Uncertainty and Synchronization

Vojtěch Vorel

Supervised by Prof. Václav Koubek

Charles University
Prague, Czech Republic

CSA 2016

Deterministic Finite Automaton

- *DFA* is a triple $A = (Q, \Sigma, \delta)$:
  - Q ... finite set of *states*
  - Σ ... finite set of *letters* (the *alphabet*)
  - δ ... total function $Q \times \Sigma \to Q$ (*transition function*)
- Extended transition function:

$$\delta : 2^Q \times \Sigma^\star \to 2^Q$$

Deterministic Finite Automaton

- *DFA* is a triple $A = (Q, \Sigma, \delta)$:
  - Q ... finite set of *states*
  - Σ ... finite set of *letters* (the *alphabet*)
  - δ ... total function $Q \times \Sigma \to Q$ (*transition function*)
- Extended transition function:

$$\delta : 2^Q \times \Sigma^\star \to 2^Q$$

Full Reset Words

- $w \in \Sigma^\star$ is a *reset word* of A if

$$|\delta(Q, w)| = 1$$

- If A has some reset word, we call it *synchronizing*.

Full Reset Words

- $w \in \Sigma^\star$ is a *reset word* of A if

$$|\delta(Q, w)| = 1$$

- If A has some reset word, we call it *synchronizing*.

Subset Reset Words

- $w \in \Sigma^\star$ is a *reset word* of $S \subseteq Q$ if

$$|\delta(S, w)| = 1$$

- If S has some reset word, we call it *synchronizable.*

Subset Reset Words

- $w \in \Sigma^\star$ is a *reset word* of $S \subseteq Q$ if

$$|\delta(S, w)| = 1$$

- If S has some reset word, we call it *synchronizable.*

Bounds for Shortest Reset Words

$n = |Q|$

	Full Synchronization
Upper Bounds	$\mathcal{O}\left(n^3\right)$
Lower Bounds	$\Omega\left(n^2\right)$

Bounds for Shortest Reset Words

$n = |Q|$

	Full Synchronization	Subset Synchronization
Upper Bounds	$\mathcal{O}\left(n^3\right)$	
Lower Bounds	$\Omega\left(n^2\right)$	

Bounds for Shortest Reset Words

$n = |Q|$

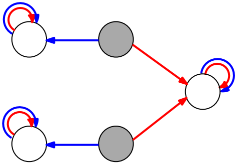
	Full Synchronization	Subset Synchronization
Upper Bounds	$\mathcal{O}(n^3)$	$2^{\mathcal{O}(n)}$
Lower Bounds	$\Omega(n^2)$	$2^{\Omega(n)}$

What is wrong with subsets

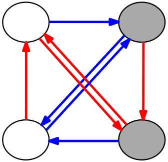
What is wrong with subsets

Greedy strategies do not work!

