Winter road maintenance in the Czech Republic

Jiří Fink

Department of Theoretical Computer Science and Mathematical Logic
Faculty of Mathematics and Physics
Charles University in Prague

April 29, 2025

Content

Winter road maintenance in the Czech Republic

Create plans for maintenance vehicles removing show from roads.

Content

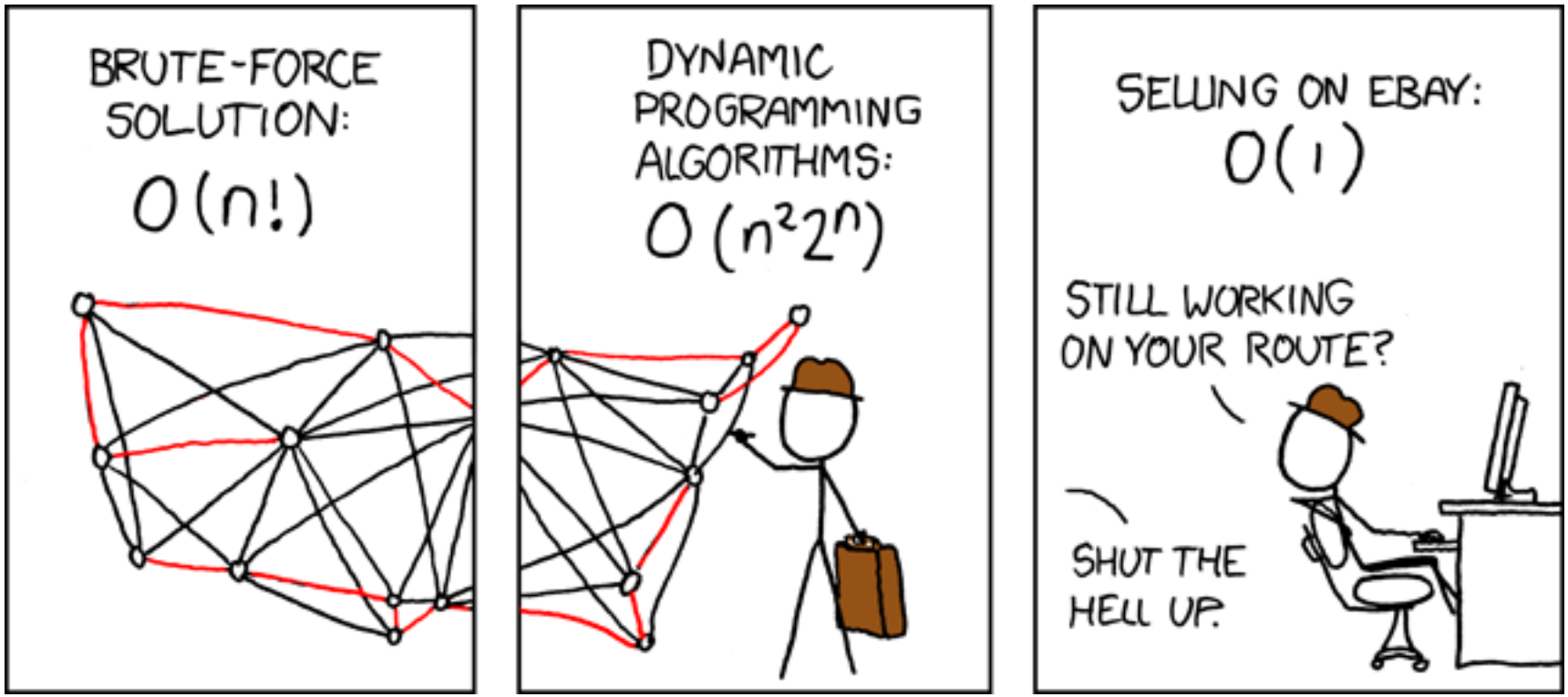
Winter road maintenance in the Czech Republic

Create plans for maintenance vehicles removing show from roads.

