## Introduction to Artificial Intelligence English practicals 10: Machine learning

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May 13th 2021

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## Machine learning

#### Supervised (task driven)

# **Unsupervised** (data driven)

#### Reinforcement (learn from mistakes)



source: towardsdatascience.com

- Ad popularity
- Spam classification
- Face recognition

source: towardsdatascience.com

- Recommendation systems
- Buying habits

source: towardsdatascience.com

- Video games
- Industrial simulations
- Robot navigation
- We already touched upon reinforcement learning when last time we

Grouping user logs

talked about the robot moving in the maze Marika Ivanová (MFF UK) Introduction to Artificial Intelligence

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Decide what type of problems are the following tasks:

- You have an inventory of identical items. You want to predict how many of these items will sell over the next 3 months.
- ② You'd like to examine individual customer accounts, and for each account decide if it has been hacked.
- Treat both as classification problems
- Treat problem (1) as classification, problem (2) as regression
- Treat problem (1) as regression, problem (2) as classification
- Ireat both as regerssion problems

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- Treat problem (1) as regression, problem (2) as classification
- Treat both as regerssion problems
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Hypothesis:

$$h_w(x) = w_0 + w_1(x)$$

 $w_0, w_1 \dots$  parameters how to choose w?

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Choose  $w_0$ ,  $w_1$  so that  $h_w(x)$  is close to y for training examples (x, y)

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Choose  $w_0$ ,  $w_1$  so that  $h_w(x)$  is close to y for training examples (x, y)

$$\min_{w_0,w_1} \frac{1}{2m} \sum_{i=1}^m \left( h_w(x^{(i)}) - y^{(i)} \right)^2$$

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3 May 2021 6/16

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Cost function when we do not simplify the hypothesis, i. e,  $h_w(x) = w_0 + w_1 x$ 



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## Linear regression: gradient descent

Linear regression model:

Gradient descent:

• 
$$h_w(x) = w_0 + w_1 x_1$$
  
•  $J(w_0, w_1) = \frac{1}{2m} \sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)})^2$   
repeat until convergence {  
 $w_j \leftarrow w_j - \alpha \frac{\partial}{\partial w_j} J(w_0, w_1)$   
for  $j = 0$  and  $j = 1$   
}

$$\begin{split} \frac{\partial}{\partial w_j} J(w_0, w_1) &= \frac{\partial}{\partial w_j} \frac{1}{2m} \sum_{i=1}^m \left( h_w(x^{(i)}) - y^{(i)} \right)^2 = \frac{\partial}{\partial w_j} \frac{1}{2m} \sum_{i=1}^m \left( w_0 + w_1 x^{(i)} - y^{(i)} \right)^2 \\ \frac{\partial}{\partial w_0} J(w_0, w_1) &= \frac{\partial}{\partial w_0} \frac{1}{m} \sum_{i=1}^m \left( h_w(x^{(i)}) - y^{(i)} \right) \\ \frac{\partial}{\partial w_1} J(w_0, w_1) &= \frac{\partial}{\partial w_1} \frac{1}{m} \sum_{i=1}^m \left( h_w(x^{(i)}) - y^{(i)} \right) x^{(i)} \end{split}$$

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## Linear regression: gradient descent



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## Linear regression: gradient descent



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#### Decision trees

- **(1)** Select the most important attribute based on entropy H(V)
- ② Divide samples based on this attribute
- 3 Repeat 1) and 2) until all samples belong to the same category

$$H(V) = -\sum_{i=1}^{c} p_i \log_2 p_i$$



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- H(parent) =
- *H*(*balance* < 50) =
- *H*(*balance* > 50) =

Provost, Foster; Fawcett, Tom. Data Science for Business: What You Need to Know about Data Mining and Data-Analytic Thinking

May 2021 11 / 16

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- $H(parent) = -\frac{16}{30} \log \frac{16}{30} \frac{14}{30} \log \frac{14}{30} \approx 0.99$ • H(balance < 50) =
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May 2021 11 / 16

