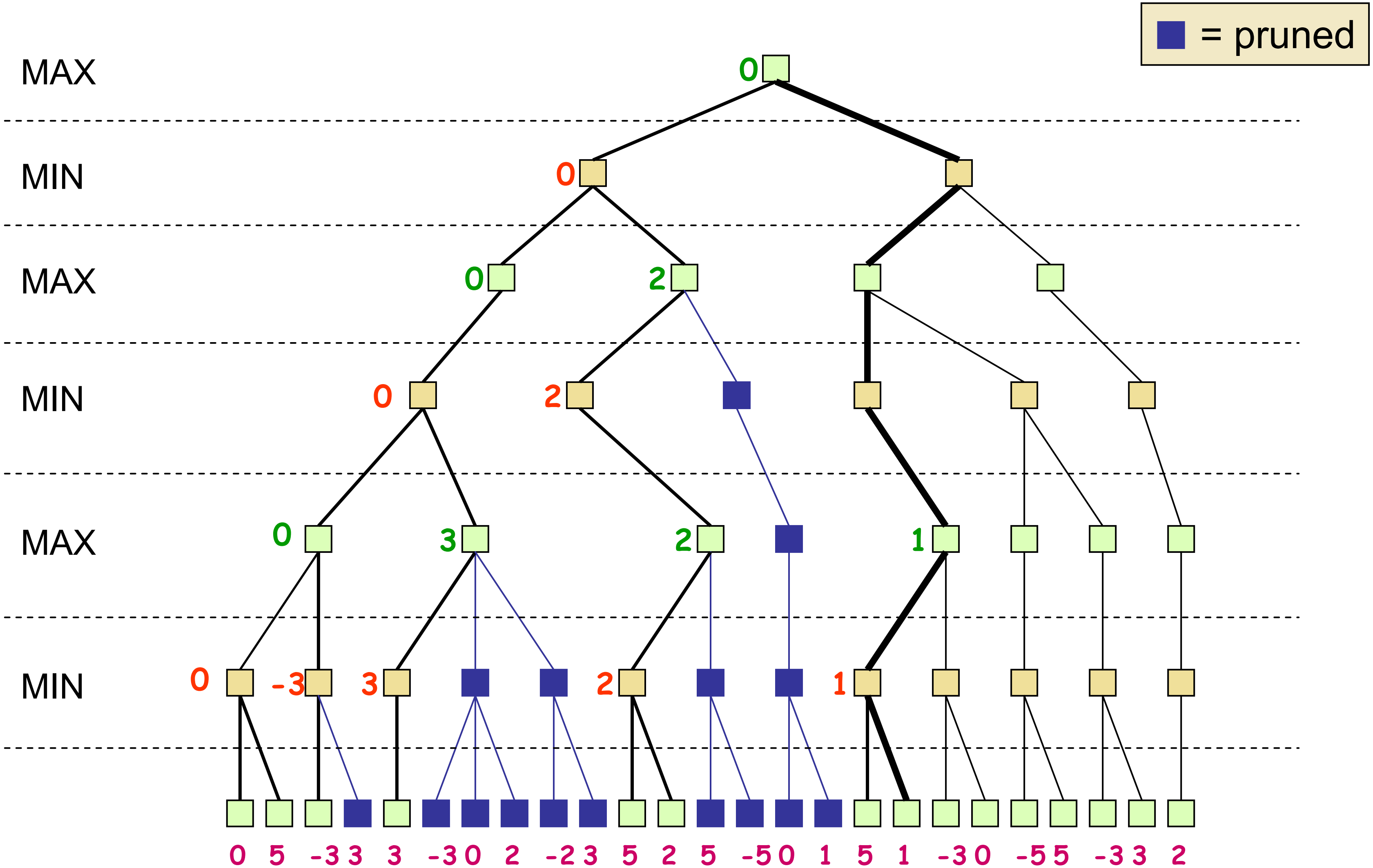
Alpha-Beta Pruning – Example



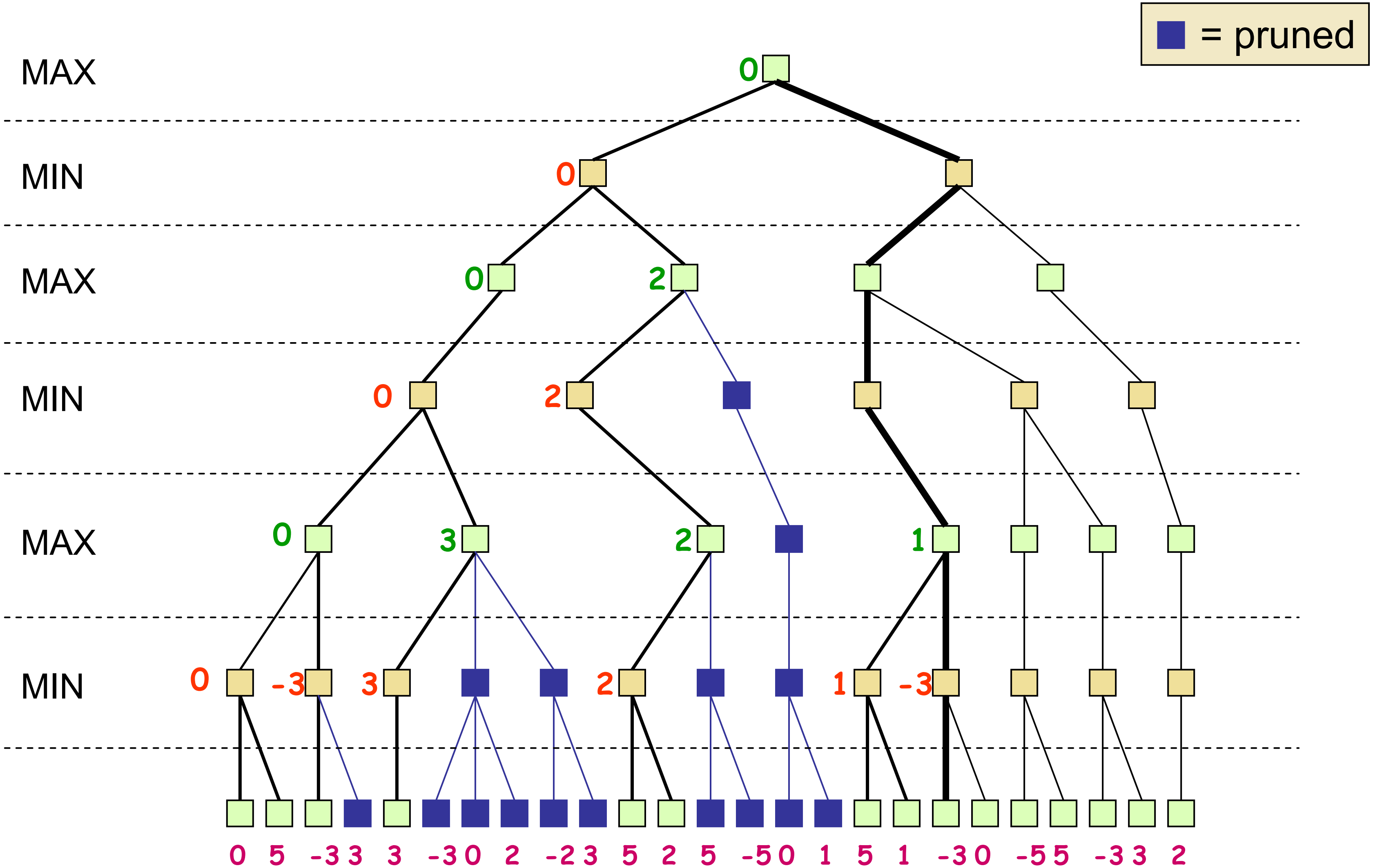
