The the third homework are Problems 6 and 7.

Problem 1. Is the following language (partially) deciable?

$$\{(M, w);\ M \text{ terminates on input w and the tape of } M \text{ is empty after the computation}\}$$

Problem 2. Are the following languages (partially) decidable where M is a (code of a) Turing machine and $k \in \mathbb{N}$ and $|w|$ is the length of a word w.

1. $\{w;\ |w| \leq k\}$
2. $\{(M, k);\ |L(M)| \leq k\}$
3. $\{M;\ |L(M)| \geq 10\}$
4. $\{M;\ L(M) = \{w;\ |w| = 10\}\}$
5. $\{(M, k);\ L(M)$ contains a word of length $k\}$

Problem 3. Are the following problem (partially) decidable?

1. For a given Turing machine M, a state q of M and an input w, does M enter the state q during computation $M(w)$?
2. For a given Turing machine M and a state q of M, is there an input w such that M enter the state q during computation $M(w)$?
3. For a given Turing machine M and a state q of M, does M enter the state q during computation $M(w)$ for every word w?

Problem 4. Consider a Turing machine M with the following time complexities $f(n)$ and an integer $k \in \mathbb{N}$. For which functions f the language $L(M)$ belongs to P or to EXP?

1. $f(n) = n^k$
2. $f(n) = k^n$
3. $f(n) = n^n$
4. $f(n) = k^{k \log n}$
5. $f(n) = n^{\log \log n}$
6. $f(n) = k^{n^k}$
7. $f(n) = k^{n^n}$
8. $f(n) = n^{n^k}$
9. $f(n) = n^{n^n}$

Problem 5. For each pair of complexity classes below try to decide if we can prove a relation between them (by using theorems and propositions from the lecture). That is, for every pair of classes A and B decides whether $A \subseteq B$, $B \subseteq A$, $A \setminus B = \emptyset$ and $B \setminus A = \emptyset$. Note that some relations may not be known.

1. $\mathsf{DSPACE}(n)$ and $\mathsf{DSPACE}(n^3)$
2. $\mathsf{DSPACE}(n^3)$ and $\mathsf{DTIME}(2^{n^3})$
3. $\mathsf{DTIME}(2^{n^3})$ and $\mathsf{NSPACE}(n \log n)$
4. $\mathsf{NSPACE}(n \log n)$ and $\mathsf{NTIME}(n \log n)$
5. $\mathsf{NTIME}(n \log n)$ and $\mathsf{DSPACE}(n)$
6. $\mathsf{DSPACE}(n)$ and $\mathsf{DTIME}(2^{n^3})$
7. $\mathsf{DSPACE}(n^3)$ and $\mathsf{NSPACE}(n \log n)$
8. $\mathsf{DTIME}(2^{n^3})$ and $\mathsf{NTIME}(n \log n)$
9. $\mathsf{NSPACE}(n \log n)$ and $\mathsf{DSPACE}(n)$
10. $\mathsf{NTIME}(n \log n)$ and $\mathsf{DSPACE}(n^3)$

Problem 6. For each pair of complexity classes below try to decide if we can prove a relation between them (by using theorems and propositions from the lecture). That is, for every pair of classes A and B decides whether $A \subseteq B$, $B \subseteq A$, $A \setminus B = \emptyset$ and $B \setminus A = \emptyset$. Note that some relations may not be known.

1. $\mathsf{DSPACE}(n)$ and $\mathsf{DTIME}(2^{n^3})$
2. $\mathsf{DSPACE}(n^3)$ and $\mathsf{NSPACE}(n \log n)$
3. $\mathsf{DTIME}(2^{n^3})$ and $\mathsf{NTIME}(n \log n)$
4. $\mathsf{NSPACE}(n \log n)$ and $\mathsf{DSPACE}(n)$

Problem 7. Let A be the language of properly nested parentheses. For example, $(())$ is in A but $)($ is not. Let B be the language of properly nested parentheses and brackets. For example, $([()]()[])$ is in B but $([)]$ is not. Show that both A and B are in L.