Problem 1 (2 points). Construct a Turing machine which recognizes language $L = \left\{0^{(2^n)};\ n \in \mathbb{N}\right\}$, i.e. words containing only zeros whose length is a power of two. Write precisely your Turing machine!

Problem 2 (4 points).

- Prove that one-tape Turing machines which are allowed to write on every cell at most twice are equivalent to (standard) Turing machines.
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Problem 3 (3 points). For a deterministic finite automata A, let $L(A)$ denotes the set of all words accepted by A. Decide whether the language $\{A;\ A \text{ is deterministic finite automata and } L(A) \text{ is an infinite language}\}$ is decidable.

Problem 4 (2 points). Consider the language L of all pairs (M, w) where a Turing machine M on an input w ever attempts to move its head left at any point during its computation on w. Determine whether L is decidable.

Problem 5 (2 points). Decide whether the Post correspondence problem is decidable over

1. the unary alphabet $\Sigma = \{1\}$,
2. the binary alphabet $\Sigma = \{0, 1\}$.

Problem 6 (3 points). Consider a language $L = \{(M, N);\ M, N \text{ are Turing machines and } L(M) \setminus L(N) \neq \emptyset\}$. Are languages L and $\bar{L}$ decidable or partially decidable?

Problem 7 (4 points). For each pair of complexity classes below try to decide if we can prove a relation between them (by using theorems and propositions from the lecture). That is, for every pair of classes A and B decides whether $A \subseteq B$, $B \subseteq A$, $A \setminus B = \emptyset$ and $B \setminus A = \emptyset$. Note that some relations may not be known.

1. $\mathsf{DSPACE}(n^3)$ and $\mathsf{DTIME}(2^{n^3})$
2. $\mathsf{DTIME}(2^{n^3})$ and $\mathsf{NSPACE}(n \log n)$
3. $\mathsf{NSPACE}(n \log n)$ and $\mathsf{NTIME}(n \log n)$
4. $\mathsf{NTIME}(n \log n)$ and $\mathsf{DSPACE}(n)$
5. $\mathsf{NTIME}(n \log n)$ and $\mathsf{DSPACE}(n^3)$

Problem 8 (1 point). Decide whether the following problem is polynomial or NP-complete.
Instance CNF formula ϕ
Question Are there two different assignments u, v of truth values to variables in ϕ which satisfy ϕ, i.e. $\phi(u)$ and $\phi(v)$ both evaluate to true.

Problem 9 (3 points). Consider the following parametrized problem.
Parameter: $k \in \mathbb{N}$
Instance: A set of n points in a plane.
Question: Are there k lines covering all points?
Construct FPT algorithm and kernelization.

Problem 10 (4 points). A path in an oriented graph $G = (V, E)$ is sequence $v_1, \ldots, v_k$ of distinct vertices of V where $(v_i, v_{i+1}) \in E$ is an edge oriented from v_i to v_{i+1} for every $i = 1, \ldots, k-1$. An oriented path $v_1, \ldots, v_k$ forms a cycle if $(v_k, v_1) \in E$. Decide whether the following problems are NP-complete or coNP-complete or they belong to P.

1. Determine whether a given oriented graph G contains a cycle <u>or</u> a Hamiltonian path.
2. Determine whether a given oriented graph G contains a cycle <u>and</u> a Hamiltonian path.

Problem 11 (3 points). A factory has machines M which can produce products P. One machine can only work on one product and each product $p \in P$ has a subset $M_p \subseteq M$ machines that must be working on p to produce p. A management needs to decide whether the factory is able to be manufacturing at least k products. Does this problem belong to Por is it NP-complete.