Mathematical modelling of devices and flows in energy systems*

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Abstract

In the future of Smart Grids, many different devices have to be integrated into one overall system. These devices are of various types, they are related to various forms of energy, they have to exchange these different forms of energy, and together they have to work towards common objectives. To handle such a system, a general model is required, which allows the integration of all these different devices, energy forms and objectives and which is able to incorporate also future developments for which the concrete setting can not be foreseen yet.

In this paper, we present such a flexible model which handles different types of devices and which supports arbitrary ways of energy flows between devices. Instead of considering complex operations of devices we introduce simple basic devices which can be easily combined to obtain advanced functionality. The flexibility of the resulting model is demonstrated by considering some advanced devices but also by giving a reduction between Binary Linear Programming and the proposed model where variables and constrains are in a one-to-one correspondence to devices and energy flows.

1 Introduction

Modern society is substantially depending on energy. Humans are consuming energy during lots of their activities: Lighting is used to see our work or entertainment, heating or cooling is used to keep us in a comfort environment and other energy is used for communication or transportation. The energy needed for these activities has various forms, e.g. electricity, gas and hot water and in the past this energy was produced mainly in large plants far away from the consumption whereby the production was to a large extend based on fossil fuels. Recent developments and environmental needs have led to a fundamental change of the system. Nowadays more and more energy is generated based on renewable sources and the generation is partly done by small units near the consumption; e.g. PV-panels on roofs of family houses.

The mentioned changes of the energy system lead to new challenges as now a larger part of the energy generation is based on non-controllable sources (e.g. wind and sun). To compensate for this reduced flexibility on the production side, new occurring flexibilities on the consumer side need to be taken into account. This process is called demand side management (DSM) and several DSM approaches have been proposed in the literature (see e.g. [20, 4, 14, 21, 9]). Next to these changes in flexibility, in the future also the number of relevant devices for generating, consuming, storing and converting of energy will increase a lot. To deal with these new developments, the concept of Smart Grids is seen as an essential building block to support efficient control of producing, transporting and consuming energy in the future (for definitions and further information on Smart Grids, see e.g. [1, 2]). The main goal of Smart Grids is to allow a screening and control of the energy flow and the involved consuming and producing devices in order to realize a matching of supply and demand and to stabilize the grid. This implies that many different devices have to be integrated, whereby these devices may be quite different from nature, are related to various forms

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of energy, exchange these different forms of energy with each other and with the grid, and have to operate taking into account operational restrictions and objectives. To set up such a system, an overall model for Smart Grids is needed. This model has to support the integration of all sorts of different devices, energy forms and objectives whereby at the time the model is developed not even all concrete devices, energy forms and objectives are known. This asks for a general and flexible model.

In this paper we develop such a general and flexible mathematical model which describes flows of different forms of energy between various types of devices. The model has two major elements: devices and flows of energy. On the device side it is clear that the difference in the functionality of the devices may ask for different mathematical models. However, it is not a good choice to introduce for each new individual device a detailed model. For this, we have chosen to introduce more general classes of devices that can produce, transport, consume or convert different types of energy and may even be able to do several of these aspects simultaneously. The second important element within Smart Grids is that energy is exchanged between devices; e.g. one device can produce energy, which is then transported by another devices and finally is consumed by yet another device. These energy exchanges can create very complex relations between devices. Therefore, a flexible model is necessary to capture the complexity of interconnections of different devices.

At this point, it should be noticed that for the applicability of a general model it is important not to consider all technical details, but to focus on the main aspects which are needed for an efficient control in Smart Grids. Furthermore, the model should allow to focus only on that part of the grid, which is e.g. relevant for the considered particular case. For example, determining every single cable in a building is unnecessary as long as transportation losses can be estimated, or the energy sources used by a heat pump may be important for some studies and irrelevant for others. Therefore, we need a model which allows to abstract from engineering details but can be flexibly adopted for different cases.

Many of the papers found in the literature do not consider such flexibility in modelling. Some papers concentrate only on a single type of device (e.g. a heat pump in [7], a combined heat and power generator in [5], a single flexible smart device in [16], or an appliance with fixed total electricity demand [22]). Other papers have flexibility in types of devices but they are restricted to a single form of energy which is distributed in a treelike structure (see e.g. [15, 20]). Another example of a restricted approach is given in [18] where load shifting is studied by shifting given consumption profiles to reach a desired total profile, but where e.g. storage devices are not considered. Furthermore, a lot of papers study very specific subjects; e.g., using energy from photovoltaic panels for charging electrical vehicles [26].

In literature, different concepts for Smart Grids and technical demonstration are studied (see e.g. [13] for a survey). Furthermore, many software programs were implemented to monitor and measure production, transmission and consumption resources, to calculate the impact of different Smart Grid investments and to test communication standards (see e.g. [24]). The algorithms supporting Smart Grid, in general rely on a central controller (see e.g. [15, 20]) or are completely distributed and independent (e.g. frequency and voltage response systems [23]). In contrary to these more technically oriented approaches, we are looking for a mathematical description of devices and energy flows. Although this paper mainly concern about house devices, our model can easily be extended for the industrial sector where demand side management is also an important topic (see e.g. [17]).

The underlying basic idea of the model presented in this paper was already introduced by Molderink [19]. Here, we extend that basic concept to a general model. Hereby we limit ourselves by modelling most properties of devices using linear constraints. This allows us to use Mixed Integer Linear Programming solvers to find optimal or close to optimal solutions. The main purpose of the paper is to discuss how devices and energy flows between them can be modelled in a quite general form and how these models can be used in various study cases and be the base for algorithms to control Smart Grids. As starting point for our modelling approach we use some simple elementary types of devices and show how they can be extended and combined to model all sort of advanced devices. Furthermore, we show that although the basic devices are quite simple and intuitive, they are quite powerful. More precisely, we show that our model is at least as expressive as Binary Linear Programming.

The concept presented in this paper may have various applications. First, the presented model can form the base of software programs designed to simulate or control the behaviour of smart devices in future Smart Grids. We see also based on our cooperations with industrial partners the rich that commercial companies might prefer chose a different approach and implement a huge device library with plenty of parameters and constrains to model all sorts of existing appliances. Creating such a device library is time consuming and also not easy to be maintained. Furthermore, these is a high risk that such specific implementation hidden the integration of new such of devices which may occurs in the future. Instead, it is more convenient to implement only a small but flexible set of devices with basic properties and then provide the user the opportunity to model more complex scenarios as combinations of basic devices as presented in Section 4. A second application is more of education nature. The presented approach can be used to explain non-OR specialists and students the power of a good mathematical modelling, especially that of Mix Integer Linear Programming (MILP), in the context of Smart Grids.

It may be the case that in some study cases, there are more specialized algorithms that find optimal solution in a more efficient way than by using generic MILP solvers based on the presented model. However, developing such ad-hoc algorithms usually take a lot of time, so generic software can be used to obtain preliminary results on small-scale cases to observe whether considered approaches lead to promising results. Furthermore, it may take quite an effort to adapt such ad-hoc approaches if an extension of the current version has to be carried out in the future.

The paper is organized as follows. Section 2 presents the model of devices and energy streams. Section 3 shows how an arbitrary instance of Binary Linear Programming can be reduced to the device model. Section 4 shows that some specific properties and devices can be obtained as a combination of basic devices. Finally in Section 5, we give a short overview of one of our previous studies [12] where the model of this paper was used. Important symbols of the model are listed in Appendix.

2 The model

The presented model of energy systems consists of various types of devices which exchange arbitrary types of energy between them. Examples of possible devices are fridges, heat pumps, PV, TV, gas pipes, etc. Devices can consume, produce, store or transport energy or convert energy into different types of energy. Every type of energy can be produced and consumed by many devices. In that way, flows of energy between devices form complex relations between devices. As in general energy does not disappear, we require that these flows of energy fulfil the conservation law, so we need to take this into account when connecting devices. On the other hand, the model should not be too detailed and describe all technical details, e.g. it may not be necessary to take every single electrical cable in a building into account. To realize this, we use pools which provide us an abstraction from technical implementations of interconnections of devices. Every pool is connected to a set of devices, it maintains energy flow of only one type of energy and it ensures that the total amount of energy coming into the pool equals the total amount of energy going out of the pool at every time. The devices themselves are not required to fulfill the conservation law if they consume or produce energy and the way of production and consumption is not relevant for the model, e.g. a TV consumes electricity and produces heat but heat may not be interesting in studies of electrical demands.

Figure 1 presents a fictive model a biogas station and two houses interconnected via a transportation network. Meaning of all devices of the proposed model is presented in the remainder of this section. Figure 1 also gives images of basic devices used in other figures in this paper.

