Propositional and Predicate Logic – Second Homework

The deadline for this homework is December 16, 2024 at 10:40. You can either hand me the solution on paper before the tutorials on Dec. 16, or send it via e-mail to `Martin.Pilat@mff.cuni.cz` (a scan/photo of your handwriting is OK).

The points obtained for this homework count towards the points required to obtain the credit for the seminar.

1. Let $\mathcal{A} = \langle \mathbb{Q}, +, \cdot \rangle$ be a structure in language $L = \langle +, \cdot \rangle$ with equality, where $\mathbb{Q}$ is the set of rational numbers, "+" is the addition of rational numbers and "$\cdot$" is the multiplication of rational numbers.

   (a) Find the substructure of $\mathcal{A}$ generated by the set $\{1\}$, i.e. $\mathcal{A}\langle 1 \rangle$. (*0.5 point*)

   (b) Are there any other substructures of $\mathcal{A}$? (*0.5 point*)

   (c) Are all the substructures of $\mathcal{A}$ elementarily equivalent? (*1 point*)

2. Let $F(x, y)$ represent that "there is a flight from x to y" and let $C(x, y)$ represent that "there is a connection from x to y". Assume that

   (a) From Prague you can fly to Bratislava, London and New York, and from New York to Paris,

   (b) $(\forall x)(\forall y)(F(x, y) \to F(y, x))$,

   (c) $(\forall x)(\forall y)(F(x, y) \to C(x, y))$,

   (d) $(\forall x)(\forall y)(\forall z)((C(x, y) \land F(y, z)) \to C(x, z))$.

   Prove by tableau method that there is a connection from Bratislava to Paris. (*1 point*)

3. Let $T = \{(\exists y_1)(\exists y_2)(\forall x)(x = y_1 \lor x = y_2), \neg(\exists x)(f(x) = x)\}$ be a theory in language $L = \langle f \rangle$ with equality, where f is a unary function symbol.

   (a) Is T a conservative extension of the theory $T_0 = \{(\exists y_1)(\exists y_2)(\forall x)(x = y_1 \lor x = y_2)\}$ in language $L' = \langle\rangle$ with equality? (*1 point*)

   (b) Are theories T and T_0 complete? Give an explanation. (*1 point*)