

## Propositional and Predicate Logic - Homework on Propositional Logic

The deadline for this homework is November 18, 2024, at 10:40 (before the tutorials). You can submit your solution to my email [Martin.Pilat@mff.cuni.cz](mailto:Martin.Pilat@mff.cuni.cz) (a scan on your handwriting is OK), or on paper before the tutorials.

You can get up to 5 points for the homework that count towards the credit requirements.

Let  $\varphi, \psi$  are two propositions over the set of variables  $\mathbb{P} = \{p, q, r, s, t\}$ :

$$\varphi : (p \vee r) \wedge (\neg p \vee q) \wedge (\neg q \vee \neg r), \quad (1)$$

$$\psi : (\neg s \vee \neg r) \wedge (\neg t \vee s) \wedge \neg p. \quad (2)$$

1. Using implication graph, decide if the proposition  $\varphi \wedge \psi$  is satisfiable. If it is, find a satisfying assignment. (1 point)
2. Transform  $\varphi$  into DNF. (1 point)
3. Count the number of non-equivalent propositions  $\theta$  over  $\mathbb{P}' = \{p, q, r\}$  such that  $\varphi \models \theta$ . (1 point)
4. Let variables  $r, s, t$  represent (in this order) that "Rachel / Sarah / Tom is at school" and let  $\mathbb{P} = \{r, s, t\}$ . We know that
  - (i) If Tom is not at school, Sarah is also not there.
  - (ii) Rachel does not go to school without Sarah.
  - (iii) If Rachel is not at school, Tom is there.

We want to use the tableau method to show that these facts imply that Tom is at school. Therefore:

- (a) Write propositions  $\varphi_1, \varphi_2, \varphi_3$  over  $\mathbb{P}$  expressing the facts (i)-(iii). (1 point)
- (b) Use the tableau method to show that  $T = \{\varphi_1, \varphi_2, \varphi_3\}$  implies that Tom is at school, i.e.  $T \models t$ . (1 point)