Alpha-Beta Pruning – Example



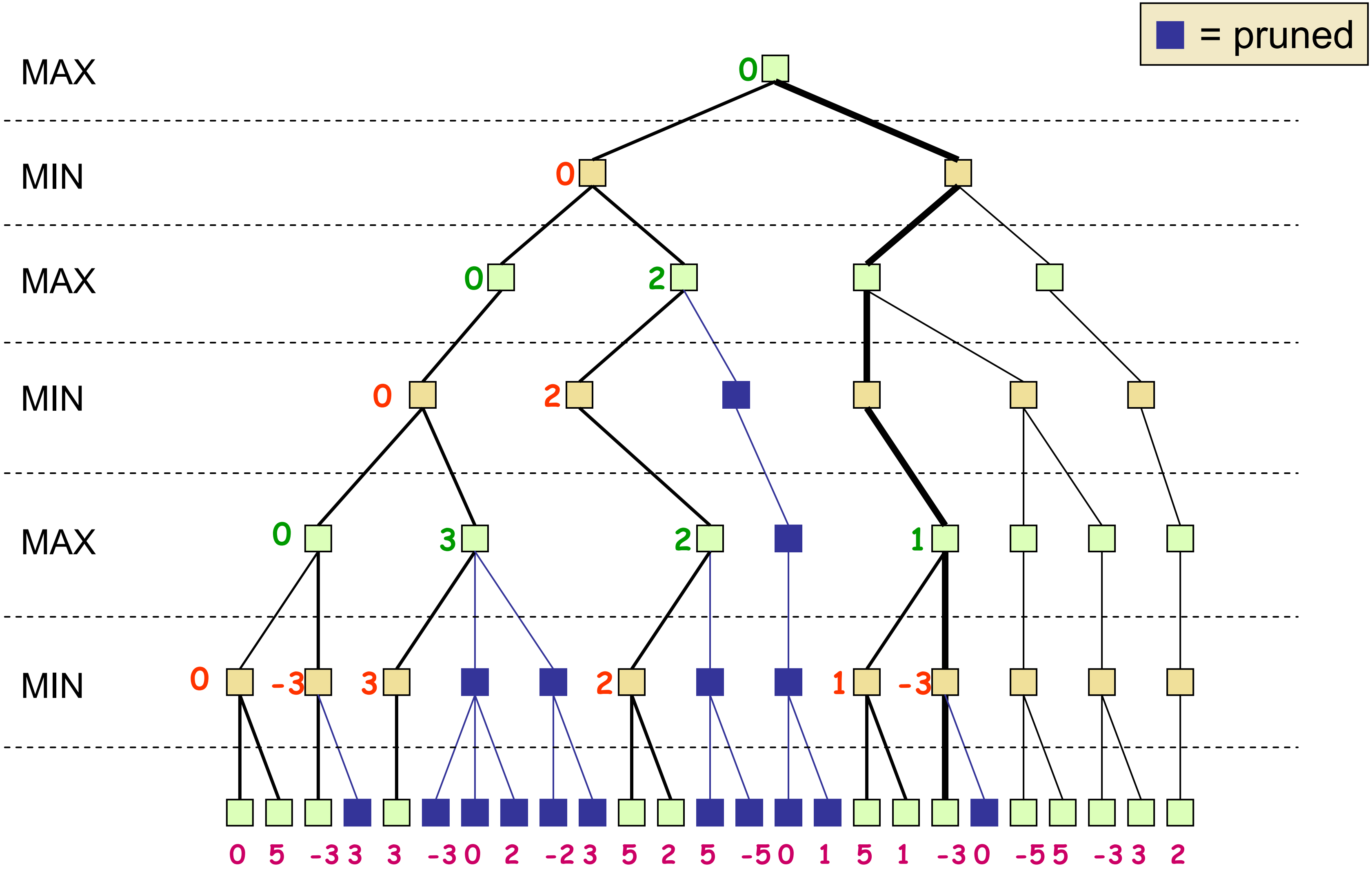
Alpha-Beta Pruning – Example