Table of content

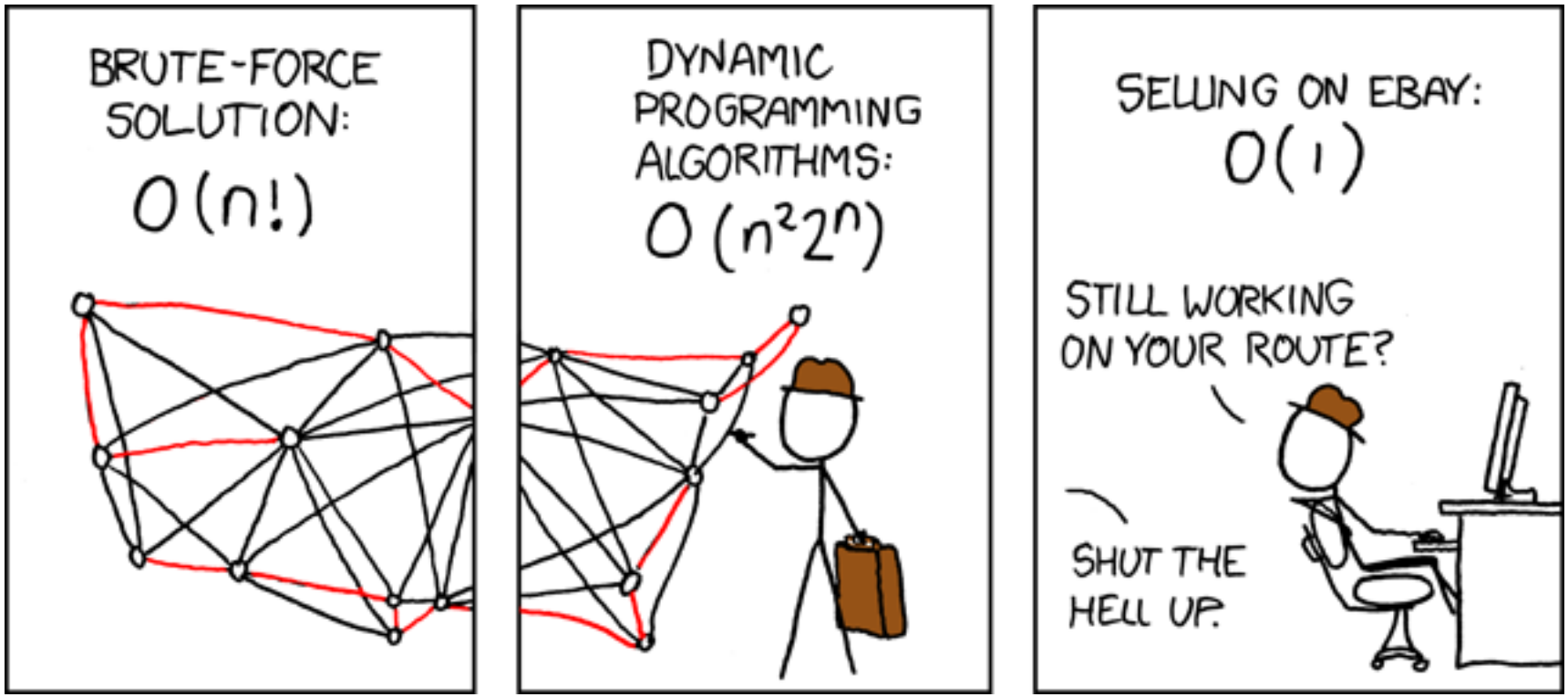
- Related problems
- Practical motivation
- Problem description
- Algorithms
- Winter road maintenance
- Results in Plzeň Region