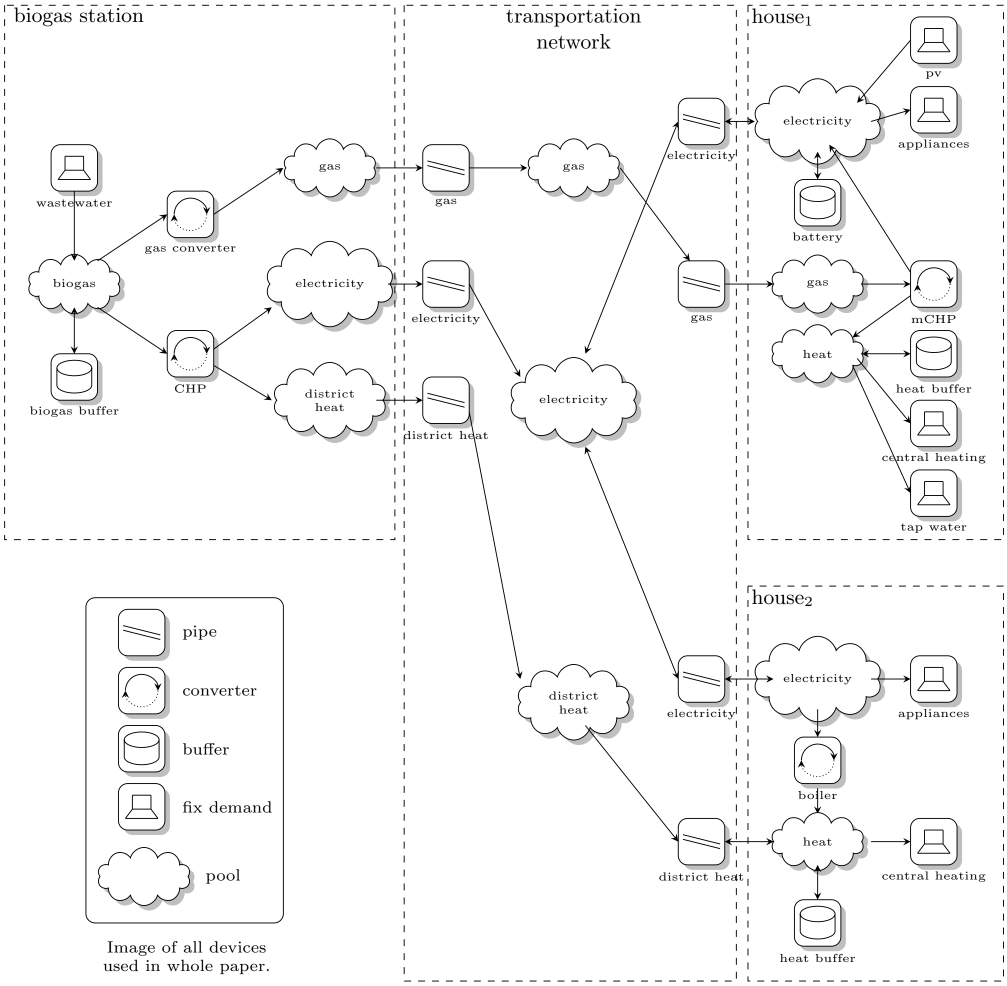

Figure 1: Example of a model consisting of a biogas station, two houses and a transportation network.

2.1 Notations

In order to obtain a simple but powerful model, we do not allow any direct connection of neither two devices nor two pools. Therefore, a model can be represented as a bipartite graph whose one partite is formed by a set D of *devices* and the other one is formed by a set P of *pools*. A device and a pool are connected by an edge whenever the device produces energy into the pool or consumes energy from the pool. Let $D_p \subset D$ be the set of devices which are connected to a pool $p \in P$ and let $P_d \subset P$ be the set of pools which are connected to a device $d \in D$.

In the strict mathematical formulation we do not need to take care of the types of used energy nor the physical units in which the energy is measured. However, we always assume that one pool can be used for only one type of energy and all flows to the pool are measured in the same physical unit.

In general, time in the real world is continuous, and therefore many physical observations are described by differential equations. However, if we would choose such an analytical model for the control of Smart Grids, we would end up in a model which would be too hard to be solved analytically. Hence, we simplify the model by considering a discretization of time leading to a partition of the planning horizon into a set $\mathcal{T} = \{1, \ldots, T\}$ of T consecutive time intervals.

Note that in this paper the index t always stands for time interval, p for a pool and d for a device, although it might be that only a particular type of device is discussed.

2.2 Pools

The total amount of energy produced into a pool has to be equal the total amount of energy consumed from the pool in every time interval. To incorporate this conservation law into the model, we denote by $e_{d,p,t}$ the amount of produced (consumed) energy by a device $d \in D$ into (from) a pool $p \in P$ in time interval $t \in \mathcal{T}$. For convenience, we consider that $e_{d,p,t} > 0$ if a device d produces energy into a pool p and $e_{d,p,t} < 0$ if a device d consumes energy from a pool p. The conservation law is incorporated into the model by a constraint $\sum_{d \in D_p} e_{d,p,t} = 0$ for every pool $p \in P$ and time interval $t \in \mathcal{T}$.

2.3 Overview of devices

While all pools have the same physical and mathematical behaviour, we introduce various types of devices with different behaviour. In this section, we explain the basic ideas of the proposed model using the following types of devices.

- *Non-controllable device*: represents a fixed production or consumption pattern of energy which we can neither control nor schedule.
- *Converter*: converts one or more types of energy into one or more other types of energy.
- *Pipe*: transports energy between different places (pools).
- *Buffer*: stores energy over time.
- *Time shiftable device*: has a fix consumption profile during its working process and has to start its processing in a time interval chosen from a given prespecified subset of time intervals, e.g. a washing machine and a dish washer.

Since different types of devices may have different operation modes or states and different restrictions, every type of device has its own parameters, variables and constraints. These data allow to compute for every device d the energy flows $e_{d,p,t}$ based on its internal states for every time interval $t \in \mathcal{T}$ and every pool $p \in P_d$.

2.4 Non-controllable device

We first describe the simplest type of device which is called a *non-controllable device*. It is used to model all energy production and demand which we cannot control nor change (e.g. PV, wind mill, TV, PC, ...). Of course, if such devices enable some kind of control in particular case studies, the model of a non-controllable device is not appropriate.

The production of a non-controllable device d is determined by parameters $F_{d,p,t}$ which give the amount of produced energy into a pool p in time interval t (it is negative if energy is consumed). The energy flow of a non-controllable device is simply expressed by the equation $e_{d,p,t} = F_{d,p,t}$.

We may restrict ourselves to the case that every non-controllable device is connected to exactly one pool because it is always possible to split a non-controllable device connected to several pools into several single-connected non-controllable devices.

2.5 Converter

We restrict ourself to a simple model for a converter which has only two operational states — it is either on or off during a time interval. In practice, most converters are usually connected to two or three pools (e.g. a Combined Heat and Power unit is connected to gas, electricity and heat). In order to obtain a general model, we consider that a converter can be connected to an arbitrary number of pools. When the converter is running it consumes (or produces) a fix amount of energy from (to) every pool that the converter is connected to whereby this amount may differ per pool.

To incorporate the operational states into our model, we use a decision variable $x_{d,t} \in \{0, 1\}$ indicating whether a converter d is turned on in time interval $t \in \mathcal{T}$ or not. The production (or consumption) of energy by the converter d is determined by a parameter $L_{d,p}$ which is the amount of produced energy into the pool p when the converter is turned on ($L_{d,p}$ is negative if energy is consumed). If start up and shutdown profiles exists, where the production or consumption differs from these values, we have to add extra elements to the model.

The energy flow between a converter and a pool is computed by the flow formula $e_{d,p,t} = L_{d,p}x_{d,t}$ for every time interval $t \in \mathcal{T}$ and pool $p \in P_d$ connected to the converter d. This formula guaranties that the device produces no energy ($e_{d,p,t} = 0$) when it is turned off ($x_{d,t} = 0$) in time interval t. On the other hand, when the converted is running ($x_{d,t} = 1$), then it is producing $e_{d,p,t} = L_{d,p}$ energy into pool p.

Minimal running time

If that the converter d has to run at least R_d^U time intervals whenever it starts up, we need to guarantee in our model that all variables $x_{d,t+2}, \ldots, x_{d,t+R_d^U}$ are equal 1 whenever $x_{d,t} = 0$ and $x_{d,t+1} = 1$. We can achieve this by adding a constraint

$$(R_d^U - 1)(x_{d,t+1} - x_{d,t}) \leq \sum_{i=2}^{R_d^U} x_{d,t+i}$$

for every time interval t.