What is wrong with subsets



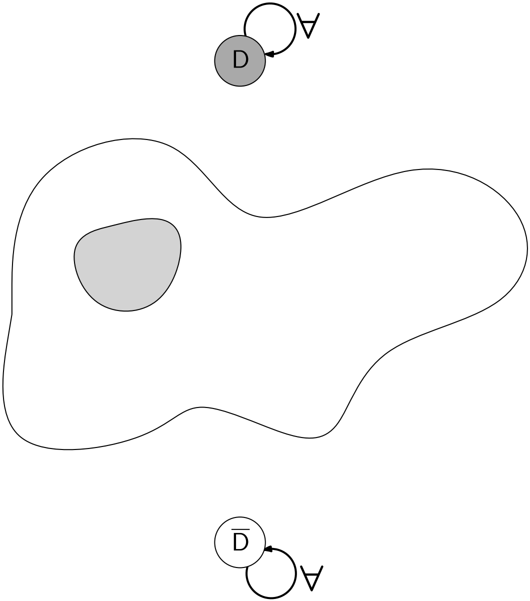
What is wrong with subsets



Bounds for Shortest Reset Words

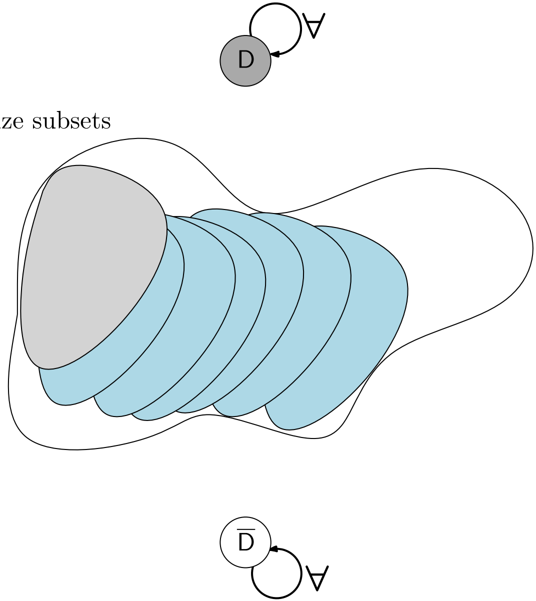
	Full Synchronization	Subset Synchronization
Upper Bounds	$\mathcal{O}(n^3)$	$2^{\mathcal{O}(n)}$
Lower Bounds	$\Omega(n^2)$	$2^{\Omega(n)}$

Proofs of the Lower Bound $2^{\Omega(n)}$



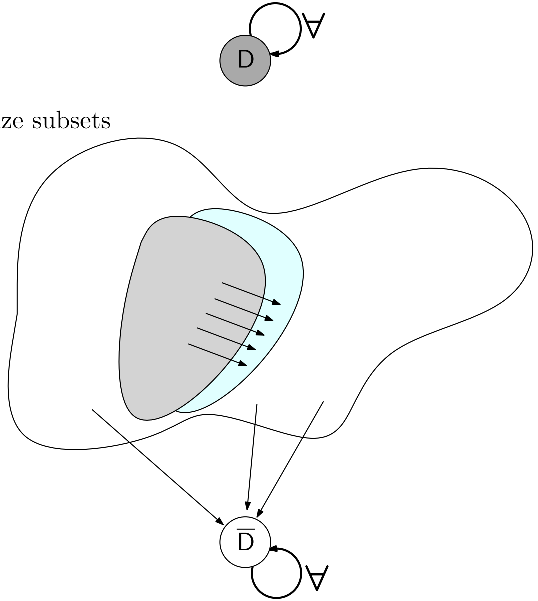
Proofs of the Lower Bound $2^{\Omega(n)}$

letters $\equiv$ half-size subsets



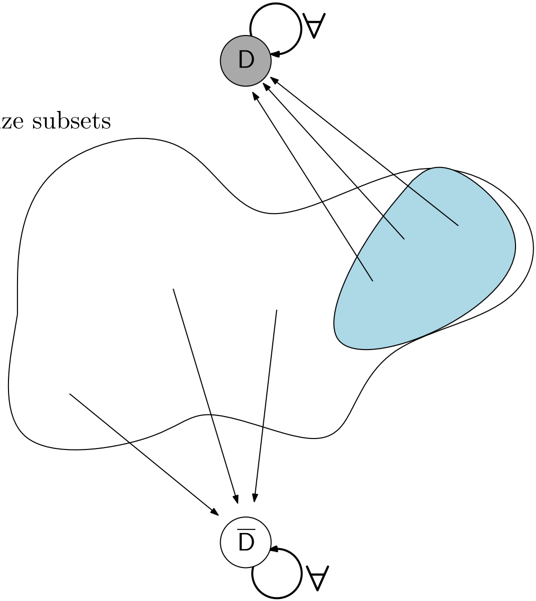
Proofs of the Lower Bound $2^{\Omega(n)}$