Travelling Salesman Problem



Source: `https://xkcd.com/399/`

Travelling Salesman Problem

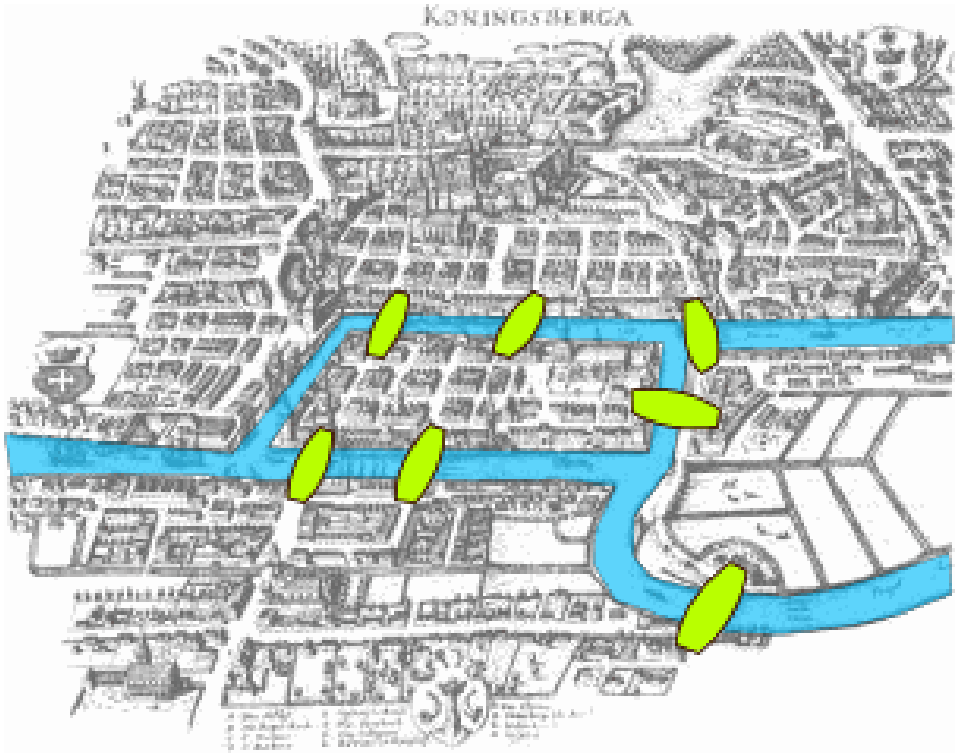


Source: https://xkcd.com/399/

Capacitated Vehicle Routing Problem

- Packages of given weights have to be delivered from a depot.
- Cars have limited capacity and have to return to the depot.
- The goal is to minimize the total length of all routes.

Seven Bridges of Königsberg (Euler, 1736)



Source: `https://en.wikipedia.org/wiki/Seven_Bridges_of_Konigsberg`

Capacitated Arc Routing Problem (Golden, Wong, 1981)

Input

- A graph (V, E)
- A depot $g \in V$
- Demand d_e and traversing cost c_e for every edge $e \in E$
- A set C of cars with capacity K

Capacitated Arc Routing Problem (Golden, Wong, 1981)

Input

- A graph (V, E)
- A depot $g \in V$
- Demand d_e and traversing cost c_e for every edge $e \in E$
- A set C of cars with capacity K

Feasible solutions

- Every car has assigned a closed tour starting at the depot
- Every edge is serviced by one car
- The total demand of serviced edges of each car is at most car's capacity

Capacitated Arc Routing Problem (Golden, Wong, 1981)

Input

- A graph (V, E)
- A depot $g \in V$
- Demand d_e and traversing cost c_e for every edge $e \in E$
- A set C of cars with capacity K

Feasible solutions

- Every car has assigned a closed tour starting at the depot
- Every edge is serviced by one car
- The total demand of serviced edges of each car is at most car's capacity

Objective

Minimize the total cost which is the sum of all costs of traversed edges over all cars

Practical motivation of our problem

- Postal and Newspaper Delivery

Practical motivation of our problem

- Postal and Newspaper Delivery
- Waste Collection

Practical motivation of our problem

- Postal and Newspaper Delivery
- Waste Collection
- Road Cleaning and Marking

Practical motivation of our problem

- Postal and Newspaper Delivery
- Waste Collection
- Road Cleaning and Marking
- Money Collection

Practical motivation of our problem

- Postal and Newspaper Delivery
- Waste Collection
- Road Cleaning and Marking
- Money Collection
- Security Patrol Routing

Practical motivation of our problem

- Postal and Newspaper Delivery
- Waste Collection
- Road Cleaning and Marking
- Money Collection
- Security Patrol Routing
- Outdoor Activities

Practical motivation of our problem

- Postal and Newspaper Delivery
- Waste Collection
- Road Cleaning and Marking
- Money Collection
- Security Patrol Routing
- Outdoor Activities
- Post-Disaster Operations

Practical motivation of our problem

- Postal and Newspaper Delivery
- Waste Collection
- Road Cleaning and Marking
- Money Collection
- Security Patrol Routing
- Outdoor Activities
- Post-Disaster Operations

Source: `https://www.tsk-praha.cz`

Variants of Routing Problems (RP)

- Time Windows RP
  Service can be provided only during given time constraints

Variants of Routing Problems (RP)

- Time Windows RP
  Service can be provided only during given time constraints
- Split Delivery RP Demand can be serviced by several vehicles

Variants of Routing Problems (RP)

- Time Windows RP
  Service can be provided only during given time constraints
- Split Delivery RP Demand can be serviced by several vehicles
- Location RP
  Consider both location of facilities and route planning in one problem

Variants of Routing Problems (RP)

- Time Windows RP
  Service can be provided only during given time constraints
- Split Delivery RP Demand can be serviced by several vehicles
- Location RP
  Consider both location of facilities and route planning in one problem
- Periodic RP
  Repeated service is required