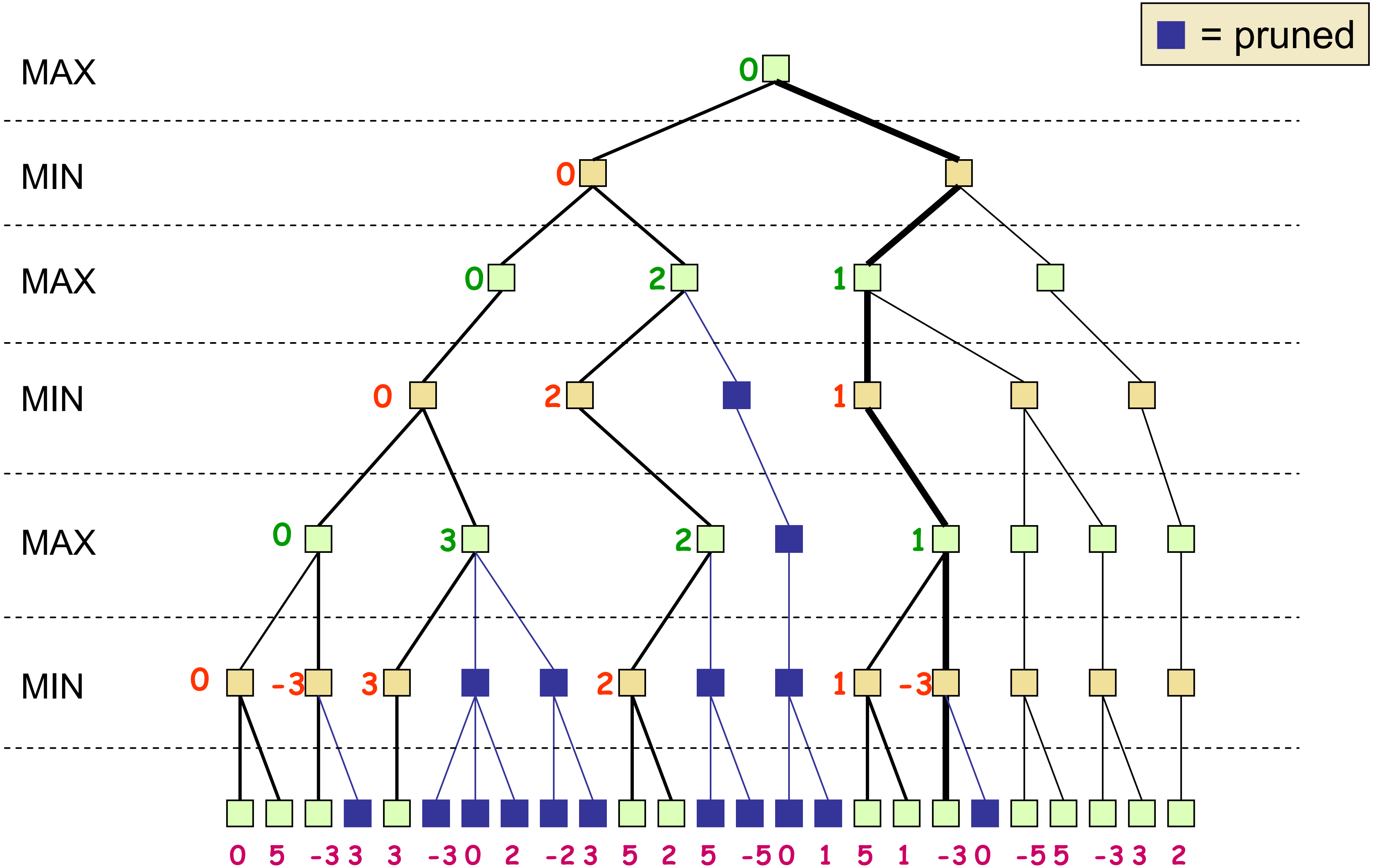
Alpha-Beta Pruning – Example