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- $H(parent) = -\frac{16}{30} \log \frac{16}{30} \frac{14}{30} \log \frac{14}{30} \approx 0.99$
- $H(balance < 50) = -\frac{12}{13} \log \frac{12}{13} \frac{1}{13} \log \frac{1}{13} \approx 0.39$
- *H*(*balance* > 50) =

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Introduction to Artificial Intelligence

May 2021 11 / 16

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May 2021 11 / 16

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Weighted average of entropy for each node:

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May 2021 11 / 16

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Weighted average of entropy for each node:

•  $H(balance) = \frac{13}{30*0.39+17} \times 0.79 \approx 0.62$ 

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May 2021 11 / 16

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<sup>13</sup> Information gain:

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Weighted average of entropy for each node:

•  $H(balance) = \frac{13}{30*0.39+17} \times 0.79 \approx 0.62$ Information gain:

 Gain(parent, balance) = H(parent) - H(balance) = 0.99 - 0.62 = 0.37

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May 2021 11 / 16

$$H(V) = -\sum_{i=1}^{c} p_i \log_2 p_i$$



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Introduction to Artificial Intelligence

May 2021 12 / 16

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$$H(V) = -\sum_{i=1}^{c} p_i \log_2 p_i$$

• 
$$H(residence = OWN) = -\frac{7}{8}\log\frac{7}{8} - \frac{1}{8}\log\frac{1}{8} \approx 0.54$$



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Introduction to Artificial Intelligence

May 2021 12 / 16

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May 2021 12 / 16

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Weighted average of entropy for each node:



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Weighted average of entropy for each node:

*H*(residence) = 8/30 \* 0.54 + 10/30 \* 0.97 + 12/30 \* 0.98 ≈ 0.86

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Marika Ivanová (MFF UK)

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Information gain:



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May 2021 12 / 16

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Information gain:

• Gain(parent, residence) =H(parent) - H(residence) = 0.99 - 0.86 = 0.13



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## Unsupervised learning: clustering



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## Unsupervised learning: clustering



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## Unsupervised learning: clustering example

#### Google news

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Top stories		
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COVID-19	Hamas has launched rockets at Tel Aviv after a major Israeli airstrike in Gaza   DW News	
P U.S.	At least 35 killed in Gaza as Israel ramps up airstrikes in response to rocket attacks	
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Business	- Opinion   Why So Much Rests on the Fate of a Tiny Neighborhood in East Jerusalem	
Technology	The New York Times - 41 minutes ago - Opinion	
Entertainment	Expert panel says mistakes led to coronavirus pandemic, but stops	
No Sports	short of holding countries, leaders to account	
Science	The Washington Post - 1 hour ago	
Ky Health	WCNC Charlotte. Always On. Streaming News for May 12, 2021     WCNC - 2 hours ago	
Language & region	Covid pandemic was preventable, says WHO-commissioned report The Quardian - 48 minutes ago	
English (United States) Settings	Panel suggests WHO should have more power to stop pandemics     For News - 51 minutes ago	
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Simple, widely used clustering algorithm The number of clusters must be given

**Input:** k, set of points  $a_1, \ldots, a_n$ 

Place centroids  $c_1, \ldots, c_k$  at random locations

Repeat until convergence:

• for each point *a<sub>i</sub>*:

find nearest centroid:  $c_j = \arg \min_j d_{ij}$ assign the point  $a_i$  to cluster  $C_j$ 

• for each cluster  $C_j$ ,  $j = 1, \ldots, k$ :

new centroid  $c_j$  = mean of all points  $a_i$  assigned to cluster  $C_j$  in the previous step

$$c_j(\ell) = rac{1}{n} \sum_{a_i \in C_j} a_i(\ell) ext{ for } \ell = 1, \dots, m$$

Stop when none of the cluster assignments change

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