Variants of Routing Problems (RP)

- Time Windows RP
  Service can be provided only during given time constraints
- Split Delivery RP Demand can be serviced by several vehicles
- Location RP
  Consider both location of facilities and route planning in one problem
- Periodic RP
  Repeated service is required
- Mixed RP
  Road may be one-way, two-way or mixed

Variants of Routing Problems (RP)

- Time Windows RP
  Service can be provided only during given time constraints
- Split Delivery RP Demand can be serviced by several vehicles
- Location RP
  Consider both location of facilities and route planning in one problem
- Periodic RP
  Repeated service is required
- Mixed RP
  Road may be one-way, two-way or mixed
- Windy RP
  Traversing cost depends on traversing direction

Variants of Routing Problems (RP)

- Time Windows RP
  Service can be provided only during given time constraints
- Split Delivery RP Demand can be serviced by several vehicles
- Location RP
  Consider both location of facilities and route planning in one problem
- Periodic RP
  Repeated service is required
- Mixed RP
  Road may be one-way, two-way or mixed
- Windy RP
  Traversing cost depends on traversing direction
- Time-Dependent RP
  Cost or demand may depends on time

Variants of Routing Problems (RP)

- Time Windows RP
  Service can be provided only during given time constraints
- Split Delivery RP Demand can be serviced by several vehicles
- Location RP
  Consider both location of facilities and route planning in one problem
- Periodic RP
  Repeated service is required
- Mixed RP
  Road may be one-way, two-way or mixed
- Windy RP
  Traversing cost depends on traversing direction
- Time-Dependent RP
  Cost or demand may depends on time
- Stochastic and Dynamic Routing Problems

Variants of Routing Problems (RP)

- Time Windows RP
  Service can be provided only during given time constraints
- Split Delivery RP Demand can be serviced by several vehicles
- Location RP
  Consider both location of facilities and route planning in one problem
- Periodic RP
  Repeated service is required
- Mixed RP
  Road may be one-way, two-way or mixed
- Windy RP
  Traversing cost depends on traversing direction
- Time-Dependent RP
  Cost or demand may depends on time
- Stochastic and Dynamic Routing Problems
- VeRoLog Solver Challenge, Kaggle, . . .

Overview of methods

Complexity

Capacitated Arc Routing Problem is strongly NP-hard. Related problems are

- Knapsack, Bin Packing, 3-partitioning, and
- Steiner Tree Problem.

Overview of methods

Complexity

Capacitated Arc Routing Problem is strongly NP-hard. Related problems are

- Knapsack, Bin Packing, 3-partitioning, and
- Steiner Tree Problem.

Theorem (Golden, Wong, 1981)

$\frac{3}{2}$-approximation is NP-hard even if all demands are equal to one.

Overview of methods

Complexity

Capacitated Arc Routing Problem is strongly NP-hard. Related problems are

- Knapsack, Bin Packing, 3-partitioning, and
- Steiner Tree Problem.

Theorem (Golden, Wong, 1981)

$\frac{3}{2}$-approximation is NP-hard even if all demands are equal to one.

Exact algorithm and lower bounds

- Integer linear programming
- Cutting planes
- Column generation
- Lagrangian relaxation
- Branch-Price-and-Cut Algorithms

Heuristic algorithms for Capacitated Arc Routing Problem

- Golden, DeArmon, Baker (1982)
  A greedy path-scanning heuristic

Heuristic algorithms for Capacitated Arc Routing Problem

- Golden, DeArmon, Baker (1982)
  A greedy path-scanning heuristic
- Brandão and Eglese (2008)
  A tabu search algorithm

Heuristic algorithms for Capacitated Arc Routing Problem

- Golden, DeArmon, Baker (1982)
  A greedy path-scanning heuristic
- Brandão and Eglese (2008)
  A tabu search algorithm
- Morgan and Mumford (2010)
  A weight coded genetic algorithm

Heuristic algorithms for Capacitated Arc Routing Problem

- Golden, DeArmon, Baker (1982)
  A greedy path-scanning heuristic
- Brandão and Eglese (2008)
  A tabu search algorithm
- Morgan and Mumford (2010)
  A weight coded genetic algorithm
- Mei, Tang, Yao (2011)
  A memetic algorithm