Similarly, if a converter has minimal off time of length R_d^D, then we have to add a constraint

$$(R_d^D - 1)(1 + x_{d,t+1} - x_{d,t}) \geq \sum_{i=2}^{R_d^D} x_{d,t+i}$$

to our model.

These constraints might have to be adapted slightly for the beginning and the end of the planning horizon. The exact formulas depend on whether we allow the converter to be running before and after the planning horizon, however the resulting constrains are similar to the onces given; for more details, see e.g. [6].

Start up and shutdown profiles

Some types of converters do not produce immediately the full amount of energy into a pool when they start but they need some time to reach full production (e.g. microCHP devices). This start up time may be longer than the length of one time interval, so a start up profile needs to be considered.

Let us consider that the converter d has a start up profile $P_{d,p,1}^U, \ldots, P_{d,p,\hat{P}_{d,p}^U}^U$ of length $\hat{P}_{d,p}^U$. If the converter starts at time interval t, than it produces $P_{d,p,i}^U$ energy into pool p in the time interval $t + i - 1$ for $i = 1, \ldots, \hat{P}_{d,p}^U$.

To incorporate such profiles into our model, we create new variables $u_{d,t} \in \{0, 1\}$ such that $u_{d,t} = 1$ whenever $x_{d,t} = 1$ and $x_{d,t-1} = 0$, i.e. $x_{d,t} = 1$ if the device is started in the time interval t . We obtain this property by adding constraints

- $u_{d,t} \geq x_{d,t} - x_{d,t-1}$,
- $u_{d,t} \leq x_{d,t}$ and
- $u_{d,t} \leq 1 - x_{d,t-1}$.

The amount of produced energy now is given by

$$e_{d,p,t} = L_{d,p} x_{d,t} + \sum_{i=1}^{\hat{P}^U_{d,p}} \left(P^U_{d,p,i} - L_{d,p}\right) u_{d,t-i+1}.$$

Similarly, we can consider a shutdown profile $P^D_{d,p,1}, \ldots, P^D_{d,p,\hat{P}^D_{d,p}}$ of length $\hat{P}^D_{d,p}$. Again, we create new variables $v_{d,t} \in \{0,1\}$ such that $v_{d,t} = 1$ whenever $x_{d,t} = 0$ and $x_{d,t-1} = 1$. We obtain this property by constraints

- $v_{d,t} \geq x_{d,t-1} - x_{d,t}$,
- $v_{d,t} \leq x_{d,t-1}$ and
- $v_{d,t} \leq 1 - x_{d,t}$.

The amount of produced energy considering both start up and shutdown profiles is now given by

$$e_{d,p,t} = L_{d,p} x_{d,t} + \sum_{i=1}^{\hat{P}^U_{d,p}} \left(P^U_{d,p,i} - L_{d,p}\right) u_{d,t-i+1} + \sum_{i=1}^{\hat{P}^D_{d,p}} P^D_{d,p,i} v_{d,t-i+1}.$$

Note, that we assume that a converter with a start up profile has a minimal running time $R^U_d \geq \max\left\{\hat{P}^U_{d,p} \, ; \, p \in P_d\right\}$ and a converter with a shutdown profile has a minimal off time $R^D_d \geq \max\left\{\hat{P}^D_{d,p} \, ; \, p \in P_d\right\}$ which in general is the case in practice.

2.6 Pipes

The main purpose of the device called *pipe* is to model transportation of energy between different places (pools in our model). In the simplest case, a pipe d is connected to two pools p_1 and p_2 and $x_{d,t}$ specifics the amount of energy transported from the pool p_1 into the pool p_2 in time interval t, whereby $x_{d,t}$ is a decision variable of the model. We assume that the transportation time is negligible (or much shorter than the length of one time interval) which is true for electrical cables, but e.g. for transcontinental gas pipelines a more complex model may be needed. In this simple setting, the energy flows are $e_{d,p_1,t} = -x_{d,t}$ and $e_{d,p_2,t} = x_{d,t}$. Note that the control variable $x_{d,t}$ may be considered as continuous for pipes while it is binary for converts. We use the same symbol for control variables of a pipe and a converter to emphasise the relation between them. This relation is also discussed later in this section.

Capacity

Note, that the amount of transported energy $x_{d,t}$ can be negative which means that the energy is flowing the reverse direction. For some pipes this may be an undesired behaviour in reality. Furthermore, pipes usually have limited capacity. Hence, we introduce a lower bound B^-_d and an upper bound B^+_d which express the capacity of a pipe d. The pipe d now has to fulfil the constraints $B^-_d \leq x_{d,t} \leq B^+_d$ for every time interval $t \in \mathcal{T}$.

A typical example of the usage of a lower bound is occurring when a pipe which can transport energy only in one direction is modelled. In this case, we set $B^-_d = 0$. The lower bound or upper bound of the capacity of a buffer usually do not vary over time. If we need to model a pipe with fluctuating bounds on the capacity, we can use parameters $B^-_{d,t}$ and $B^+_{d,t}$ instead of B^-_d and B^+_d to express a lower bound and an upper bound of the capacity in the time interval t.

Transportation factor

In the most simple formulation, a pipe is connected to two pools p_1 and p_2. However, these two pools need to be distinguished to specify the direction of the flow of energy. Since it is better not to rely on distinguishing pools connected to the same pipe, we introduce transportation factors (which are later also used for other purposes).

For every connection between a pipe d and a pool p we introduce a transportation factor $T_{d,p}$ which is a linear factor between the amount of transported energy by the pipe $x_{d,t}$ and the energy flow $e_{d,p,t}$. Formally, the energy flow formula becomes $e_{d,p,t} = T_{d,p}x_{d,t}$. For example, a simple pipe d from a pool p_1 to pool p_2 has transportation factors $T_{d,p_1} = -1$ and $T_{d,p_2} = 1$. Note that by using transportation factors, the variable $x_{d,t}$ no longer needs to represent the actual amount of transported energy.

Using the transportation factors, we now can also consider pipes that are not connected to exactly two pools. A pipe where the flow is split in two different flows with a constant fraction between these two flows is a practical example of three-sided pipe.

One-side pipes

Pipes connected to only one pool can e.g. be used to model a source for energy which is produced (or consumed) outside our model (grid). For example, we may have the possibility to buy (or sell) energy on a market and we prefer to model the market as one point where energy enters the modelled system instead of incorporating a full model of the market.

The one-sided pipes have some relation with non-controllable devices. A non-controllable device d with production parameter $F_{d,p,t}$ can directly be modelled using a one-side pipe $\bar{d}$ connected to the pool p with capacity bounds $B^-_{\bar{d},t} = B^+_{\bar{d},t} = F_{d,p,t}$ and the transportation factor $T_{\bar{d},p} = 1$. But as non-controllable devices occur quite often, they are introduced as separate type for easier understanding within case studies. This modelling of a non-controllable device using a pipe is an example of how a device can be used to model other devices. More such examples are given in Section 4.

To get some feeling for the power of one-sided pipes, consider a production (market) with some flexibility on the amount of produced energy. This can be modelled by a one-side pipe d which is connected to a pool p with transportation factor $T_{d,p} = 1$ and the flexibility is modelled using the bounds B^-_d and B^+_d.

Multiple-side pipes

A natural way to model transportation losses is to connect a pipe to three pools: the first one for the source of energy, the second for the destination and the third for the losses. This motivates the introduction of multiple-side pipes.

Formally, a pipe connected to three or more pools has a well-defined behaviour: For every connection to a pool p a parameter $T_{d,p}$ represents a transportation factor. The variable $x_{d,t}$ represents an operational state of the pipe d in time interval t and the energy flows are given by $e_{d,p,t} = T_{d,p}x_{d,t}$ for each $p \in P_d$.

This formulation looks very similar to the basic definition of a converter. The main difference is that the variable $x_{d,t}$ is binary for converters but it can have arbitrary value for pipes. However, if we have a converter which allows a continuous control on production and consumption, such a converter may be (mathematically) modelled using a pipe. Note, that this approach has one disadvantage in the requirement that the ratio between the produced energy into the different pools has to be independent on the operational state $x_{d,t}$ (the ratio follows from the transportation factors $T_{d,p}$). This requirement essentially says that efficiency of the converter is constant which may not be true for all such devices in reality. However, the current model of a converter can easily be extended to a converter which has a discrete set of operational states.