letters ≡ half-size subsets



Proofs of the Lower Bound $2^{\Omega(n)}$

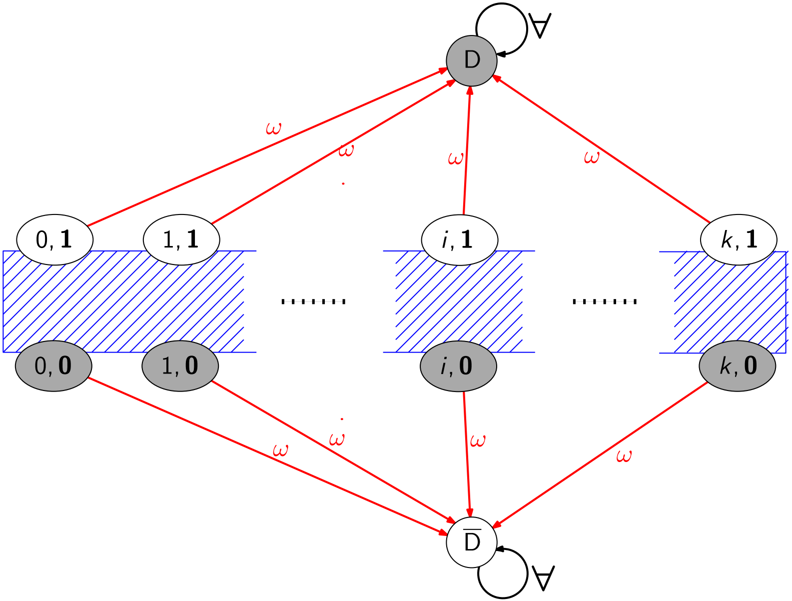
letters $\equiv$ half-size subsets



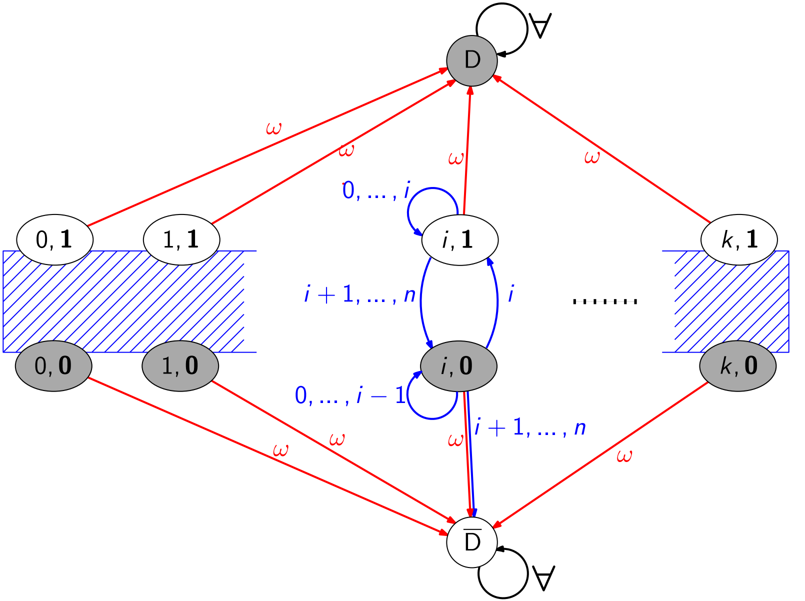
Proofs of the Lower Bound $2^{\Omega(n)}$

	alphabet size	strong connectivity	min. length of reset words
Subset listing construction	$2^{\theta(n)}$	no	$2^{\theta(n)}$

Proofs of the Lower Bound $2^{\Omega(n)}$



Proofs of the Lower Bound $2^{\Omega(n)}$



Proofs of the Lower Bound $2^{\Omega(n)}$

	alphabet size	strong connectivity	min. length of reset words
Subset listing construction	$2^{\theta(n)}$	no	$2^{\theta(n)}$
Basic radix construction	$\theta(n)$	no	$2^{\theta(n)}$

Proofs of the Lower Bound $2^{\Omega(n)}$

	alphabet size	strong connectivity	min. length of reset words
Subset listing construction	$2^{\theta(n)}$	no	$2^{\theta(n)}$
Basic radix construction	$\theta(n)$	no	$2^{\theta(n)}$

High-order permutation constr.	2	no	$2^{\theta(\sqrt[3]{n}\log n)}$

Bounds for Shortest Reset Words

	Full Synchronization	Subset Synchronization
Upper Bounds	$\mathcal{O}\left(n^3\right)$	$2^{\mathcal{O}(n)}$
Lower Bounds	$\Omega\left(n^2\right)$	$2^{\Omega(n)}$