Heuristic algorithms for Capacitated Arc Routing Problem

- Golden, DeArmon, Baker (1982)
  A greedy path-scanning heuristic
- Brandão and Eglese (2008)
  A tabu search algorithm
- Morgan and Mumford (2010)
  A weight coded genetic algorithm
- Mei, Tang, Yao (2011)
  A memetic algorithm
- Chen, Hao, Glover (2016)
  A hybrid metaheuristic algorithms

Heuristic algorithms for Capacitated Arc Routing Problem

- Golden, DeArmon, Baker (1982)
  A greedy path-scanning heuristic
- Brandão and Eglese (2008)
  A tabu search algorithm
- Morgan and Mumford (2010)
  A weight coded genetic algorithm
- Mei, Tang, Yao (2011)
  A memetic algorithm
- Chen, Hao, Glover (2016)
  A hybrid metaheuristic algorithms
- Bensedira, Layeb (2017)
  A cuckoo search algorithm

Heuristic algorithms for Capacitated Arc Routing Problem

- Golden, DeArmon, Baker (1982)
  A greedy path-scanning heuristic
- Brandão and Eglese (2008)
  A tabu search algorithm
- Morgan and Mumford (2010)
  A weight coded genetic algorithm
- Mei, Tang, Yao (2011)
  A memetic algorithm
- Chen, Hao, Glover (2016)
  A hybrid metaheuristic algorithms
- Bensedira, Layeb (2017)
  A cuckoo search algorithm
- Vidal (2017)
  An algorithm based on a structural neighborhood decomposition

Heuristic algorithms for Capacitated Arc Routing Problem

- Golden, DeArmon, Baker (1982)
  A greedy path-scanning heuristic
- Brandão and Eglese (2008)
  A tabu search algorithm
- Morgan and Mumford (2010)
  A weight coded genetic algorithm
- Mei, Tang, Yao (2011)
  A memetic algorithm
- Chen, Hao, Glover (2016)
  A hybrid metaheuristic algorithms
- Bensedira, Layeb (2017)
  A cuckoo search algorithm
- Vidal (2017)
  An algorithm based on a structural neighborhood decomposition
- Ting and Tsai (2018)
  Ant Colony Optimization with Path Relinking

Heuristic algorithms for Capacitated Arc Routing Problem

- Golden, DeArmon, Baker (1982)
  A greedy path-scanning heuristic
- Brandão and Eglese (2008)
  A tabu search algorithm
- Morgan and Mumford (2010)
  A weight coded genetic algorithm
- Mei, Tang, Yao (2011)
  A memetic algorithm
- Chen, Hao, Glover (2016)
  A hybrid metaheuristic algorithms
- Bensedira, Layeb (2017)
  A cuckoo search algorithm
- Vidal (2017)
  An algorithm based on a structural neighborhood decomposition
- Ting and Tsai (2018)
  Ant Colony Optimization with Path Relinking
- Arakaki and Usberti (2019)
  A path-scanning heuristic

Recent trends in CARP

- Corberán, Eglese, Hasle, Plana, Sanchis (2020)
  Arc routing problems: A review of the past, present, and future

Recent trends in CARP

- Corberán, Eglese, Hasle, Plana, Sanchis (2020)
  Arc routing problems: A review of the past, present, and future
- Liu, Tang, Yao (2021)
  Robust Optimization in Uncertain CARP (survey)

Recent trends in CARP

- Corberán, Eglese, Hasle, Plana, Sanchis (2020)
  Arc routing problems: A review of the past, present, and future
- Liu, Tang, Yao (2021)
  Robust Optimization in Uncertain CARP (survey)
- Wei, Sanne Wøhlk, Ada Che (2024)
  A multi-level capacitated arc routing problem with intermediate facilities in waste collection

Recent trends in CARP

- Corberán, Eglese, Hasle, Plana, Sanchis (2020)
  Arc routing problems: A review of the past, present, and future
- Liu, Tang, Yao (2021)
  Robust Optimization in Uncertain CARP (survey)
- Wei, Sanne Wøhlk, Ada Che (2024)
  A multi-level capacitated arc routing problem with intermediate facilities in waste collection
- Cao, Zhang, Wang, Zhao, Gu, Zhang (2021)
  A memetic algorithm based on two_Arch2 for multi-depot heterogeneous-vehicle capacitated arc routing problem