Losses

Transportation losses are common in reality. A simple model of such transportation losses is based on the assumption that losses are linear in the amount of transported energy. If a pipe d models the transport of energy from a pool p_1 to a pool p_2 without transportation losses, we use transportation factors $T_{d,p_1} = -1$ and $T_{d,p_2} = 1$. However, if we simply decrease the parameter T_{d,p_2}, we model the fact that $1 - T_{d,p_2}$ of the amount of transported energy is lost. Furthermore, we may connect the pipe to a pool $\bar{p}$ of losses and set $T_{d,\bar{p}} = 1 - T_{d,p_2}$. In this way we may get information of the efficiency of an energy system.

Note that a pipe with a transportation factor $T_{d,p}$ different from 1 and -1 for some pool $p \in P_d$ may have an undesired behaviour if the amount of transported energy $x_{d,t}$ is negative. For instance, a pipe with losses now increases the amount of transported energy if energy flows in the reverse direction. In such cases we should set the lower bound $B_d^- = 0$ on the capacity to obtain the proper one-directional pipe and model, if needed the opposite direction by an own pipe.

2.7 Buffers

Buffers are devices used for storing energy. Typical examples of buffers are electrical batteries or hot water storages but buffers can e.g. also be used for modelling fridges.

A buffer d is connected to one "main" pool p which is used for charging and discharging of the buffer. However, a buffer may also be connected to a pool of losses.

The amount of energy stored in a buffer d in the beginning of time interval t is called the state of charge and is denoted by $c_{d,t}$. Since the amount of energy which the buffer d produces into the pool p during time interval t is given by $e_{d,p,t}$, the state of charge decreases by the amount of $e_{d,p,t}$. Hence, the basic charging formula is $c_{d,t+1} = c_{d,t} - e_{d,p,t}$ and the energy flow is $e_{d,p,t} = c_{d,t} - c_{d,t+1}$. Note that $c_{d,T+1}$ denotes the state of charge at the end of the last time interval.

Capacity

Common buffers do not allow to store a negative amount of energy and their capacity is limited. Therefore, we consider that the state of charge $c_{d,t}$ of a buffer d can be restricted by a lower bound $B_{d,t}^-$ and an upper bound $B_{d,t}^+$ on the capacity of the buffer at time interval t. We might have chosen to consider lower and upper bounds that do not vary over time, but it is at least helpful to use varying capacity to be able to incorporate the initial state of charge and to require a minimal state of charge at the end of the planning horizon.

Note, that if the capacity of a buffer is infinite or irrelevant during the planning horizon (e.g. the thermal storage of a heat pump), then it may be an option to replace the buffer by a one-side pipe.

Lower bound on the capacity following from a real world case

The lower bounds on the capacity are often a consequence of technical properties of the buffers. But next to these technical properties there maybe also desired lower levels on the content of the buffer (we call them set points) which result from the usage of the buffer. E.g. a buffer for hot tap water should be filled up to a certain level to be prepared to fulfill the demand inside the house. If this demand occurs, the content of the buffer may fall below this set point, but then a connected converter should start running to fill up the buffer again.

To model this, let d be a buffer which is connected to a pool p and let $\bar{d}$ be a converter that is producing energy to the pool p. Furthermore, let $B_{d,t}^\star$ be a set point on the capacity of the buffer d which indicates when the converter $\bar{d}$ has to be turned on. This means that the converter $\bar{d}$ has to be running ($x_{\bar{d},t} = 1$) when the state of charge of the buffer d is below the set point ($c_{d,t} < B_{d,t}^\star$). This can formulated using the inequality $Zx_{\bar{d},t} \geq B_{d,t}^\star - c_{d,t}$ where Z is a sufficiently large number; that is always greater than the difference $B_{d,t}^\star - c_{d,t}$.

The above introduced condition models the real usage of buffers more realistically than modifying the lower bound only. Note that it is possible to use both the classical lower bound $B^-_{d,t}$ and the lower bound $B^\star_{d,t}$ on one buffer d simultaneously but they also may be considered one by one.

Losses

Buffers often have losses during charging, storing and discharging of energy. An often used way to model storing losses is to assume that losses are linear in the current state of charge. If L^s_d denotes this linear factor of these losses, the amount of lost energy during time interval t is $L^s_d c_{d,t}$. Now, the charging formula is $c_{d,t+1} = c_{d,t} - e_{d,p,t} - L^s_d c_{d,t}$ which gives the energy flow $e_{d,p,t} = (1 - L^s_d)c_{d,t} - c_{d,t+1}$.

Furthermore, it is possible to model charging losses that are linear in the amount of charged energy. For this we split the variable $e_{d,p,t}$ in variables $e^+_{d,p,t}$ and $e^-_{d,p,t}$ which denotes the amount of outgoing and incoming energy to a pool p in time interval t for charging and discharging a buffer d, respectively. Now, the energy flow is $e_{d,p,t} = e^-_{d,p,t} - e^+_{d,p,t}$. Let L^c_d be the linear factor of the charging losses. Since $L^c_d e^+_{d,p,t}$ energy is lost, the state of charge of the buffer is increased only by $(1 - L^c_d)e^+_{d,p,t}$, meaning that the charging formula now is

$$c_{d,t+1} = (1 - L^s_d)c_{d,t} + (1 - L^c_d)e^+_{d,p,t} - e^-_{d,p,t}.$$

Note that we also need to add non-negativity restrictions $e^+_{d,p,t} \geq 0$ and $e^-_{d,p,t} \geq 0$.

The discharging losses are modelled similarly using a parameter L^d_d. For simplicity of the notation, we consider that the amount of energy entering the pool p is $e^-_{d,p,t}$ while the state of charge of the buffer decreased by $(1 + L^d_d)e^-_{d,p,t}$. The charging formula now becomes

$$c_{d,t+1} = (1 - L^s_d)c_{d,t} + (1 - L^c_d)e^+_{d,p,t} - (1 + L^d_d)e^-_{d,p,t}.$$

Note that discharging losses may also be modelled via the charging losses by adapting the capacity of the buffer and adding all losses to the charging process.

In the mathematical model of buffers with losses we implicitly assume that we do not allow charging and discharging in the same time interval (i.e. we assume that not both energy flows $e^+_{d,p,t}$ and $e^-_{d,p,t}$ are positive) as this is impossible in reality. To prevent this possibility, we have to add a binary variable $b_{d,t}$ and constraints $Zb_{d,t} \geq e^+_{d,p,t}$ and $Z(1 - b_{d,t}) \geq e^-_{d,p,t}$ for every time interval t where Z is a sufficiently large number.

Maximal charging energy

The amount of charging or discharging may be bounded due to physical properties of a buffer or limited capacity of a connection. This can be modelled by constraint $-M^c_d \leq e_{d,p,t} \leq M^d_d$ where M^c_d gives the maximal charging and M^d_d the maximal discharging.

2.8 Time shiftable devices

Time shiftable devices are devices consuming energy whose consumption can be scheduled in time. Time shiftable devices can have various operational restrictions which depends on the type of the appliances and human interaction. In this paper, we restrict us to the basic case and do not discuss more advanced properties and the possibility to preempt the operation of the device.

In the simplest case, a time shiftable device d has an energy profile $P_{d,p,1}, \ldots, P_{d,p,L_d}$ of length L_d. A controller of the time shiftable device decides the time interval $t_d \in \mathcal{T}$ in which the time shiftable device starts. When the appliance is scheduled to start in time interval t_d, then it produces $P_{d,p,t-t_d+1}$ energy into pool p in time interval $t = t_d, \ldots, t_d + L_d - 1$. Examples of such appliances are intelligent washing machines, driers and dish washers.

Let $x_{d,t}$ be a variable indicating whether the appliance d starts in time interval t. Observe that the energy flow from this device is $P_{d,p,i}x_{d,t-i+1}$ in time interval t if the appliance d starts at

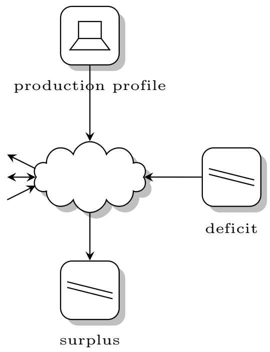

Figure 2: Surplus and deficit pipes are used to model deviation from the production profile.

time $t-i+1$. This implies that the energy flow formula is $e_{d,p,t} = \sum_{i=1}^{L_D} P_{d,p,i} x_{d,t-i+1}$. Hereby, we assume that the appliance has to start exactly once, leading to the restriction $\sum_{t\in\mathcal{T}} x_{d,t} = 1$.