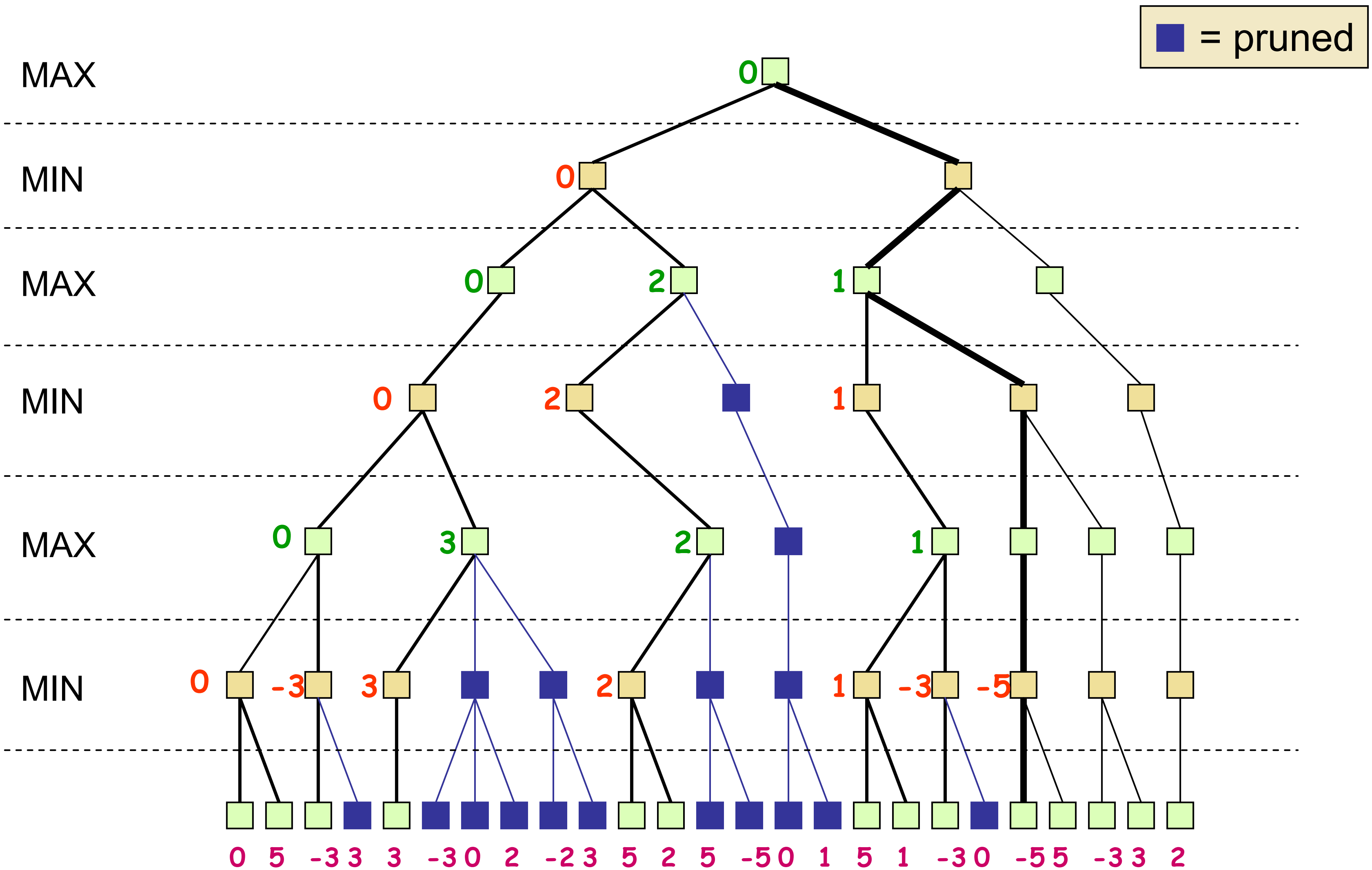
Alpha-Beta Pruning – Example



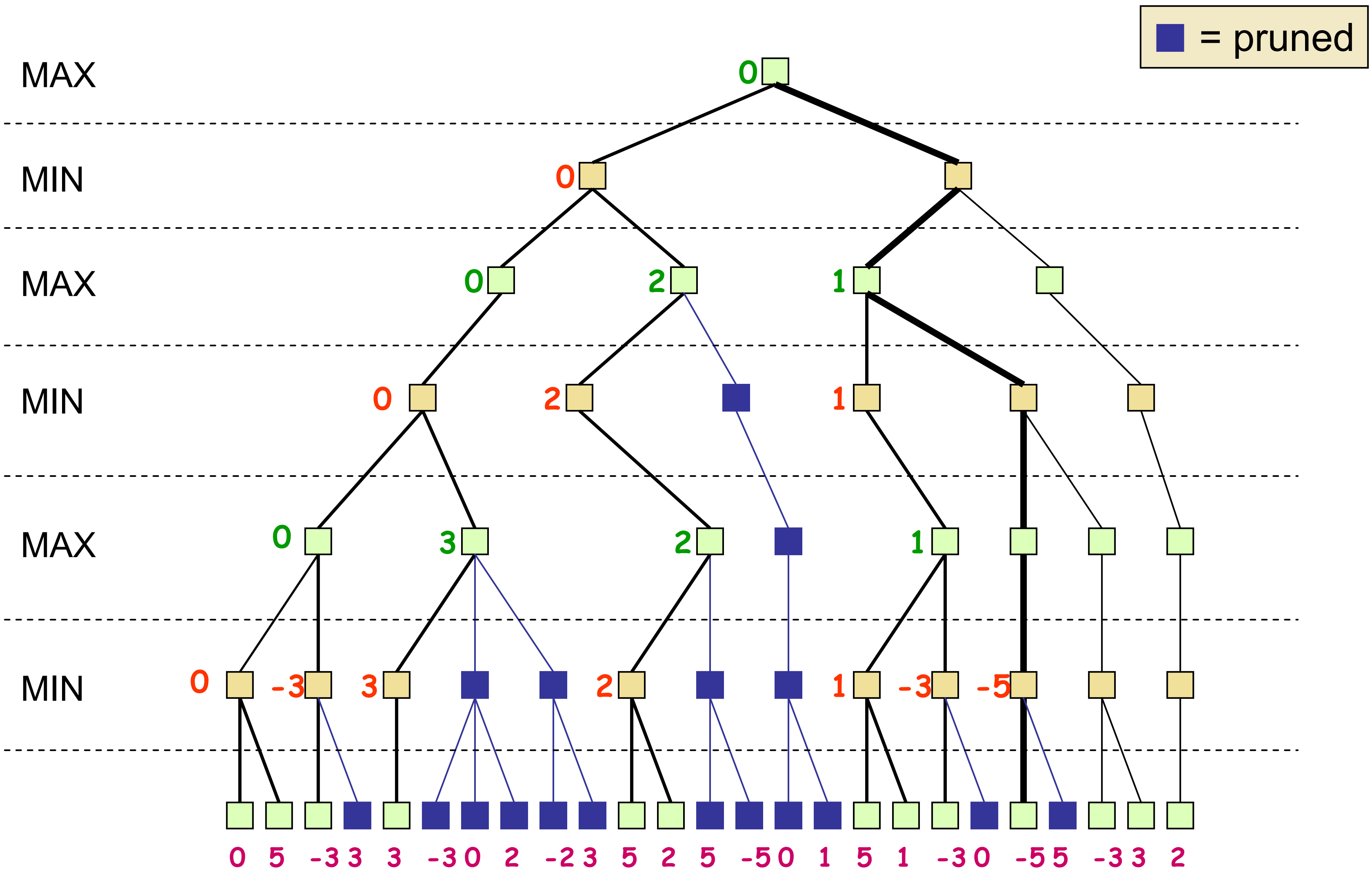