Recent trends in CARP

- Corberán, Eglese, Hasle, Plana, Sanchis (2020)
  Arc routing problems: A review of the past, present, and future
- Liu, Tang, Yao (2021)
  Robust Optimization in Uncertain CARP (survey)
- Wei, Sanne Wøhlk, Ada Che (2024)
  A multi-level capacitated arc routing problem with intermediate facilities in waste collection
- Cao, Zhang, Wang, Zhao, Gu, Zhang (2021)
  A memetic algorithm based on two_Arch2 for multi-depot heterogeneous-vehicle capacitated arc routing problem
- Wang, Mei, Zhang, Yao (2021)
  Genetic Programming With Niching for Uncertain Capacitated Arc Routing Problem

Recent trends in CARP

- Corberán, Eglese, Hasle, Plana, Sanchis (2020)
  Arc routing problems: A review of the past, present, and future
- Liu, Tang, Yao (2021)
  Robust Optimization in Uncertain CARP (survey)
- Wei, Sanne Wøhlk, Ada Che (2024)
  A multi-level capacitated arc routing problem with intermediate facilities in waste collection
- Cao, Zhang, Wang, Zhao, Gu, Zhang (2021)
  A memetic algorithm based on two_Arch2 for multi-depot heterogeneous-vehicle capacitated arc routing problem
- Wang, Mei, Zhang, Yao (2021)
  Genetic Programming With Niching for Uncertain Capacitated Arc Routing Problem
- 7560 research papers about CARP since 2021

Variant of Capacitated Arc Routing Problem

- Every road has to be serviced in both direction by one car

Variant of Capacitated Arc Routing Problem

- Every road has to be serviced in both direction by one car
- Multiple depots

Variant of Capacitated Arc Routing Problem

- Every road has to be serviced in both direction by one car
- Multiple depots
- Chemical, inert and snowplow service

Variant of Capacitated Arc Routing Problem

- Every road has to be serviced in both direction by one car
- Multiple depots
- Chemical, inert and snowplow service
- Different road priorities

Winter road maintenance in the Czech Republic

Variant of Capacitated Arc Routing Problem

- Every road has to be serviced in both direction by one car
- Multiple depots
- Chemical, inert and snowplow service
- Different road priorities
- Cars cannot make U-turn at some crossroads

Winter road maintenance in the Czech Republic

Variant of Capacitated Arc Routing Problem

- Every road has to be serviced in both direction by one car
- Multiple depots
- Chemical, inert and snowplow service
- Different road priorities
- Cars cannot make U-turn at some crossroads
- More constraints at mountains
- . . .

Winter road maintenance in the Czech Republic

Variant of Capacitated Arc Routing Problem

- Every road has to be serviced in both direction by one car
- Multiple depots
- Chemical, inert and snowplow service
- Different road priorities
- Cars cannot make U-turn at some crossroads
- More constraints at mountains
- . . .

Objectives

- Maximize social happiness
- Minimize number of cars
- Minimize deadheads

Winter road maintenance in the Czech Republic

Variant of Capacitated Arc Routing Problem

- Every road has to be serviced in both direction by one car
- Multiple depots
- Chemical, inert and snowplow service
- Different road priorities
- Cars cannot make U-turn at some crossroads
- More constraints at mountains
- . . .

Objectives

- Maximize social happiness
- Minimize number of cars
- Minimize deadheads

The most difficult step in practice

Obtain all data

Every road has to be serviced in both directions

Observation

Every connected graph contain a tour traversing every edge in both direction exactly once.

Every road has to be serviced in both directions

Observation

Every connected graph contain a tour traversing every edge in both direction exactly once.

Proof

- The statement holds for trees
- For general graph, use a spanning tree

Every road has to be serviced in both directions

Observation

Every connected graph contain a tour traversing every edge in both direction exactly once.

Proof

- The statement holds for trees
- For general graph, use a spanning tree

Goal

- Cover edges by connected subgraphs
- Every subgraph contains a depot
- Every subgraph has limited length of edges

Greedy algorithm

- Used to create an initial solution
- Choose an uncovered edge that is the most distant from any covered edge
- Attach the edge to the closest subgraph with a sufficient remaining capacity
- Create a new subgraph in the closest depot if necessary

Variants of local search

- **Local search** algorithms move from solution to solution by applying local changes, until a solution deemed optimal is found or a time bound is elapsed.
- For a **very large-scale neighborhood search**, the neighborhood is large and its needs to be explored systematically.
- **Problem size reduction methods** optimally solves some subproblem while the rest of solution is preserved.

Local search and knapsack

Variants of local search

- **Local search** algorithms move from solution to solution by applying local changes, until a solution deemed optimal is found or a time bound is elapsed.
- For a **very large-scale neighborhood search**, the neighborhood is large and its needs to be explored systematically.
- **Problem size reduction methods** optimally solves some subproblem while the rest of solution is preserved.