The time shiftable device may also have limitations on the time intervals it can start. In this case, we may set $x_{d,t} = 0$ whenever the appliance cannot start in time interval t.

Furthermore, a time shiftable device may require to consume more than one type of energy (e.g. washing machines may consume electricity and hot water), leading to given profiles for each type of energy. However, we can assume that consuming profiles for all connected pools have the same length since we can fill up the shorter profile by zeros.

2.9 Objective functions

In order to optimize production and consumption of all devices objective functions need to be modelled. In this section we describe some objectives which can be used and optimized.

Section 2.6 shows that a simple energy market can be modelled using a single-side pipe. Let $G_{d,t}$ be the price on a pipe (market) d in time interval t. As the amount of bought energy on the market is the amount of transported energy $x_{d,t}$ by the pipe d in time interval t, the total cost for energy in this simple case is given by $\sum_{t\in\mathcal{T}} G_{d,t} x_{d,t}$. Similarly, we can consider a price $G_{d,t}$ for running a converted d in time interval t leading to the same mathematical formula $\sum_{t\in\mathcal{T}} G_{d,t} x_{d,t}$ determining the total operational cost.

Another common goal is to minimize the deviation of the used energy from a given prespecified production profile. The aim is to adopt the consumption of all considered devices in such a way that the total consumption over time is as close as possible to the given production profile. Figure 2 shows that this deviation can be measured using a surplus pipe d_s and a deficit pipe d_d that are both connected to the same pool p thereby modelling the production profile. The lower bound on the capacity of the both pipes equals zero and the transportation factors of pipes are $L_{d_s,p} = -1$ and $L_{d_d,p} = 1$. The objective function now is $\min \sum_{t\in\mathcal{T}} (x_{d_s,t} + x_{d_d,t})$.

We may also be interested in minimizing the peak of the energy consumption instead of the total sum. This is typical in situations where electrical cables or gas pipes need to dimensioned for the maximal flow. In this case we introduce for a pipe d a new variable p_d which measures the maximal flow over time. Constraints $x_{d,t} \le p_d$ for every time interval $t \in \mathcal{T}$ guarantee that p_d is never smaller than the flow in the pipe. If energy can flow in both directions, we also should add a constraint $-x_{d,t} \le p_d$. Now, we can minimize the peak by using the objective function $\min p_d$.

We can combine the last two approaches to minimize the maximal fluctuation. For this, we use peak variables p_{d_d} and p_{d_s} for the deficit pipe p_d and the surplus p_s and constraints $x_{d_d,t} \le p_{d_d}$ and $x_{d_s,t} \le p_{d_s}$ for every t. The objective function now gets $\min(p_{d_d} + p_{d_s})$.

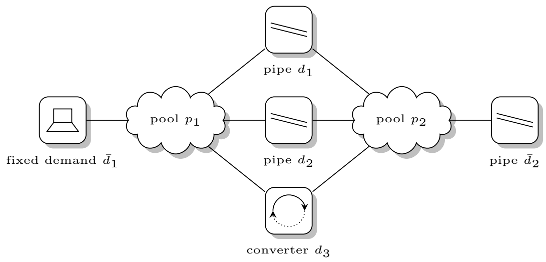

Figure 3: Example of reduction of an instance of binary LP into our model.

3 Relation to Binary Linear Programming

The previous section presents a model for energy flows in Smart Grids. The basic elements of this model are devices and pools together with connections between them. Although only these basic structures are used, the resulting model is already very flexible and powerful as we show in this section. We do this by presenting a quite direct polynomial reduction of the binary linear programming problem in the developed model whereby using only one time interval. The idea of the reduction is to replace every continuous variable by one pipe and every binary variable by one converter. Then, every equation is replaced by one pool and every right hand side of a constraint by one non-controllable device.

Small example

We first present the reduction on a small example to give the reader a feeling of the used construction. For this, consider the following binary LP.

$$\begin{array}{rcccccccc} \min & c_1\,x_1 & + & c_2\,x_2 & + & c_3\,x_3 & & \\ \text{s.t.} & a_{1,1}\,x_1 & + & a_{1,2}\,x_2 & + & a_{1,3}\,x_3 & = & b_1 \\ & a_{2,1}\,x_1 & + & a_{2,2}\,x_2 & + & a_{2,3}\,x_3 & \geq & b_2 \\ & & & \multicolumn{3}{c}{x_1 \in \mathbb{R},\, x_2 \geq 0,\, x_3 \in \{0,1\}} & & \end{array}$$

For the two constraints we introduce pools p_1 and p_2, for the continuous variable x_1 we introduce a pipe d_1 and for the continuous variable x_2 we introduce a pipe d_2 having a lower bound on the capacity $B^-_{d_2} = 0$. Both pipes d_1 and d_2 are connected to both pools p_1 and p_2 with the transportation factor $T_{d_j,p_i} = a_{i,j}$ for $i, j \in \{1, 2\}$. For the binary variable x_3 we introduce a converter d_3 which is connected to both pools p_1 and p_2 with the production factors $L_{d_3,p_i} = a_{i,3}$ for $i \in \{1, 2\}$. For the right hand side of the equality we introduce a non-controllable device $\bar{d}_1$ with the production parameter $F_{\bar{d}_1,p_1} = -b_1$. For the right hand side of the inequality we introduce a pipe $\bar{d}_2$ connected to the pool p_2 with the transportation factors $T_{\bar{d}_2,p_2} = 1$ and a lower bound on the capacity of $T_{\bar{d}_2,p_2} = -b_2$. For the objective function we introduce prices $G_{d_j,1} = c_j$ for $j \in \{1, 2, 3\}$. The model obtained by this reduction is presented on Figure 3.

General reduction

For the general reduction, let us consider a binary linear programming problem of the basic form

$$\min\left\{c^T x\,;\ Ax = b,\, x \geq 0,\, x_j \in \{0,1\}\ \text{ for } j \in I\right\}$$

where I is the set of binary variables.

The reduction is as follows: For every equation $\sum_j A_{i,j} x_j = b_i$ of $Ax = b$ we introduce a pool p_i, for every binary variable x_j with $j \in I$ we introduce a converter d_j which is connected to every pool p_i with the production factor $L_{d_j,p_i} = A_{i,j}$ and for every continuous variable x_j with $j \notin I$

we introduce a pipe d_j having a lower bound on the capacity $B^-_{d_j} = 0$. The pipe d_j is connected to every pool p_i with the transportation factor $T_{d_j,p_i} = A_{i,j}$. We do not need to connected a device d_j and a pool p_i if $A_{i,j} = 0$ since such a connection has no influence on the model. For the right hand side b_i we connect a non-controllable device $\bar{d}_i$ having a production parameter $F_{d_i,p_i,1} = -b_i$ to pool p_i. For the objective function, we introduce prices $G_{d_j,1} = c_j$ for every variable x_j.

Note that if we allow an advanced converter whose operational state can be any integer instead of only 0 and 1, we are also able to reduce a general Mixed Integer Linear Problem (MILP) to the model. Such an advanced converter can be useful to model e.g. a valve.

Methods from LP

Based on the presented reduction, we have one-to-one correspondence between variables and constraints of the binary LP and devices and pools of the model. This natural correspondence is very useful since we can use knowledge from LP theory within the energy model. Some of this knowledge has already been used without explicit notion. In the following, we summarize these methods and discuss the relation between the model and LP theory.

An unbounded variable x can be replaced by two non-negative variables x^+ and x^- in order to obtain the basic form where all variables are non-negative. A similar approach is used to transform the absolute value $|x|$ into an LP. Note that we use such a method in Section 2.9 to model deviation from a given production profile. Furthermore, in Section 2.7 we use the positive $e^+_{d,p,t}$ and negative $e^-_{d,p,t}$ parts of the energy flow $e_{d,p,t}$ to model charging and discharging losses of a buffer. We also model those losses using two pipes which represent positive and negative parts of the energy flow between a pool and a buffer. Generally, we can replace a bi-directional pipe by two one-directional pipes, but we have to be careful whether flows in both pipes can be positive in the same time interval.

In LP theory, non-negative slack variables are used to replace inequalities with equalities in the constraints. For this purpose, we can use within our energy model a pipe with the capacity constraint $B^-_d = 0$. In the presented example above we use the pipe $\bar{d}_2$ to model the inequality.

Finally, the classical algorithm for solving LP is the Simplex method which is based on Gaussian elimination. In Section 2.7 we note that buffers without any capacity constraints can be replaced by a pipe. The correctness of this replacement can e.g. be proven using Gaussian elimination.