Bounds for Shortest Reset Words

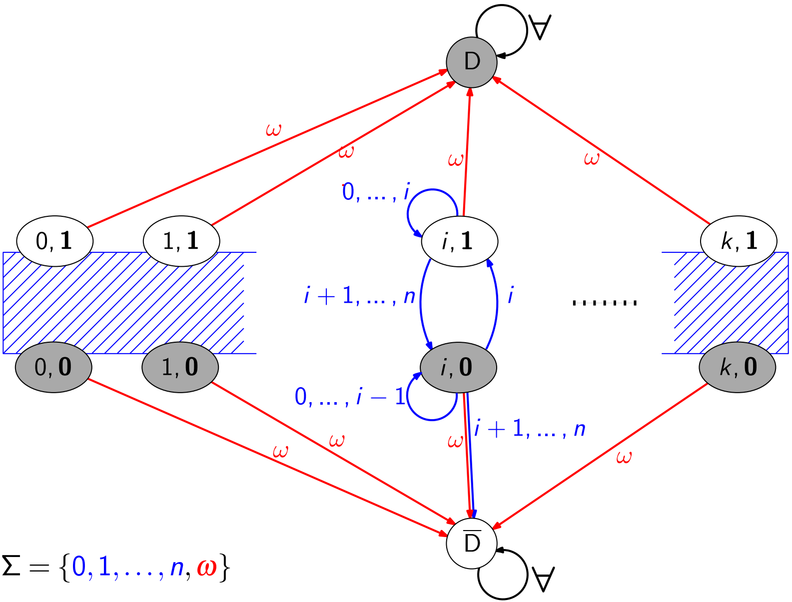
With constant-size alphabet:

	Full Synchronization	Subset Synchronization
Upper Bounds	$\mathcal{O}(n^3)$	$2^{\mathcal{O}(n)}$
Lower Bounds	$\Omega(n^2)$	

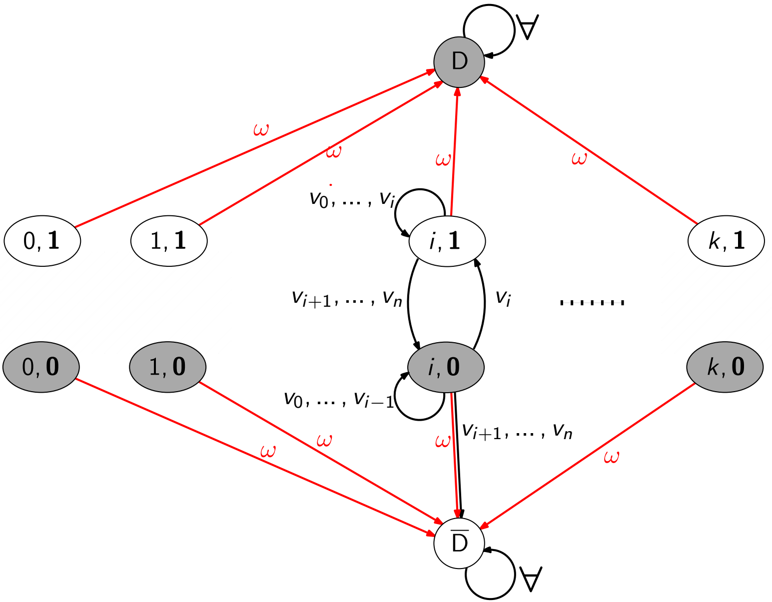
Bounds for Shortest Reset Words

With constant-size alphabet:

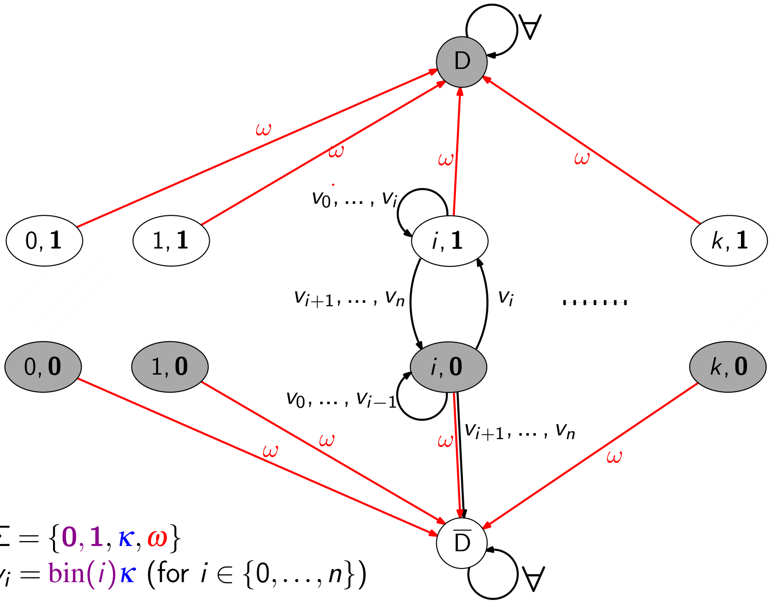
	Full Synchronization	Subset Synchronization
Upper Bounds	$\mathcal{O}(n^3)$	$2^{\mathcal{O}(n)}$
Lower Bounds	$\Omega(n^2)$	$2^{\Omega(n)}$