Knapsack

- Every subgraph is stored as a tree.
- Two trees sharing a common vertex u can exchange subtrees.
- For two trees, consider all common vertices and all combinations of subtrees.
- Using knapsack, minimize the size of one tree while preserving the capacity of the second tree.
- If two trees are small enough, merge them into one.

Case study: Plzeň region

Input size

- 1719 vertices
- 2280 edges
- 34 depots

Case study: Plzeň region

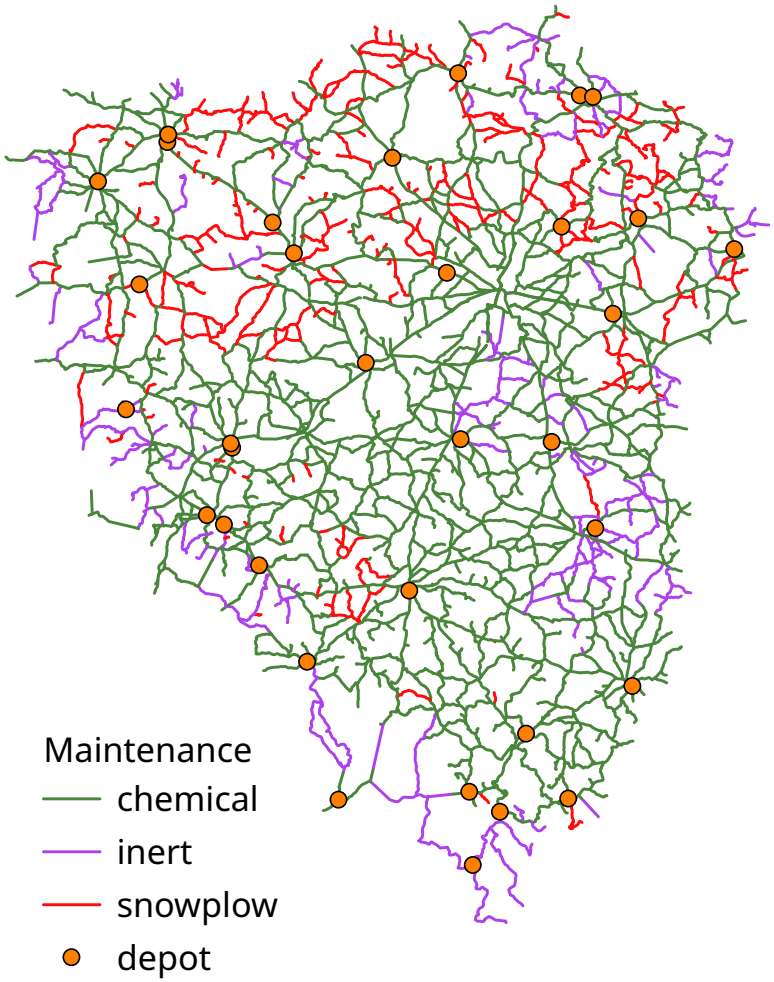
Input size

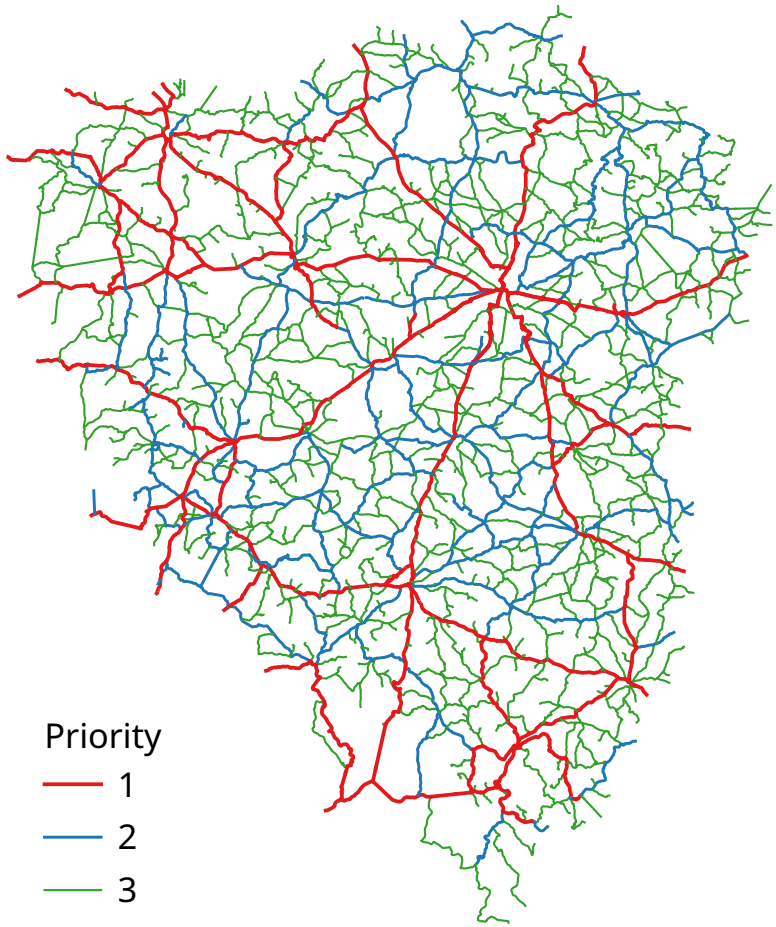
- 1719 vertices
- 2280 edges
- 34 depots