4 Combination of devices

The previous section indicates the strength of the model by presenting a relation to Binary Linear Programming. In this section we present some further examples demonstrating this strength. More precisely, we show how some advanced devices can be obtained by using combinations of basic devices presented in Section 2.

Examples of this section proofs the strength of the model in various ways. Section 4.1 shows that the same combination of a converter, a buffer and a non-controllable device be used for purposes unrelated on the first sight. Section 4.2 shows that losses of a buffer can be modelled using pipes and the main observation is that MILP solver should not require significantly more computational time to solve the model with the combination, although the model looks more complex. Section 4.3 shows that basic thermodynamics can be modelled using simple devices. Section 4.4 discusses modelling devices that are not connected during the whole planning horizon (e.g. electrical cars). Section 4.5 shows that energy flows $e_{d,p,t}$ do not necessarily mean only amount of energy but they can also be used for purely mathematical purposes to model mutual restrictions between devices.

4.1 A combination of a converter, a buffer and a non-controllable device

Several complex devices can be modelled using combinations of basic devices and some of such combinations may be handy in different situations. One of the most useful combinations is formed

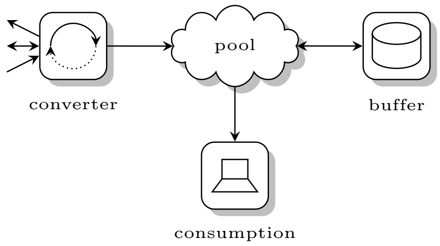

Figure 4: Combination of a converter, buffer and non-controllable consumption.

by a converter, a buffer and a non-controllable device which are connected to a common pool, see Figure 4. This combination can be used to model the following elements.

Hot water supply: The converter and buffer form a model of a simple electrical or gas boiler to produce hot water. The added non-controllable device represents the consumption of hot water.

Heating: The given combination may be used as a very simple model of house heating. Hereby, the converter represents a simple heater. The capacity of the buffer corresponds to the heat capacity of a house and the state of charge of the buffer is related to the temperature inside the house. Heat losses may be modelled using the non-controllable device if we assume that the temperature differences inside the house do not have significant influence on the losses.

Fridges and freezers: A fridge essentially works in the opposite way than heating, so it may be modelled similarly. However, we should be careful with the correct interpretation of all parameters. The state of charge of the buffer again represents the temperature inside the fridge, but a higher state of charge means lower temperature. The converter does not produce heat to the fridge but it decreases the temperature inside the fridge, so the converter increases the state of charge of the buffer (fridge). The non-controllable device decreases the state of charge of the fridge due to thermal loss and usage of the fridge by humans.

Inventory: The considered combination is also related to basic Inventory control problems (see e.g. [25]). A buffer may represent an inventory and a converter may represent orders. However, the way converters are defined leads to a situation, where it is only possible to order a fix amount which is not a typical situation in inventory management.

Specialized algorithms for this combination of a converter, a buffer and a non-controllable device are developed in [10, 11].

4.2 Losses of a buffer using pipes

Section 2.7 presents how charging and discharging limits and losses of a buffer can be incorporated into the model of a buffer. In this section, we show that our energy model is strong enough to model these limits and losses without the necessity to introduce a special constrains in the model of a buffer. Our goal is to model charging and discharging limits and losses of a buffer using pipes and a simple buffer without constrains for charging and discharging limits and losses. Next, we show that this approach should not lead to an increase of the computational time needed to solve the model using MILP solvers.

Figure 5(a) presents a buffer d connected to a pool p. Hereby this buffer and the pool are only a part of a grid and other devices may also be connected to the pool p. If we now have charging and discharging bounds for the buffer d, we can model this using a pipe $\bar{d}$ with bounds on its capacity (see Figure 5(b)). Hereby, we set the transportation factors to be $T_{\bar{d},p_1} = -1$ and

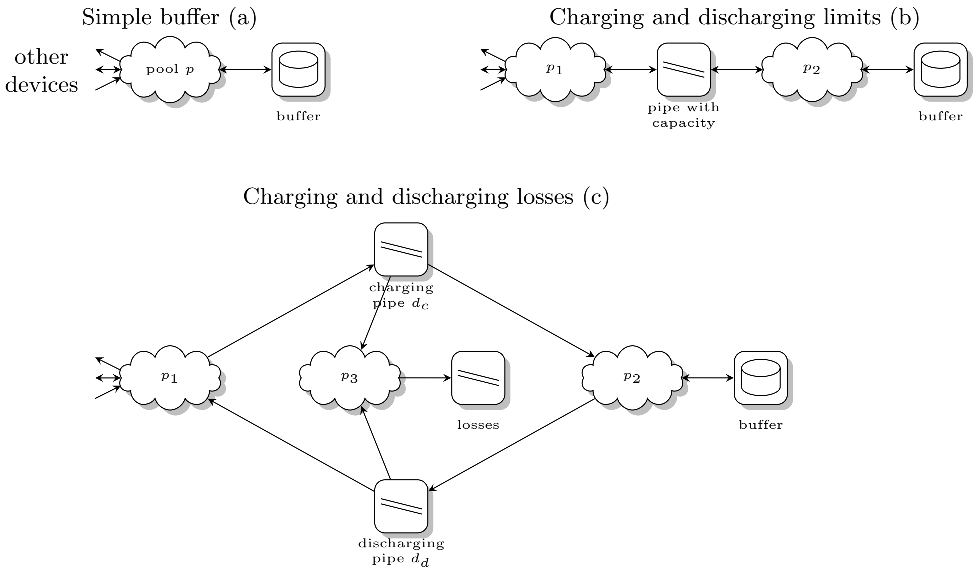

Figure 5: Modelling charging and discharging limits and losses using pipes.

$T_{\bar{d},p_2} = 1$ and the charging limit of the buffer is modelled by the upper bound $B^+_{\bar{d}}$ on the capacity of the pipe and the discharging limit is modelled by the negative value of the lower bound $B^-_{\bar{d}}$.

If we further want to model charging and discharging losses of the buffer d we can do this as indicated in Figure 5(c). The pipe d_c is only used for charging while the pipe d_d is only used for discharging. Therefore, both pipes have lower bounds $B^-_{d_c} = B^-_{d_d} = 0$ on their capacity and upper bounds may be used for charging and discharging limits of the buffer. Charging losses L^c_d of the buffer are modelled using a transportation factor $T_{d_c,p_2} = 1 - L^c_d$ of the charging pipe d_c and discharging losses L^d_d of the buffer are modelled using a transportation factor $T_{d_d,p_2} = -(1 + L^d_{d_d})$ of the discharging pipe d_d. The transportation factors to the pool p_1 are $T_{d_d,p_1} = -1$ and $T_{d_c,p_1} = 1$. In order to sum up both charging and discharging losses, we can connect both charging and discharging pipes to a pool p_3 of losses which also is connected to a pipe of losses. The transportation factors are $T_{d_c,p_3} = L^d_d$ and $T_{d_d,p_3} = L^c_d$.

Computational time

In the following we compare the approach chosen in Section 2.7 for charging and discharging losses of a buffer using the parameters L^c_d and L^d_d with the above approach, where losses are modelled by pipes. More precisely, we investigate which approach is more efficient from an algorithmic point of view. As there may exist specialized algorithms for both ways, the given arguments are only an indication and not necessarily a finite answer. Nevertheless, we can at least compare corresponding MILP models to see which problem might be solved faster by an MILP solver.

A simple example of our aim is given in Section 2.6 where a non-controllable device $\bar{d}$ is modelled using a pipe d with tight bounds $B^+_{d,p,t} = B^-_{d,p,t} = F_{\bar{d},p,t}$. Although the pipe introduces new variables $x_{d,t}$ constrained by inequalities $B^-_{d,p,t} \leq x_{d,t} \leq B^+_{d,p,t}$, these variables can be eliminated by MILP presolver, so the dominating computational time spent on finding an integer solution should not be influenced by using the pipe d instead of the non-controllable device $\bar{d}$.

The analysis of the buffer is more complicated, so we first list all constraints of both modelling approaches. The constraints for the model which uses an advanced buffer can be summarized as follows.

Charging formula:	$c_{d,t+1} = c_{d,t} + (1 - L^c_d)e^+_{d,p,t} - (1 + L^d_d)e^-_{d,p,t}$
Energy flow:	$e_{d,p,t} = e^-_{d,p,t} - e^+_{d,p,t}$
Non-negativity:	$e^+_{d,p,t},\ e^-_{d,p,t} \geq 0$

The constraints for the model which uses pipes is as follows.