$\Sigma = \{0, 1, \ldots, n, \omega\}$

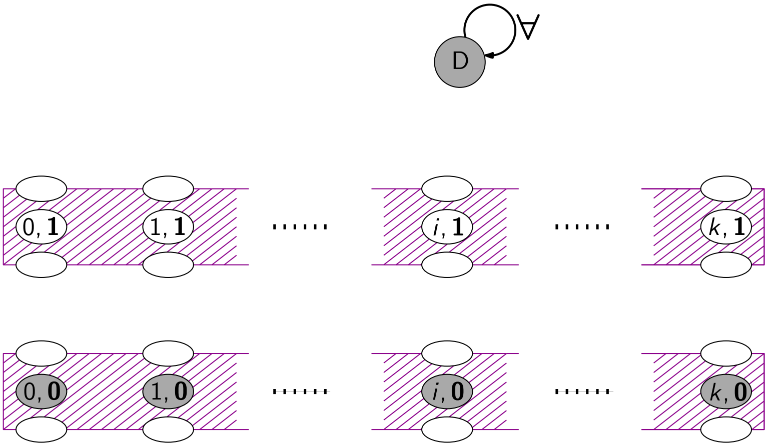

D
∀
ω
$v_0, \dots, v_i$
$0, \mathbf{1}$
$1, \mathbf{1}$
$i, \mathbf{1}$
$k, \mathbf{1}$
$v_{i+1}, \dots, v_n$
v_i
$0, \mathbf{0}$
$1, \mathbf{0}$
$i, \mathbf{0}$
$k, \mathbf{0}$
$v_0, \dots, v_{i-1}$
$v_{i+1}, \dots, v_n$
$\overline{\mathrm{D}}$
∀

	alphabet size	strong connectivity	min. length of reset words
Subset listing construction	$2^{\theta(n)}$	no	$2^{\theta(n)}$
Basic radix construction	$\theta(n)$	no	$2^{\theta(n)}$
High-order permutation constr.	2	no	$2^{\theta\left(\sqrt[3]{n}\log n\right)}$
Extended radix construction	2	no	$2^{\theta\left(\frac{n}{\log n}\right)}$



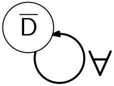
$\Sigma = \{\mathbf{0}, \mathbf{1}, \kappa, \omega\}$

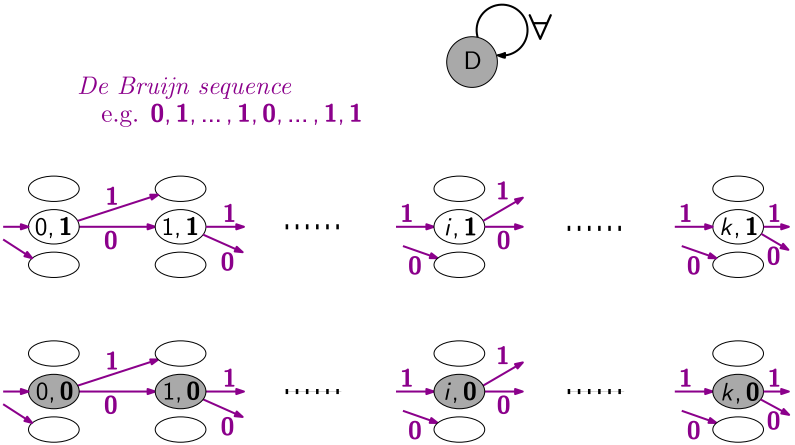
$v_i = \mathrm{bin}(i)\kappa$ (for $i \in \{0, \ldots, n\}$)



