

Implementation of algorithms and data structures

1. seminar

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Goals

- Learn how to implement advanced algorithms and data structures without tedious debugging
- Learn how to write and use various tests
- Learn to structure code and write API

Entrance expectations

- Knowledge of some programming language (e.g. C/C++, Python, Java, C#)
- Theoretical knowledge algorithms and data structures from bachelor study
- Experience in implementing basic algorithm (e.g. graph search)

Web

<https://ktiml.mff.cuni.cz/~fink/>

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Recodex

- Enroll into my group on Recodex
- Submit your fully working programs here

Gitlab

- Create a private fork of the repository
<https://gitlab.mff.cuni.cz/finkj1am/implementation>
- Give me (finkj1am) access (reporter)

Implement

- 3 testing assignments
- Red-black tree
- Goldberg algorithm for network flow problem

Bonus: Blossom algorithm for maximum matching in general unweighted graphs

Assignments

1 Red-black trees

Martin Mareš, Tomáš Valla: Průvodce labyrintem algoritmů, CZ.NIC, 2017
Robert Sedgwick: Left-leaning Red-Black Trees, doi:10.1.1.139.282

2 Network flows (Goldberg algorithm)

Martin Mareš, Tomáš Valla: Průvodce labyrintem algoritmů, CZ.NIC, 2017
Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein:
Introduction to algorithms, The MIT Press, 2001

http://mitp-content-server.mit.edu:18180/books/content/sectbyfn?collid=books_pres_0&id=8030&fn=Chapter%2026.pdf

3 Maximum matching in general unweighted graphs (Blossom algorithm)

Cunningham, Cook, Pulleyblank, Schrijver: Combinatorial optimization, John Wiley & Sons, 1997

- Unit testing
- Fuzzy testing
- Integration testing
- System testing
- Acceptance testing
- Installation testing
- Regression testing
- Continuous testing
- Destructive testing
- Software performance testing
- Security testing
- VCR testing
- Internationalization and localization
- ...

Black-box testing

- No knowledge of codes is used
- Test fulfilling the specification
- First write tests, code later

White-box testing

- Requires knowledge of codes
- Covers every part of code
- Reversed engineering: Searching for dangerous inputs for our code
- Verifies the internal structures

Testing reported bugs

For a reported bug, a test is created before fixing.

Development cycle

- 1 Write tests
- 2 Run tests and check that all fails
- 3 Implement the program
- 4 Use tests for debugging
- 5 Refactorization and clean up
- 6 Write documentation

Advantages

- Tests may contain bugs, we verify that tests fails as expected
- Writing unit tests verifies usability of interface

Unit testing

Motivation

Verify correctness of new features and preserving functionality after changes in codes.

Description

- Software testing method by which individual units of source code are tested to determine whether they are fit for use
- Tests should be independent

Advantages

- Unit tests can be run repeatedly
- Fast discovery of a bug when code is changed
- Example of usage of a library

Limitations and disadvantages

- Unit tests only proves that a program contains a bug
- Unit tests cannot prove that a program is correct (halting problem)
- Unit tests are not supposed to verify integration of modules
- Unit tests uses API without verifying correctness of internal data

Unit testing: Sum of two numbers

```
1 public class TestAdder {
2     @Test
3     public void testSumPositiveNumbersOneAndOne() {
4         Adder adder = new AdderImpl();
5         TEST(adder.add(1, 1) == 2);
6     }
7     @Test
8     public void testSumPositiveNumbersOneAndTwo() {
9         Adder adder = new AdderImpl();
10        TEST(adder.add(1, 2) == 3);
11    }
12    @Test
13    public void testSumPositiveNumbersTwoAndTwo() {
14        Adder adder = new AdderImpl();
15        TEST(adder.add(2, 2) == 4);
16    }
17    @Test
18    public void testSumZeroNeutral() {
19        Adder adder = new AdderImpl();
20        TEST(adder.add(0, 0) == 0);
21    }
22    ...
23 }
```

Unit testing: Graph

```
1 public abstract class AbstractGraphTest {
2     MutableGraph<Integer> graph;
3     @Before
4     public void init() {
5         graph = createGraph();
6     }
7     @After
8     public void validateGraphState() {
9         validateGraph(graph);
10    }
11    @Test
12    public void nodes_oneNode() {
13        addNode(N1);
14        TESTThat(graph.nodes().containsExactly(N1);
15    }
16    @Test
17    public void nodes_noNodes() {
18        TESTThat(graph.nodes().isEmpty());
19    }
20    @Test
21    public void adjacentNodes_oneEdge() {
22        putEdge(N1, N2);
23        TESTThat(graph.adjacentNodes(N1).containsExactly(N2);
24        TESTThat(graph.adjacentNodes(N2).containsExactly(N1);
25    }
26 }
```

Source: Wikipedia: Unit testing

<https://github.com/google/guava/blob/master/guava-tests/test/com/google/common/graph/AbstractGraphTest.java>

How create unit tests?