Charging pipe:	$e_{d_c,p_1,t} = -x_{d_c,t}$	since $T_{d_c,p_1} = -1$
	$e_{d_c,p_2,t} = (1 - L^c_d)x_{d_c,t}$	since $T_{d_c,p_2} = (1 - L^c_d)$
	$x_{d_c,t} \geq 0$	
Discharging pipe:	$e_{d_d,p_1,t} = x_{d_d,t}$	since $T_{d_d,p_1} = 1$
	$e_{d_d,p_2,t} = -(1 + L^d_d)x_{d_d,t}$	since $T_{d_d,p_2} = -(1 + L^d_d)$
	$x_{d_d,t} \geq 0$	
Buffer:	$c_{d,t+1} = c_{d,t} - e_{d,p_2,t}$	
Pool p_2:	$e_{d_c,p_2,t} + e_{d_d,p_2,t} + e_{d,p_2,t} = 0$	
Contribution to p_1:	$e_{d_c,p_1,t} + e_{d_d,p_1,t}$	

On the first sight, these sets of constraints look completely different. Moreover, the second model uses more variables and constraints. But we should notice that variables $x_{d_c,t}$ and $x_{d_d,t}$ represent variables $e^+_{d,p,t}$, $e^-_{d,p,t}$, respectively. Furthermore, most MILP solvers first run a presolver which may eliminate variables $e_{d_c,p_1,t}$, $e_{d_c,p_2,t}$, $e_{d_d,p_1,t}$, $e_{d_d,p_2,t}$ and $e_{d,p_2,t}$. Using these substitutions we simply observe that the contribution to the pool p_1 in the second model becomes

$$e_{d_c,p_1,t} + e_{d_d,p_1,t} = -x_{d_c,t} + x_{d_d,t} = e^-_{d,p,t} - e^+_{d,p,t}$$

which is the energy flow formula in the first model. Similarly, we derive the charging formula of the buffer as

$$\begin{aligned} c_{d,t+1} &= c_{d,t} - e_{d,p_2,t} \\ &= c_{d,t} + e_{d_c,p_2,t} + e_{d_d,p_2,t} \\ &= c_{d,t} + (1 - L^c_d)x_{d_c,t} - (1 + L^d_d)x_{d_d,t} \\ &= c_{d,t} + (1 - L^c_d)e^+_{d,p,t} - (1 + L^d_d)e^-_{d,p,t} \end{aligned}$$

Since the running time of the MILP presolver is negligible, we conclude that the running time of the MILP solver should be similar for both models. This supports our idea that modelling software for Smart Grids should only implement simple properties of basic devices and complex properties should be modelled using combinations of basic devices. From the discussion above it follows that computational time should not significantly increase when combinations of basic devices is used instead of complex properties, even though the size of MILP problem is increased.

4.3 Losses of a buffer in unsteady environment

In this section we consider an environment where unsteady temperature has significant influence on losses of a buffer which is placed in this environment; e.g. a hot water buffer is placed outside a house. For the modelling, we assume that the state of charge $c_{d,t}$ of the buffer d corresponds to the amount of energy inside the buffer and that the average temperature inside the buffer; and has a linear relation to this state of charge.

Let $T_{d,t}$ be the temperature in the buffer d in time interval t and let the state of charge be given by $c_{d,t} = C_d(T_{d,t} - T^0_d)$ where C_d is the thermal capacity and T^0_d the temperature if the state of charge is zero. As losses depend on thermal conductivity, they can be determined by Fourier's law

$$Q_{d,t} = \frac{kA\Delta t}{l}(T_{d,t} - T_{A,t}),$$

where k is the thermal conductivity of the material of the cover of the buffer whose surface area is A and thickness is l. Furthermore, in this formula, the length of one time interval is Δt and the ambient temperature in time interval t is $T_{A,t}$. Although, this formula is simple, it may not be easy to use in practice since the material of the buffer may not be homogeneous and the thickness may not be constant. In such cases, the value of $U_d = \frac{kA\Delta t}{l}$ needs to be estimated or experimentally measured.

From the law of energy conservation it follows that $C_d(T_{d,t+1} - T_{d,t}) = -e_{d,p,t} - Q_{d,t}$. The energy flow formula in the energy model now gets

$$\begin{aligned} e_{d,p,t} &= -C_d(T_{d,t+1} - T_{d,t}) - U_d(T_{d,t} - T_{A,t}) \\ &= C_d(T_{d,t} - T_d^0) - C_d(T_{d,t+1} - T_d^0) + U_d(T_{A,t} - T_d^0) - U_d(T_{d,t} - T_d^0) \\ &= \left(1 - \frac{U_d}{C_d}\right) c_{d,t} - c_{d,t+1} + U_d(T_{A,t} - T_d^0), \end{aligned}$$

so the charging formula is

$$c_{d,t+1} = \left(1 - \frac{U_d}{C_d}\right) c_{d,t} - e_{d,p,t} + U_d(T_{A,t} - T_d^0).$$

Note that the length of one time interval Δt has to be small enough to ensure that the inequality $C_d > U_d = \frac{kA\Delta t}{l}$ holds. This inequality is called stability criterion on forward approximation in finite difference methods (e.g. see [8]). It would be possible to use backward approximation instead, but that is beyond the scope of this paper. Also note that $L_d^s = \frac{U_d}{C_d}$ can be seen as losses like introduced in Section 2.7. The last term $U_d(T_{A,t} - T_d^0)$ is a fix flow which does not depend on the state of charge of the buffer. Thus, this term can be modelled using a non-controllable device $\bar{d}$ connected to a pool p with the parameter $F_{\bar{d},p,t} = U_d(T_{A,t} - T_d^0)$. The resulting formula is $e_{d,p,t} = (1 - L_d^s)c_{d,t} - c_{d,t+1}$ which gives the energy flow for a buffer with losses which does not depend on environment. Therefore, it is possible to model losses of a buffer in unsteady environment using a combination of a basic buffer and a non-controllable device.

4.4 Electrical cars

Batteries of electrical cars (PHEV) can essentially be considered as buffers in the model. The main difference is that electrical cars may not be connected to an electricity pool for the whole planning horizon.

One option to incorporate electrical cars into the model is to slightly modify the model of a buffer so that it is possible to explicitly set the energy flow $e_{d,p,t} = 0$ for every t when the buffer d is not connected to a pool p and set a new state of charge $c_{d,t}$ for the first time intervals when the car become connected.

Another possibility is adding a pipe and a non-controllable device, see Figure 6. The pipe has the capacity $B_{d,t}^+ = 0$ when the car is not connected to the electricity pool p and the non-controllable device represents the consumption for driving. We can also set up a negative value for the lower bound $B_{d,t}^-$ when the car is connected to the electricity pool in order to allow to use the energy in the battery of the car for other appliances in a house.

The advantage of the first approach is that we can explicitly set up the state of charge of the buffer in the first time interval when the car is connected to an electricity pool. In the second approach we can explicitly set up the consumption for driving by the non-controllable device and battery losses are computed automatically in the model.

4.5 Multifunctional heat pump

Some types of heat pumps are able to generate warm water for a space heating system and also hot water for tap. This generated warm and hot water is buffered and consumed later on meaning that the production is to some extend independent of the consumptions. Such a heat pump has two different operational modes for heating water and a third one for being turned off. In this section we present a model of this heat pump.

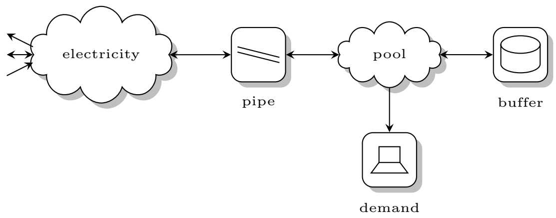

Figure 6: Model of electrical cars (PHEV).

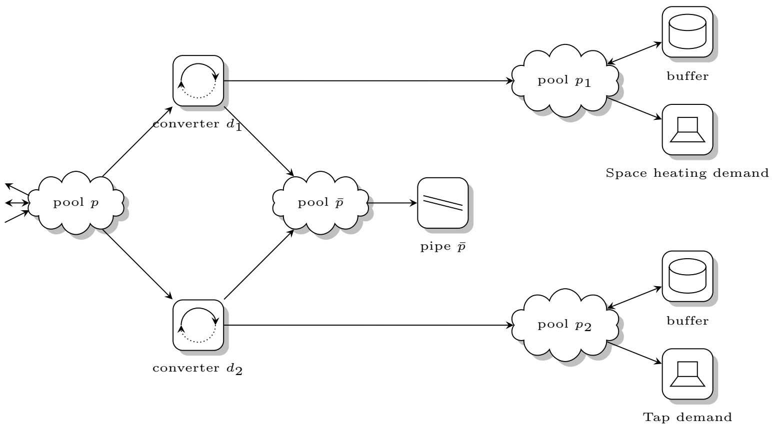

Figure 7: A model of heat pump with two operation modes for generating warm water for space heating and hot water for tap.