$\Sigma = \{\mathbf{0}, \mathbf{1}, \kappa, \omega\}$

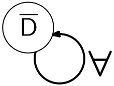
$v_i = \mathrm{bin}(i)\kappa$ (for $i \in \{0, \ldots, n\}$)

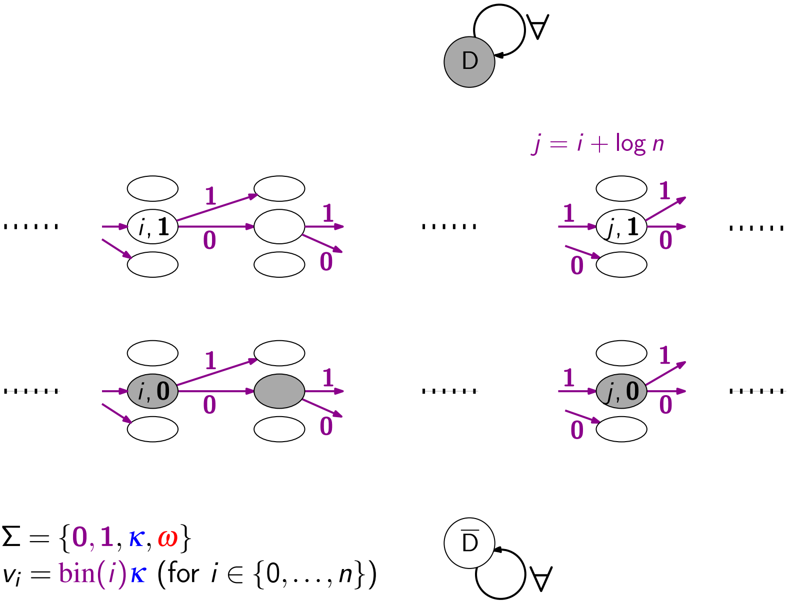




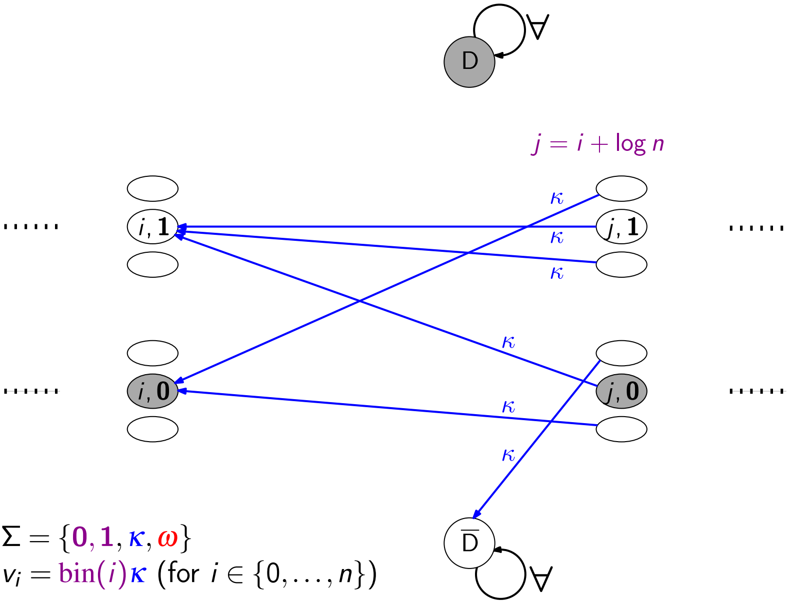
$\Sigma = \{0, 1, \kappa, \omega\}$

$v_i = \mathrm{bin}(i)\kappa$ (for $i \in \{0, \ldots, n\}$)