Alpha-Beta Pruning – Example



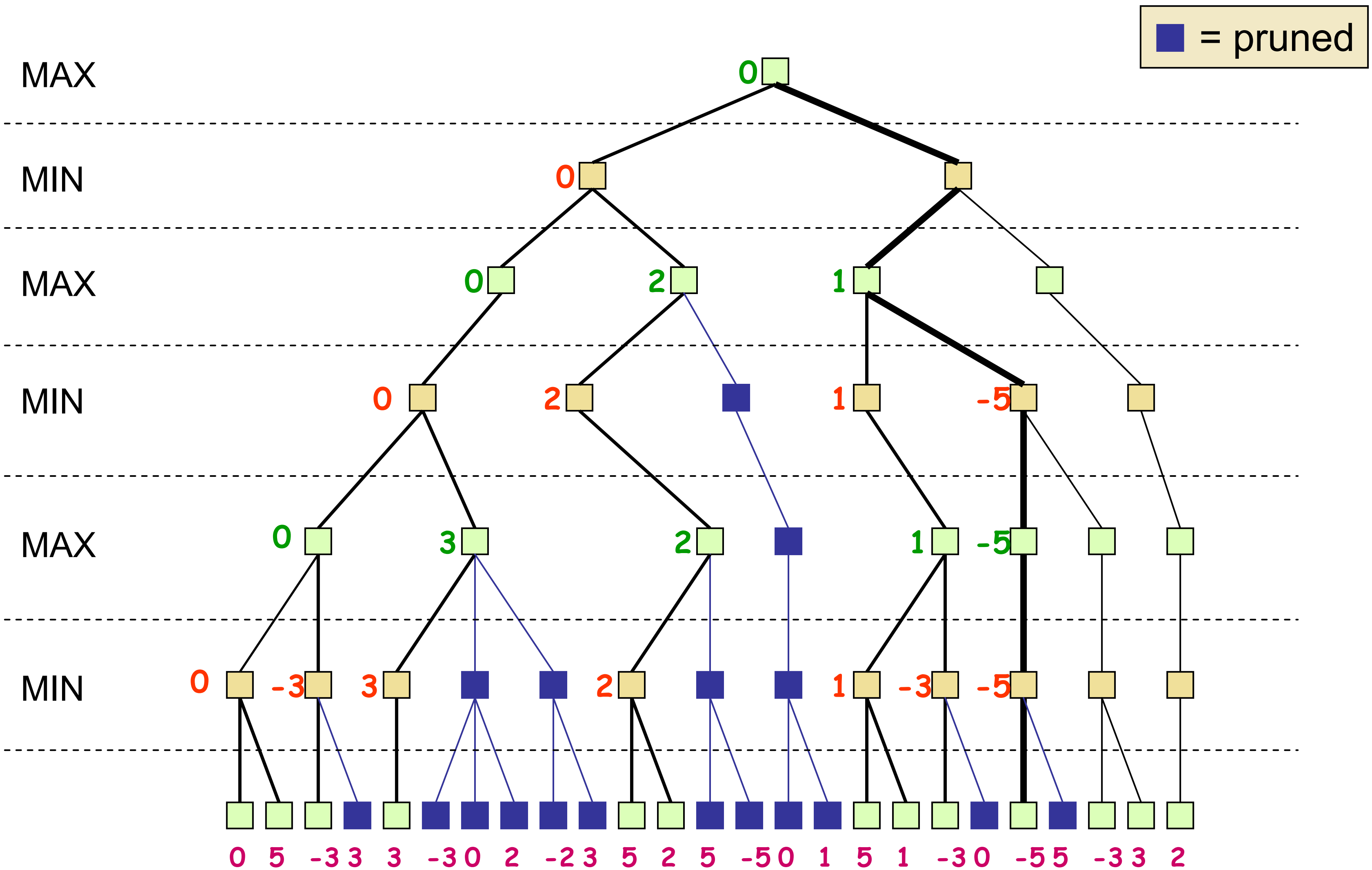
Alpha-Beta Pruning – Example