To model this, the heat pump d is connected to an electricity pool p and two pools p_1 and p_2 for warm and hot water. When the heat pump is producing warm water, it consumes $L^1_{d,p}$ electricity and produces L_{d,p_1} warm water. Similarly, when the heat pump is producing hot water, it consumes $L^2_{d,p}$ electricity and produces L_{d,p_2} hot water. For simplicity, we do not model the thermal storage from which the heat pump extracts energy to heat the water.

The straightforward approach modelling the heat pump with an MILP is the following: Let $x^1_{d,t}$ and $x^2_{d,t}$ be binary variables indicating whether the heat pump d is in time interval t in the first or the second operation mode, respectively. Since the heat pump cannot be in both modes simultaneously, we have a condition $x^1_{d,t} + x^2_{d,t} \leq 1$. The heat pump consumes electricity from a pool p and water flows to pool p_1 and p_2. In an operation mode $i \in \{1, 2\}$, the heat pump consumes $L^i_{d,p}$ electricity and produces L_{d,p_i} water, so the energy flow formula are $e_{d,p,t} = L^1_{d,p} x^1_{d,t} + L^2_{d,p} x^2_{d,t}$ and $e_{d,p_i,t} = L_{d,p_i} x^i_{d,t}$.

One way for modelling the heat pump in an energy model is extending the model by a device described type. But the heat pump can also be modelled using basic devices. For this, we use the ideas of the reduction from Binary LP which is presented in Section 3. Since we have two binary variables $x^1_{d,t}$ and $x^2_{d,t}$ for every time interval, we have to consider two converters d_1 and d_2. The inequality $x^1_{d,t} + x^2_{d,t} \leq 1$ has to be modelled using a pool $\bar{p}$ and a pipe $\bar{d}$ with production factors $L_{d_i,\bar{p}} = 1$ and transportation factor $T_{\bar{d},\bar{p}} = -1$ and the upper bound $B^+_{\bar{d}} = 1$. It remains to connect to the converter d_i to pools p and p_i and set the production factors $L_{d_i,p} = L^i_{d,p}$ and $L_{d_i,p_i} = L_{d,p_i}$ for $i \in \{1, 2\}$. The resulting model is presented on Figure 7.

Note that pool $\bar{p}$ and pipe $\bar{p}$ do not represent any physical device. Similarly, variables $e_{d_i,\bar{p},t}$ and $e_{\bar{d},\bar{p},t}$ represent rather data communication than energy flow. In this example, we use the formal mathematical formulation of our model to obtain desired variables and constrains.

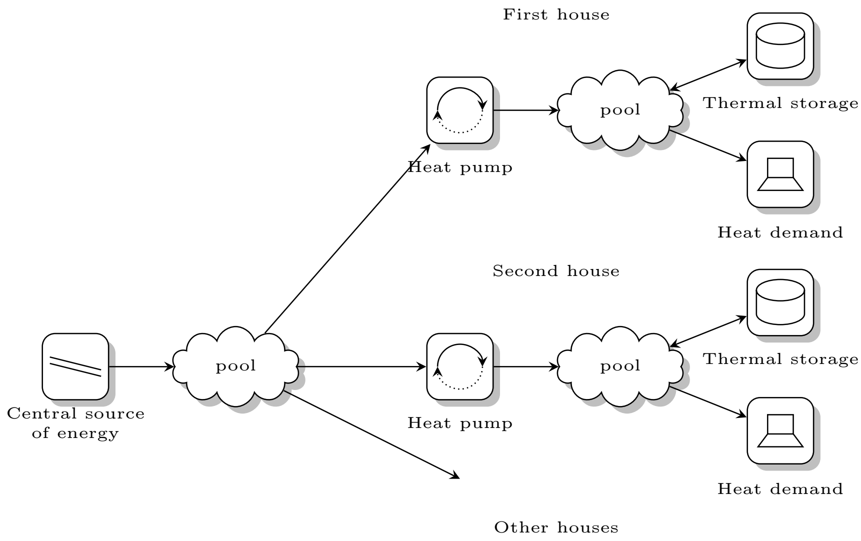

Figure 8: Model of Meppel project.

5 Study case

In this section we briefly presents one our previous study [12, 10] in which the model of this paper was used. The studied problems originate from a project called *MeppelEnergie* which plans to build a group of houses and a biogas station in Meppel, a small city in the Netherlands[1]. In this project, the houses will have a heat pump for space heating and tap demands. Due to Dutch legislation, the biogas station will provide electricity only to those heat pumps. Therefore, the heat pumps should be scheduled in such a way that they only consume, if possible, the electricity produced by the biogas station. If this is not possible, the remaining energy has to be bought on the electricity market at minimal cost.

Figure 8 presents the model studied in papers [12, 10]. It consists of a central source of energy represented by the pipe on the left. The energy is distributed though a pool to heating systems (houses) represented by a combination of converter, buffer and demand (see Section 4.1 for more details).

The study [12] shows that some central control of all heat pumps is necessary to avoid large peak loads. Therefore, our task is to design one or more algorithms to control all heat pumps. The first of our proposed algorithms is called *global MILP control* which uses an Mix Integers Linear Programming solver to find an optimal (or near to optimal) solution of the minimizing peak problem. The paper [12] shows that this approach can be used only for small number of houses. For larger number of houses, a faster algorithm for the minimizing peak problem is necessary but the problem is NP-hard. Therefore in papers [12, 10], we developed some algorithms to find a good solution of the model.

6 Conclusion

In this paper we present a mathematical modelling approach for future Smart Grids. The model considers various types of devices and energy streams within Smart Grids and is based on mathematical models of some basic devices with common operational restrictions. We indicate the advantages to consider such basic devices compared to having complex ones by showing how

[1] For more details, see websites http://www.utwente.nl/ctit/energy/projects/meppel.html and http://www.meppelwoont.nl/nieuwveense-landen/

advanced features can be obtained as a combination of the basic devices.

The model is strong in a computation sense since every instance of a Binary Linear Programming can be reduced to the model. As a consequence, we should not aim to find fast algorithms which find the optimal control for an arbitrary grid. This is even further pinpointed, as already very special versions of the model have been proven to be NP-complete (see e.g. [10]). One approach to control the model is to aim only for approximate solutions instead of an optimal one. Another possibility is to restrict the complexity of the model. One such simplification is e.g. presented in Kok [15] and Molderink [19] where it is assumed that devices are connected into a tree-like structures.

However, although the presented model is general, there are still some issues in practice which are not considered in this paper. For example, a lot of parameters on the production and consumption of energy within devices used in our model are not known when decisions need to be made (e.g. since the values have a stochastic nature). In this case, e.g. stochastic methods or prediction models have to be used (see e.g. [3]). Furthermore, different parts of Smart Grids (e.g. houses, power stations and electrical networks) are owned by different parties who may have different objectives and their goals may be contradictory. Therefore, we do not get a model with one overall objective, but different stakeholders with different objectives and constraints are involved asking for game theoretical approaches to be considered for the control of Smart Grids.

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7 Appendix

Table of the important symbols used in this paper. For simplicity, terms like "by a device d into a pool p in time interval t" are omitted in explanations of all symbols like $F_{d,p,t}$.

General, see Sections 2.1 and 2.2	
t	Index of time interval
p	Index of a pool
d	Index of a device
$\mathcal{T}$	Set of time intervals of given planning horizon
T	The number of time intervals
D	Set of all devices
D_p	Set of all devices connected to a pool p
P	Set of all pools
P_d	Set of all pools connected to a device d
$e_{d,p,t}$	The amount of produced energy by a device d into a pool p in time interval t
Non-controllable device, see Section 2.4	
$F_{d,p,t}$	Fixed amount of produced energy
Converter, see Section 2.5	
$x_{d,t}$	Binary operational state
$L_{d,p}$	The amount of produced energy when the converter d is running
Pipe, see Section 2.6	
$x_{d,t}$	Continuous operational state
$T_{d,p}$	Transportation factor; the amount of produced energy is $e_{d,p,t} = T_{d,p}x_{d,t}$
$B^-_{d,t}$, $B^+_{d,t}$	Lower and upper bounds on the operational state
Buffer, see Section 2.7	
$c_{d,t}$	State of charge
Objective function, see Section 2.9	
$G_{d,t}$	Price for operational state of a converter or a pipe