Initial questions about unit tests

- How should a unit be tested?
E.g. which library should be used to write tests?
- What should be tested?
E.g. what should be the content of unit tests?

Libraries for unit tests

- Python: unittest — Unit testing framework
- Julia: Unit testing (standard library)
- Java: JUnit 5
- C#: Unit test basics
- C/C++: Tens of libraries

Wikipedia: List of unit testing frameworks

What should be tested?

General hints

- Write test verifying that task is fulfilled
- Test boundary cases

Test sorting function

```
1 // First, small simple tests
2 TEST(sort([5,7,9,3]) == [3,5,7,9])
3 TEST(sort(['d','a','z']) == ['a','d','z'])
4 TEST(sort(["one","two","three"]) == ["one","three","two"])
5
6 // Tricky cases, some cases may depend on documentation
7 TEST(sort([]) == [])
8 TEST(sort([1,2,1,2,1]) == [1,1,1,2,2])
9 TEST(sort([1,False,5,True]) == [False, 1, True, 5])
```

Creating larger and random tests will be discussed later.

Examples of tests of binary search trees

- 1 Create a new empty tree and destroy it
- 2 Create a new empty tree, insert one element and destroy the tree
- 3 Insert more elements
- 4 Delete some elements
- 5 Delete all elements
- 6 Combine insertion and deletions
- 7 Check the counter of the number of elements
- 8 Find existing and non-existing elements
- 9 Insert existing and delete non-existing elements
- 10 Find and delete an element in an empty tree

Disadvantages of unit tests

- In non-trivial situations, all cases cannot be tested
- When a unit test fails, it does not say where a bug is
- Unit tests does not verify the correctness of stored data
- E.g. insertion of an element may be incorrect, but a bug may occur when it is deleted

What can be tested in stored data?

- All conditions given in a definition of a data structure
- Invariants
- Properties implied by proofs of correctness of an algorithm
- Values of variables which can be computed from other data
e.g. number of elements in a tree
- Values of all variables have expected values
e.g. range of integers, enumerators

Example: Doubly linked list

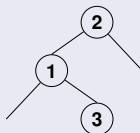
```
1 void list_test(list *l) {
2     if(!l->first) { // List is empty
3         TEST(!l->last);
4         return;
5     }
6     TEST(l->last);
7     TEST(!l->first->prev);
8     TEST(!l->last->next);
9     for(node *n = l->first; n; n = n->next) {
10         if(n != l->first)
11             TEST(n->prev && n->prev->next == n);
12         if(n != l->last)
13             TEST(n->next && n->next->prev == n);
14     }
15 }
```

Example: binary search tree

```
1 void tree_test(tree *tree) { node_test(tree->root); }
2
3 void node_test(node *node) {
4     if(!node)
5         return;
6
7     TEST(!node->left || node->left->parent == node);
8     TEST(!node->right || node->right->parent == node);
9
10    TEST(!node->left || node->left->key <= node->key);
11    TEST(!node->right || node->right->key >= node->key);
12
13    node_test(node->left);
14    node_test(node->right);
15 }
```

Question

Does this test guarantee that a binary search tree satisfying it is correct?



Testing order in a binary search tree

```
1 // Returns a pair of the minimal and the maximal key in the subtree
2 pair<int,int> order_test(node *node) {
3     int min = node->key, max = node->key, cmp;
4
5     if(node->left) {
6         min,cmp = order_test(node->left);
7         TEST(cmp <= node->key);
8     }
9
10    if(node->right) {
11        cmp,max = order_test(node->right);
12        TEST(node->key <= cmp);
13    }
14
15    return pair(min, max);
16 }
```

Example: Priority queue in an array

```
1 void heap_test(queue *q) {  
2     for(int i = 1; i < q->size; i++)  
3         // Parent is stored on position floor((i-1)/2)  
4         TEST(q->array[i].priority > q->array[(i-1)/2].priority);  
5 }
```

Hash table with separate chains

- A linked list in every bucket is correct
- Compute hash of every element to test whether it is stored in the proper bucket
- The ratio of the number of elements to buckets is within expected range

Graphs

- Incidence lists contain expected values
e.g. indices of vertices are within expected range or pointers give vertices inside the graph
- Every edge is a member of incidence lists of both end-vertices

AVL tree

Binary search tree where for every vertex, the difference between heights of the left and the right subtree is at most one.

Write tests verifying consistency of AVL tree

- There is a template on gitlab
- Implement one function testing consistency of AVL tree
- Some unit tests are given, but there are more of them on recodex