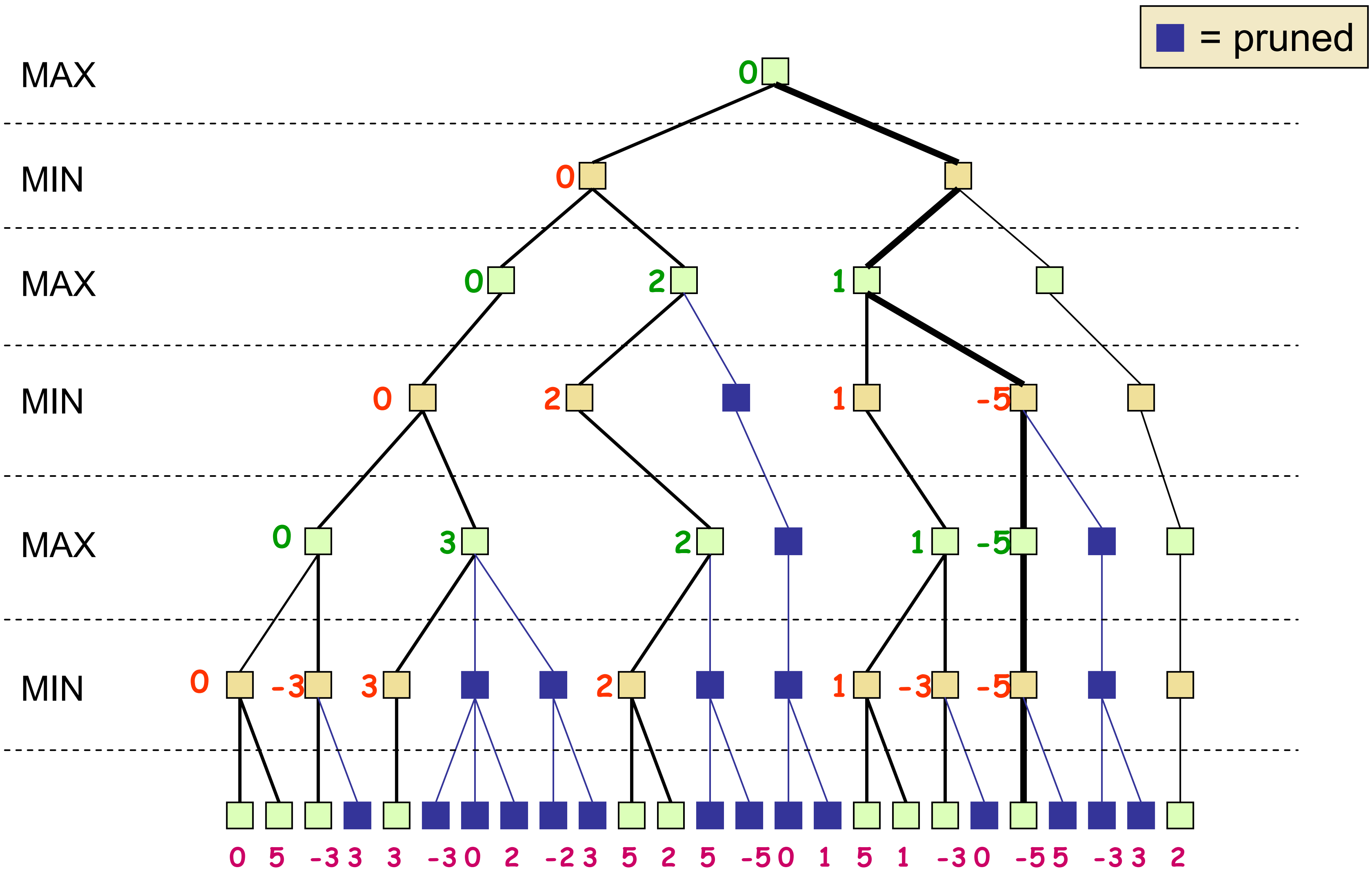
Alpha-Beta Pruning – Example



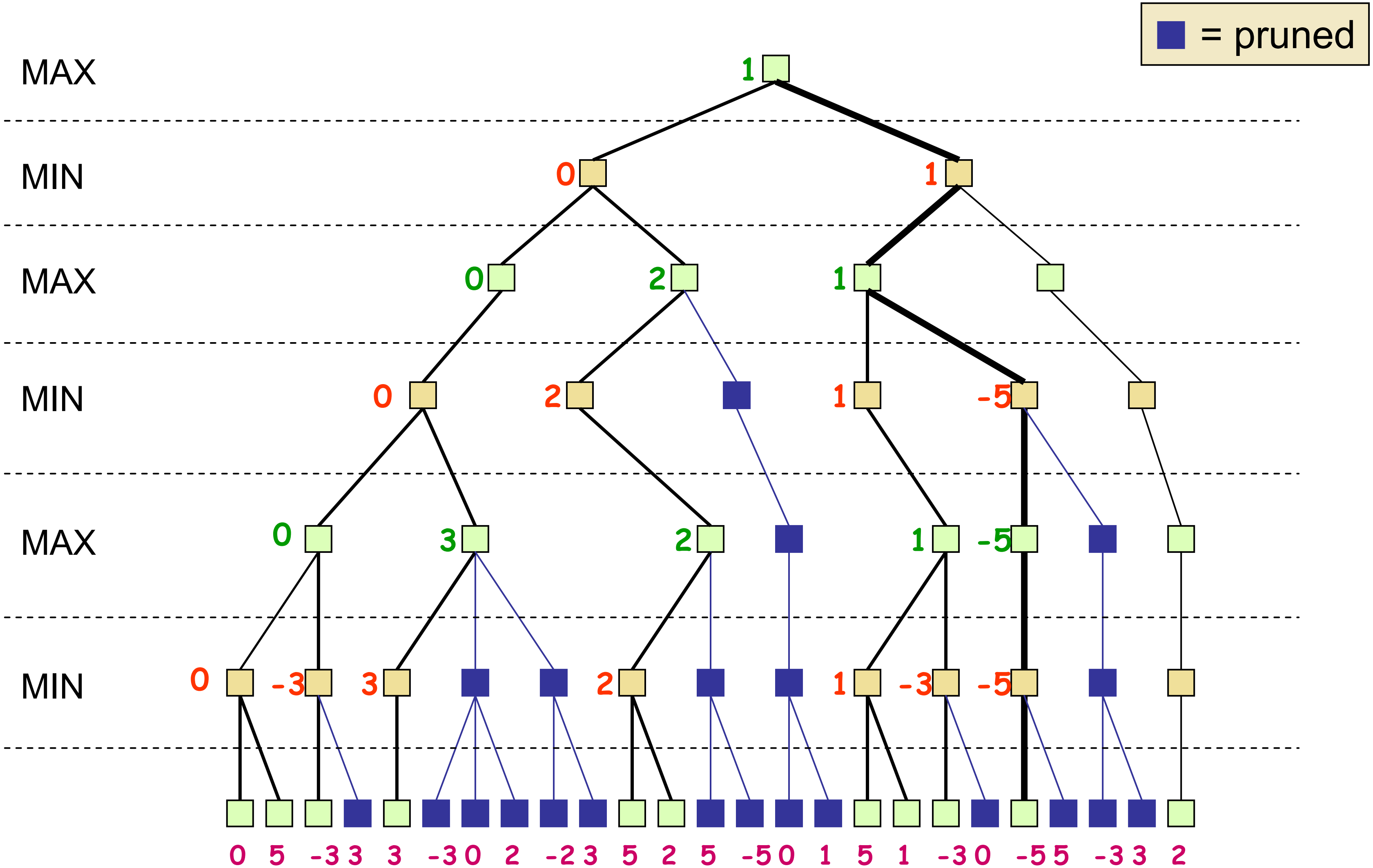