Length of roads in kilometers

maintenance/priority	1	2	3	**sum**
chemical	840	1080	1472	3392
inert	42	65	569	676
snowplow	8	15	724	747
sum	**890**	**1160**	**2765**	**4815**

Plzeň region





Results (Fink, Loebl, Pelikánová)

Number of cars

	Original solution 2018/19	Our solution	Applied solution 2019/20
Chemical	80	78	80
Inert	34	15	22
Total	114	93	102

Results (Fink, Loebl, Pelikánová)

Number of cars

	Original solution 2018/19	Our solution	Applied solution 2019/20
Chemical	80	78	80
Inert	34	15	22
Total	114	93	102

Deadheads [% of total length of all tours]

	Original solution 2018/19	Our solution	Applied solution 2019/20
Chemical	27.6	2.2	12
Inert	30.6	15.2	30.8
Total	29.5	4.3	16.6

Results (Fink, Loebl, Pelikánová)

Number of cars

	Original solution 2018/19	Our solution	Applied solution 2019/20
Chemical	80	78	80
Inert	34	15	22
Total	114	93	102

Deadheads [% of total length of all tours]

	Original solution 2018/19	Our solution	Applied solution 2019/20
Chemical	27.6	2.2	12
Inert	30.6	15.2	30.8
Total	29.5	4.3	16.6

Improvements

- Number of cars decreased by 12, i.e. 36 drivers, 15 mil CZK per year on salaries

Results (Fink, Loebl, Pelikánová)

Number of cars

	Original solution 2018/19	Our solution	Applied solution 2019/20
Chemical	80	78	80
Inert	34	15	22
Total	114	93	102

Deadheads [% of total length of all tours]

	Original solution 2018/19	Our solution	Applied solution 2019/20
Chemical	27.6	2.2	12
Inert	30.6	15.2	30.8
Total	29.5	4.3	16.6

Improvements

- Number of cars decreased by 12, i.e. 36 drivers, 15 mil CZK per year on salaries
- Deadheads decreased by 1926 km per one shift, i.e. significant CO_2 reduction

Results (Fink, Loebl, Pelikánová)

Number of cars

	Original solution 2018/19	Our solution	Applied solution 2019/20
Chemical	80	78	80
Inert	34	15	22
Total	114	93	102

Deadheads [% of total length of all tours]

	Original solution 2018/19	Our solution	Applied solution 2019/20
Chemical	27.6	2.2	12
Inert	30.6	15.2	30.8
Total	29.5	4.3	16.6

Improvements

- Number of cars decreased by 12, i.e. 36 drivers, 15 mil CZK per year on salaries
- Deadheads decreased by 1926 km per one shift, i.e. significant CO_2 reduction
- Average length of a tour decreased from 119.8 km to 115 km

Results (Fink, Loebl, Pelikánová)

Number of cars

	Original solution 2018/19	Our solution	Applied solution 2019/20
Chemical	80	78	80
Inert	34	15	22
Total	114	93	102

Deadheads [% of total length of all tours]

	Original solution 2018/19	Our solution	Applied solution 2019/20
Chemical	27.6	2.2	12
Inert	30.6	15.2	30.8
Total	29.5	4.3	16.6

Improvements

- Number of cars decreased by 12, i.e. 36 drivers, 15 mil CZK per year on salaries
- Deadheads decreased by 1926 km per one shift, i.e. significant CO_2 reduction
- Average length of a tour decreased from 119.8 km to 115 km
- Length of the longest tour decreased from 228.3 km to 187.2 km