The fifth homework consists of Problems 8 and 9.

Problem 1. Prove that the following Partition problem

Instance: Positive integers $a_1, \dots, a_k$.
Question: Can integers $a_1, \dots, a_k$ be split into two groups of the same sum?

is NP-complete if all integers are binary coded and polynomial if all integers are unary coded.

Problem 2. Consider the following variant of partition problem.

Instance: Positive integers $a_1, \dots, a_{3k}$.
Question: Can integers $a_1, \dots, a_{3k}$ be split into 3 groups so that the sum of integers in each group is the same (i.e. $\frac{1}{3}\sum_{i=1}^{3k} a_i$)?

Is this problem polynomial or NP-complete if all integers are binary coded? Is this problem polynomial or NP-complete if all integers are unary coded?

Problem 3. Decide whether the following problem is polynomial or NP-complete.
Instance: An undirected graph $G = (V, E)$.
Question: Does G contain a path throught all vertices?

Problem 4. Decide whether the following problem is polynomial or NP-complete.
Instance: A set of tasks U, processing time $d(u) \in \mathbb{N}$ associated with every task $u \in U$, number of processors m, deadline $D \in \mathbb{N}$
Question: Is it possible to assign all tasks to processors so that the (parallel) processing time is at most D?

Problem 5. Consider the following Bin packing problem.
Instance: Set of k items of rational sizes $a_1, \dots, a_k \in [0, 1]$.
Constrain: Splitting of items to pairwise disjoint bins $B_1, \dots, B_m$, which satisfy that the sum of sizes of items in every bin is at most 1.
Objective: Minimize the number of bins m.
Find approximation algorithm with best possible approximation error.

Problem 6. Decide whether the following problem is polynomial or NP-complete.
Instance CNF formula ϕ
Question Are there two different assignments u, v of truth values to variables in ϕ which satisfy ϕ, i.e. $\phi(u)$ and $\phi(v)$ both evaluate to true.

Problem 7. Prove that if there exists a polynomial algorithm deciding Satisfiability, then there also exists a polynomial time algorithm finding the assignment of logical variables so that the evaluation of a given formula is true (if such an assignment exists).

Problem 8. Prove that if there exists a polynomial algorithm deciding Hamiltonicity problem, then there also exists an algorithm which finds a Hamiltonian cycle in a given graph in polynomial time (if a Hamiltonian exists).

Problem 9. Consider the following weighted version of Sumset-Sum problem.
Instance: A set of k items A, size $s(a) \in \mathbb{N}$ and value $v(a) \in \mathbb{N}$ associated with each item $a \in A$ and size limit l.
Feasible solution: A set $A' \subseteq A$ satisfying $\sum_{a \in A'} s(a) \leq l$.
Objective: Maximize the sum of values of items in A', that is $\sum_{a \in A'} v(a)$.
Construct a pseudopolynomial algorithm that finds the optimal solution. Construct a fully polynomial approximation scheme.