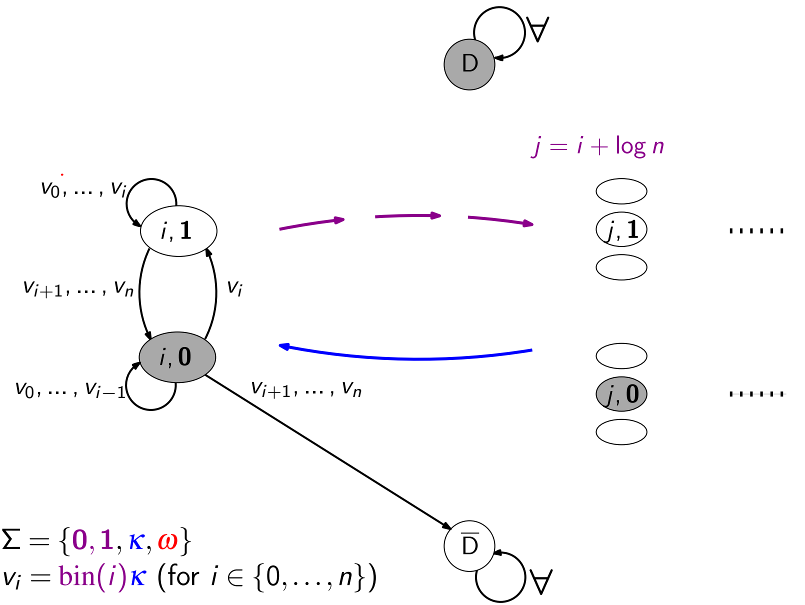


$\Sigma = \{\mathbf{0}, \mathbf{1}, \kappa, \omega\}$
$v_i = \mathrm{bin}(i)\kappa$ (for $i \in \{0, \ldots, n\}$)



$\Sigma = \{\mathbf{0}, \mathbf{1}, \kappa, \omega\}$

$v_i = \mathrm{bin}(i)\kappa$ (for $i \in \{0, \ldots, n\}$)



$\Sigma = \{\mathbf{0}, \mathbf{1}, \kappa, \omega\}$

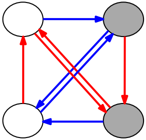
$v_i = \mathrm{bin}(i)\kappa$ (for $i \in \{0, \ldots, n\}$)

Proofs of the Lower Bound $2^{\Omega(n)}$

	alphabet size	strong connectivity	min. length of reset words
Subset listing construction	$2^{\theta(n)}$	no	$2^{\theta(n)}$
Basic radix construction	$\theta(n)$	no	$2^{\theta(n)}$
High-order permutation constr.	2	no	$2^{\theta(\sqrt[3]{n}\log n)}$
Extended radix construction	2	no	$2^{\theta(\frac{n}{\log n})}$
New radix construction	2	no	$2^{\theta(n)}$

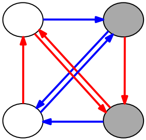
Strong Connectivity

A general technique using ***swap congruences***.



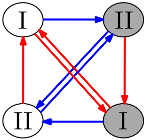
Strong Connectivity

A general technique using ***swap congruences***.



Strong Connectivity

A general technique using ***swap congruences***.



Proofs of the Lower Bound $2^{\Omega(n)}$

	alphabet size	strong connectivity	min. length of reset words
Subset listing construction	$2^{\theta(n)}$	no	$2^{\theta(n)}$
Basic radix construction	$\theta(n)$	no	$2^{\theta(n)}$
High-order permutation constr.	2	no	$2^{\theta(\sqrt[3]{n}\log n)}$
Extended radix construction	2	no	$2^{\theta(\frac{n}{\log n})}$
New radix construction	2	no	$2^{\theta(n)}$
New radix constr. + swapping	2	yes	$2^{\theta(n)}$

Avoiding Words

- $w \in \Sigma^\star$ *avoids* $q \in Q$ if

$$q \notin \delta(Q, w)$$

Bounds for Shortest Avoiding Words

	Synchronizing DFA	General DFA
Upper Bounds	$\mathcal{O}(n^3)$	$2^{\mathcal{O}(n)}$
Lower Bounds	$2n-4$	$2n-4$

Avoiding -> Short Full Reset Words

- Known upper bounds on shortest full reset words:
  - $\frac{1}{3}n^3 + \mathcal{O}(n^2)$ (Kohavi, 1970)
  - $\frac{1}{6}n^3 + \mathcal{O}(n^2)$ (Pin, 1983)
  - $\frac{7}{48}n^3 + \mathcal{O}(n^2)$ if $\mathcal{O}(n)$ for avoiding

Thank you for your attention!