Alpha-Beta Pruning – Example



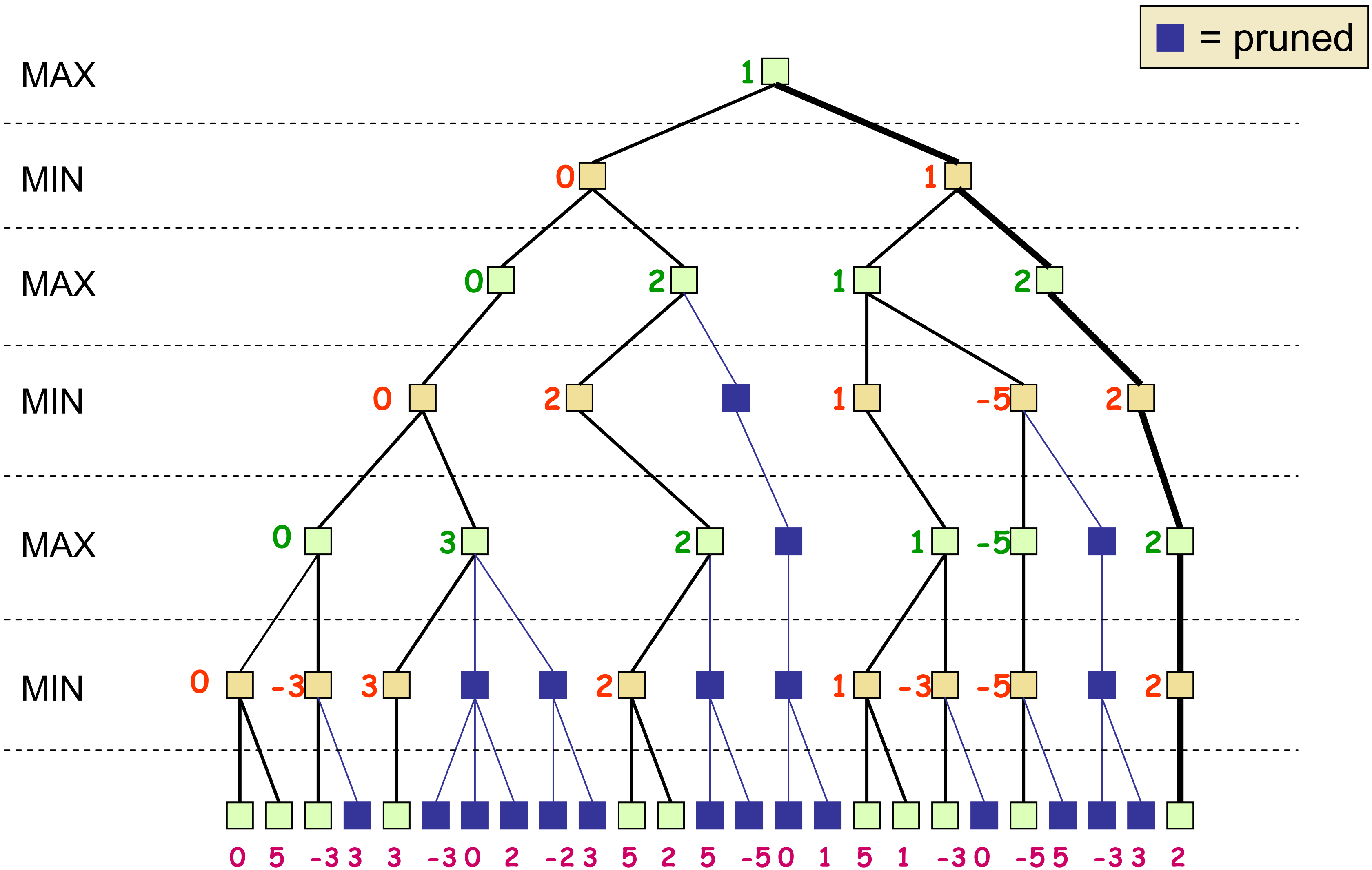
Alpha-Beta Pruning – Example



Alpha-Beta Pruning – Example



Alpha-Beta Pruning – Example



Alpha-Beta Pruning – Example